Fractional vortex dipoles of edge-screw type in self-focusing Kerr nonlinear media

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ABSTRACT

In this work we study the evolution of dark beams of finite length carrying edge-screw phase dislocations in self-focusing Kerr nonlinear media aiming to find appropriate conditions to control the process of filamentation of the background beam. In the case of a single fractional vortex dipole, geometry-controlled conditions for changing the intensity ratio of the peaks and their offset are found. Depending on their orientation, two parallel or two in-line mixed phase dislocations carried by a common background beam are predicted to perturb it and to initiate filamentation of different number of peaks with different spatial distributions.

Keyword list: self-focusing, filamentation, Kerr nonlinearity, phase dislocation, dark beam, fractional vortex dipole

1. INTRODUCTION

Propagation of optical beams in nonlinear media (NLM) has been a subject of continuing interest for more than four decades due to the possibility for creation of reconfigurable waveguides through the intensity-dependent refractive index change[1,2]. Such optically induced waveguides can guide weak signal beams and pulses[3,4], which motivates the investigation of novel techniques for manipulation of the transverse beam dynamics and opens possibilities for realization of waveguides with complex geometries. Besides their intriguing physical picture, particular interest in dark spatial solitons (DSSs) is motivated by their ability to induce gradient optical waveguides in bulk self-defocusing NLM[1,4-8]. The only known truly two-dimensional (2D) DSSs are the optical vortex solitons[5] whereas in one transverse spatial dimension the DSSs manifest themselves as dark stripes[9]. The odd initial condition required for generating a fundamental 1D DSS corresponds to a π-phase jump centered along the irradiance minimum of the stripe (e.g. to an step phase dislocation). The OVSs have a helical (screw-type) phase profile described by $exp(im\varphi)$ multiplier, where φ is the azimuthal coordinate and the integer number m is the so-called topological charge. 1D and 2D fundamental DSSs of these types have the common feature of zero transverse velocity with respect to the background beam if no perturbations are present. A variable number of quasi-2D dark spatial solitons of adjustable transverse velocities could be generated[10] by a proper choice of the initial phase profile (odd or even), the width of the crossed 1D dark beams, and the background-beam intensity.

In contrast, dark (or grey) waves are known in singular optics which slowly change their parameters, even when they are generated from perfectly odd initial conditions. Classical example are the ring dark solitary waves[11,12]. In their pioneering research Nye and Berry[13] conjectured that mixed edge–screw dislocations cannot exist. Nonetheless, an indication of their existence was found[14] for two interacting optical vortices of opposite topological charges. Moderate saturation of the medium third-order nonlinearity enabled to stabilize the snake instability of crossed 1D dark solitons and to identify 1D odd dark beams (ODBs) of finite length containing mixed-type (step-screw (SS) or edge-screw (ES)) phase dislocations[15,16]. Later on, such ODBs with SS phase dislocations were experimentally generated under controllable initial conditions by computer-generated holograms[17]. The data confirmed[17] that one can effectively control the steering dynamics of such beams by varying the magnitude and/or the length of the mixed step–screw phase jump. Although two different schemes for directional coupling of signal beams by steering fractional vortex dipole beams were proposed in Kerr media with negative nonlinearities[18], the first successful experiment was conducted only recently in biased photorefractive medium with a positive nonlinearity[19]. The key to understand this and to avoid any confusion is in the fact that the inherently restless ODB is coupled to a steering bright peak on the same background beam and this peak, being self-focused, induces the all-optical waveguide in the self-focusing NLM.

2. NUMERICAL PROCEDURE

The mixed edge-screw (ES) phase dislocation considered here consist of a one-dimensional phase step of limited length, which ends, by necessity, with pairs of phase semi-spirals on π with opposite helicities. This is why this dark beam can be called fractional vortex dipole. The phase profile of the ES dislocation can be described by

$$\Phi^{ES}(x,y)=\frac{\Delta\Phi}{2\pi}\left[\arctan\left(\frac{y}{x+b}\right)-\arctan\left(\frac{y}{x-b}\right)\right]. \tag{1}$$

In Eq. 1 $\Delta\Phi$ stands for the magnitude of the step portion of the dislocation, 2*b* for its length, and *x* and *y* denote the transverse Cartesian coordinates parallel and perpendicular to the dislocation. An increase in the ODB transverse velocity can be achieved[17] by decreasing $\Delta\Phi$, but here we refrained from exploiting this. The reason is that during the steering process even the initially "black" ODB inevitably becomes "grey", and an initially "grey" ODB causes weaker background beam modulation. All the data in this work refer to $\Delta\Phi=\pi$. Surface plot of the ES phase dislocation is shown in Fig. 1. The slowly-varying electric field amplitude of the ES ODBs (SVEA approximation) is assumed to be *tanh*-shaped and of the form

$$E^{ES}(x,y,z=0)=\sqrt{I_0}B(x,y)\tanh\left[r_{\alpha,\beta}(x,y)/a\right]\exp\left[i\Phi^{ES}(x,y)\right], \tag{3}$$

where

$$r_{\alpha,\beta}(x,y)=\left[\alpha(x+\beta b)^2+y^2\right]^{1/2} \tag{4}$$

is the effective radial coordinate and the parameters α and β are defined as follows:

$$\alpha=\begin{cases}0 & |x|\le b\\ 1 \text{ and } \beta=-1 & \text{for } x>b\\ 1 \text{ and } \beta=1 & \text{for } x\le -b\end{cases}. \tag{5}$$

In order to avoid any influence of the finite background beam of super-Gaussian form

$$B(x,y)=\exp\left\{-\left[(x^2+y^2)/w^2\right]^8\right\}, \tag{6}$$

its width *w* is chosen to exceed more that 10 times the dislocation half-length *b*.

The numerical simulations of the ODB propagation along the local Kerr NLM are carried out using the (2+1)-dimensional nonlinear Schrödinger equation (NLSE)

$$i\frac{\partial E}{\partial\left(z/L_{Diff}\right)}+\frac{1}{2}\Delta_T E+sign(n_2)\frac{L_{Diff}}{L_{NL}}|E|^2E=0, \tag{7}$$

which accounts for the evolution of the slowly-varying optical beam envelope amplitude under the combined action of nonlinearity and diffraction. Here Δ_T is the transverse part of the Laplace operator whereas $L_{Diff}=ka^2$ and $L_{NL}=1/(kn_2I)$ stand for the diffraction and nonlinear length of the dark beam and $sign(n_2)=\pm1$ for self-focusing and self-defocusing nonlinearity respectively. ($sign(n_2)=-1$ and $L_{Diff}=L_{NL}$ are necessary conditions for dark spatial soliton formation). In the above notations, *k* is the wave number inside the medium and *I* is the peak field intensity. The transverse spatial coordinates (*x* and *y*) are normalized to the ODB width *a*. The model NLSE we solved by means of the split-step Fourier method with a computational window spanning over 1024x1024 grid points. As a standard test we modeled the formation of a fundamental one-dimensional dark spatial soliton by setting the length of the one-dimensional part of the ES dislocation 2*b* several times longer than the computational window (Fig. 1, open circles and solid curve). Setting the background beam intensity 1.7 times higher than this needed to form the fundamental DSS ($I=I_{sol}^{1D}$), we got the expected DSS spatial shrinking (Fig. 1, dashed curve). Unless stated otherwise, the intensity in the following simulations is kept equal to that needed to form a fundamental 1D DSS of "infinite" length.

3. EVOLUTION IN SELF-DEFOCUSING KERR MEDIA

As shown in previous analyses[17,18] of ODBs with mixed SS dislocations, the background-beam intensity has a weak influence on the dark beam steering. Negative nonlinearity is important, however, for keeping the optically induced refractive index modulation (and dark beam profile) steep, which is crucial for all-optical guiding, deflection, and switch

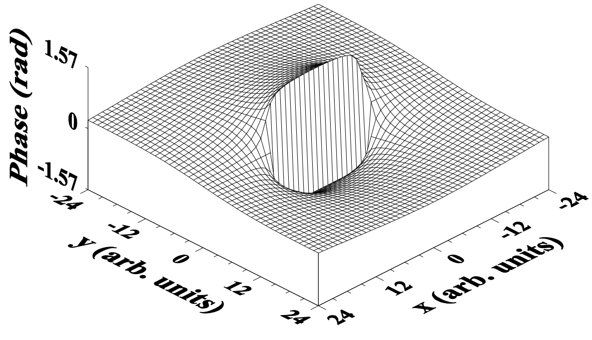

Fig. 1 Edge-screw mixed phase dislocation (fractional vortex dipole) described by Eq. 1.

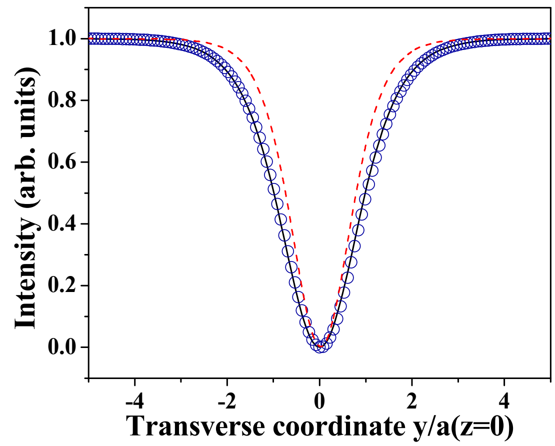

Fig. 2 Negative nonlinearity. Cross-section of the input 1D ODB (solid curve) and the fundamental 1D DSS at z/L_{Diff}=5 (open circles). Dashed curve – 1D DSS shrinking for 1.7 times higher background beam intensity.

ing of signal beams or pulses. Because the ODBs with mixed phase dislocations shorten and flatten along the nonlinear media (tending asymptotically to washout), the power density redistribution on the background creates peaks near the ODBs. The self-defocusing tends to suppress their growth and contributes, as the diffraction does, to their broadening. Under self-focusing conditions, however, the picture should be drastically different. Concerning the degrees of freedom to control the transverse velocity of fractional vortex dipoles with ES phase dislocations, refraining to vary the magnitude of the phase jump and fixing it to $\Delta\Phi = \pi$, the other known possibility is to vary the encoded dislocation length *2b* and the associated physical length-to-width ratio of the ODB (see Fig. 3). As seen in Fig. 4, the dependence of the ODB deflection vs. length-to-width ratio is linear and the shorter the ODB the larger the dark beam deflection. There are physical limitations for the horizontal scale of this graph. At initial ODB length-to-width ratio of the order of 4 or more, snake instability breaks up the dark beam into pairs of optical vortices (see Fig. 3, left inset). In contrary, when the initial dislocation length is approximately equal to the beam width, the interaction between phase semi-spirals dominates, the (fractional) topological charges annihilate and ring dark solitary wave is born (see Fig. 3, right inset).

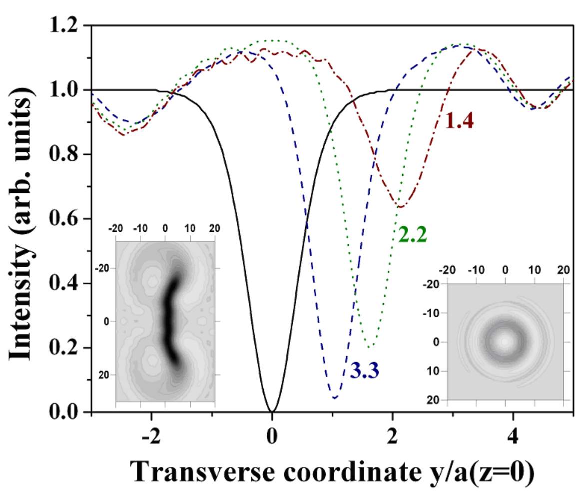

Fig. 3 Negative nonlinearity. Cross-section of the ODBs with ES phase dislocations at the entrance (solid curve) and at the exit of the NLM (z=2L_{NL}) for initial ODB length-to-width ratio 3.3 (dashed), 2.2 (dotted), and 1.4 (dash-dotted curve). Insets: snake instability (left; z/L_{NL}=2) and ring dark wave formation (right; z=L_{NL}) for ODB length-to-width ratio 8.7 and 1, respectively.

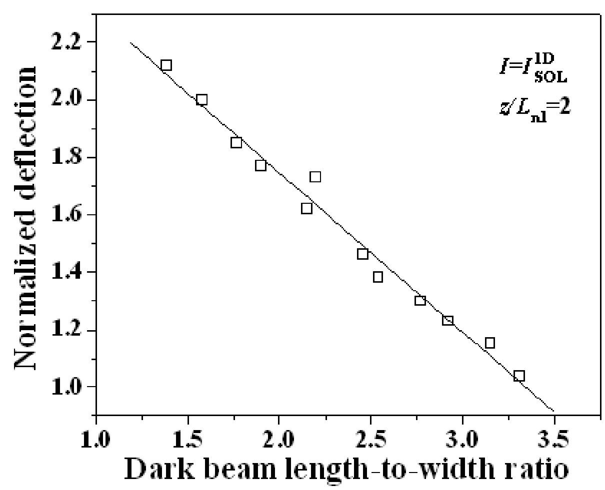

Fig. 4 Negative nonlinearity. Normalized ODB deflection Δy/a(z=0) vs. ODB length-to-width ratio. $I/I_{SOL}^{1D}=1$ and z/L_{NL}=2.

4. NONLINEAR DYNAMICS IN SELF-FOCUSING KERR MEDIA

In contrast to the evolution in self-defocusing NLM, the positive Kerr nonlinearity leads to (accelerated) dark beam broadening and to a self-focusing of the bright structures on the background[20] formed as a result of the energy density redistribution due to the ODB steering. This is clearly seen in Fig. 5, in which the initial (dashed curve) and the output ODB profiles (solid curve; after a nonlinear propagation path length $z=1L_{NL}$ in a self-focusing NLM) are compared in the case of an initial ODB length-to-width (LtW) ratio of 2.2. The comparison between this curve and the respective profile shown in Fig. 3 (the dotted curve) shows that the only parameter which remains the same as the sign of the nonlinearity becomes changed, is the transverse position of the ODB minimum. Under self-focusing conditions and at this particular distance of nonlinear propagation, the satellite bright peaks increased their peak intensity approximately 1.2 and 1.7 times, respectively. We refrained to continue the simulations to much higher distances in order to not violate the SVEA approximation under which the model NLSE (Eq. 6) is valid. In the rest of this paper we will denote the peak preceding the ODB in the steering direction as a *leading* peak, whereas the peak dominating on the other side of the ODB will be called *trailing* peak. When a single fractional vortex dipole is created on the background beam, at a fixed nonlinear propagation distance the positions of these two dominating peaks should change with changing the initial ODB length-to-width ratio. In Fig. 6 we show the estimated normalized leading peak offset Δ/a(z=0) from the dislocation position vs. ODB length-to-width ratio for $I/I_{SOL}^{1D}=1$. The saturation of this curve at relatively small dislocation lengths could be explained by the influence of the 2D self-focusing bright structures on the background beam as predicted[20] for the interaction of OVs for $n_2>0$ (see right inset in Fig. 5). Interestingly, the largest offsets of the leading peaks are to be expected for ODBs with length-to-width ratios ensuring also the highest leading-to-trailing peak intensity contrast (see Fig. 7) at this initial stage of self-focusing. The peak evolution up to this distance but at a higher intensity and at the same intensity but up to larger distance is, however, not trivial. For an ODB with an initial length-to-width ratio LtW=2.2 and $I/I_{SOL}^{1D}=1.7$, at $z/L_{NL}=1$ the peak intensities grow approximately 3.5 times but the peaks appear nearly equal in intensity. The model simulations yielded qualitatively the same result for $I/I_{SOL}^{1D}=1$ and $z/L_{NL}=1.5$ except that the peak intensities grow less - approximately 2.5 times. In both last cases the leading peak offset from the dislocation position remains the same (Δ/a(z=0)=3.9) whereas for an initial ODB with an initial length-to-width ratio LtW=2.2 Δ/a(z=0)=3.8. The comparative simulations showed that the shape of the background beam, Gaussian or super-Gaussian, does not have noticeable influence on the results presented in Figs. 6-8 if the ES dislocation is created initially in the center of the background.

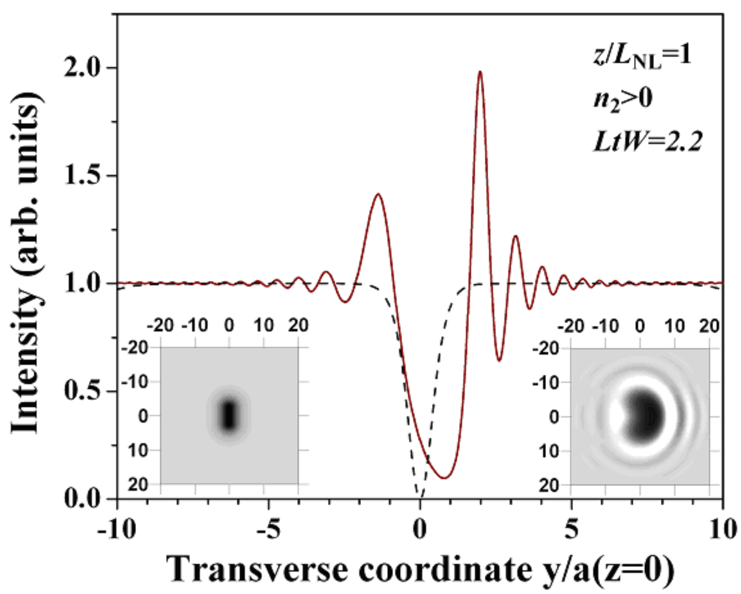

Fig. 5 Positive nonlinearity. Cross-section of the input ODB (dashed) and of the deflected beam (solid curve) for an initial length-to-width ratio of 2.2 at $z/L_{NL}=1$. Insets: Greyscale 2D images of the input (left) and output ODB intensity distributions (right). $I/I_{SOL}^{1D}=1$.

In Fig. 8 we show data obtained for a further evolution stage ($z/L_{NL}=2$) of the self-focusing peaks around the fractional vortex dipole, for which the SVEA approximation should still hold. The result predicts that by using a single fractional vortex dipole beam it should be possible to vary the ratio between the intensities of the trailing and leading peak by adjusting the initial ODB length-to-width ratio.

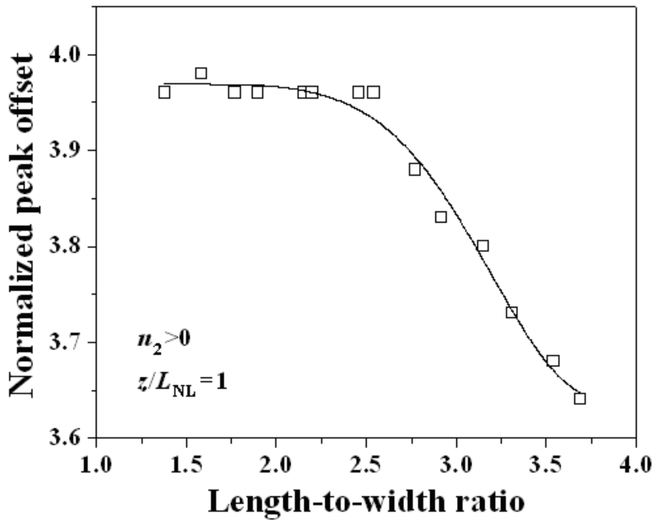

Fig. 6 Positive nonlinearity. Normalized leading peak offset Δ/a(z=0) from the dislocation position vs. ODB length-to-width ratio. $I/I_{SOL}^{1D}=1$.

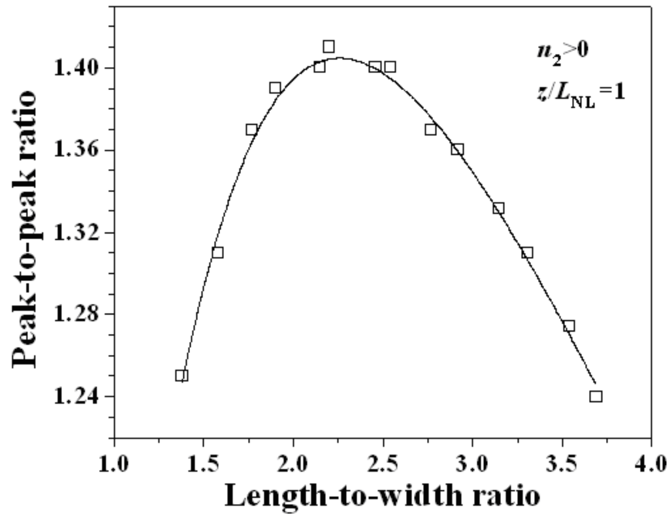

Fig. 7 Positive nonlinearity. Leading-to-trailing peak intensity ratio vs. ODB length-to-width ratio. $I/I_{SOL}^{1D}=1$.

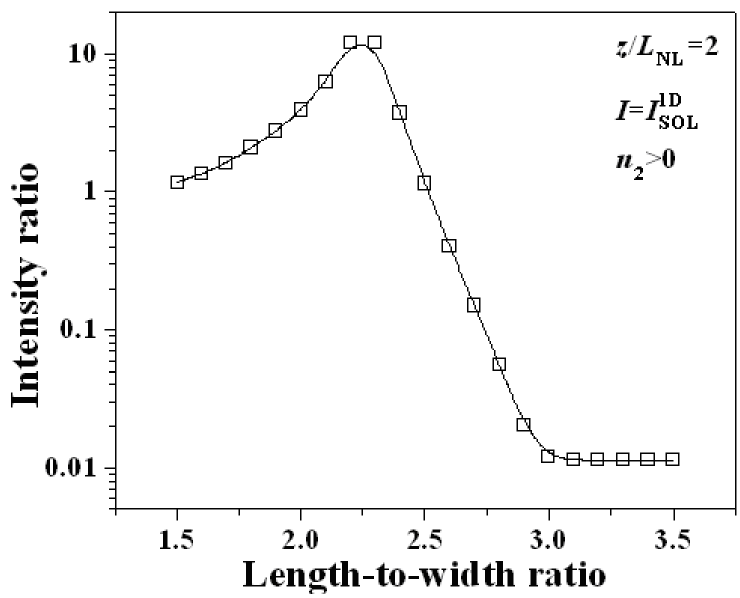

Fig. 8 Positive nonlinearity. Trailing-to-leading peak intensity ratio vs. ODB length-to-width ratio for z/L_{NL}=2 and $I/I_{SOL}^{1D}=1$.

As a next step in this analysis we concentrated on the background beam evolution in self-focusing NLM when two fractional vortex dipoles with equal or opposite phase profiles are created symmetrically with respect to its center. All further illustrations presented in this work refer to z/L_{NL}=1.75 and $I/I_{SOL}^{1D}=1$. The (0 - 2π) range in the phase profiles denoted with (b) and (d) on Figs. 9-12 are encoded in gray scale. For the sake of better visibility the respective intensity profiles denoted with (c) and (e) are shown in inverted grey scale (black corresponding to maximal peak intensity) and

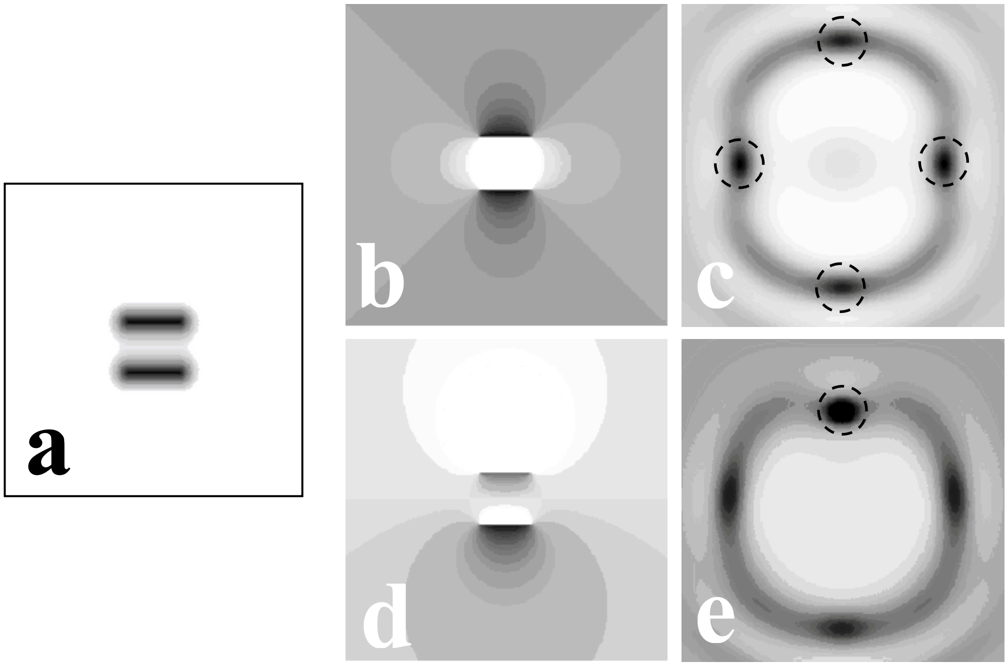

Fig. 9 Positive nonlinearity. Initial amplitude (a) and phase (b) distributions of a pair of parallel and opposite ES phase dislocations and output field intensity distribution (c). Output intensity distribution in the case of equal phase dislocations (d) is shown in frame (e). ODB length-to-width ratio 2.2 corresponding to encoded dislocation length $2b$=32 pix. Dislocation offset $\delta=2b$.

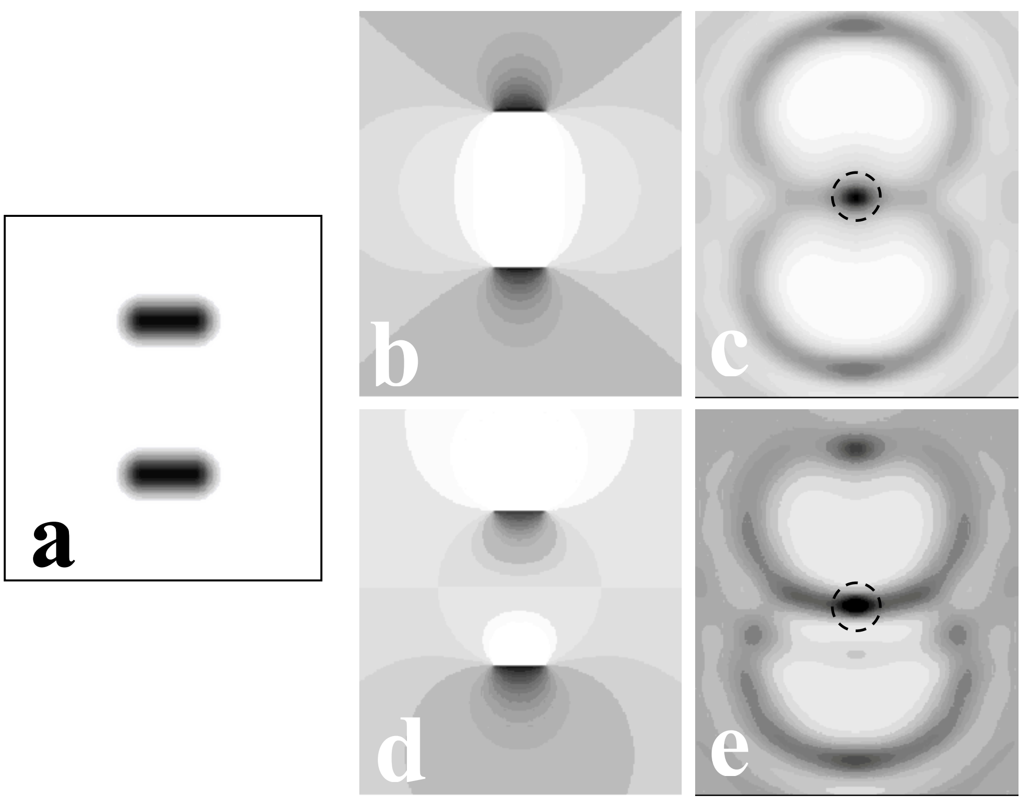

Fig. 10 The same as in Fig. 9, but for three times larger dislocation offset ($\delta=6b$).

the dominating peak(s) are surrounded in dashed circles. The dislocation mutual offset is chosen to be just large enough so that the background between the dark beams reaches its undisturbed level.

In the case of opposite phase dislocations (Fig. 9b) which, in principle, force the ODBs to start moving in opposite directions, the model predicts that four peaks with nearly equal intensities become self-focused (see Fig. 9c). When the two dislocations are equal causing ODB steering also in the same direction (downwards on Fig. 9d), one peak on the trailing side of both ODBs strongly dominates the other three formed on the background. In the qualitatively similar situation, but at a three times larger initial dislocation offset (δ=6b), independent on the direction of the ODB steering, the dominating peak is located between the fractional vortex dipoles. The relative intensities of the other two peaks located on a line perpendicular to the 1D portions of the ES dislocations, however, does depend on the mutual orientation of the dislocations.

Another interaction scheme can involve two in-line ES dislocations. As observed in self-focusing photorefractive medium[20] for a single SS dislocation, when the pair of fractional vortex dipoles are well separated (δ=4b) and steer either in opposite (Fig.11c) or in the same direction (Fig.11e), the dominating peaks are located behind the respective ODB. New features are the two peaks starting to grow between the ODBs, which can be explained by the energy-density redistribution within the surrounding background. When the ODBs steer in opposite directions (Fig. 11c) these two peaks are equally well pronounced, in the other case the leading peak of the pair is equally pronounced as compared to the peaks leading each individual dislocation. At much larger offset these satellite peaks between the ODBs will probably be negligible if present at all.

If ODBs of the same initial length-to-width ratio are initially more closely spaced (δ=2b; see Fig. 12) four or five peaks with different spatial locations dominate on the background.

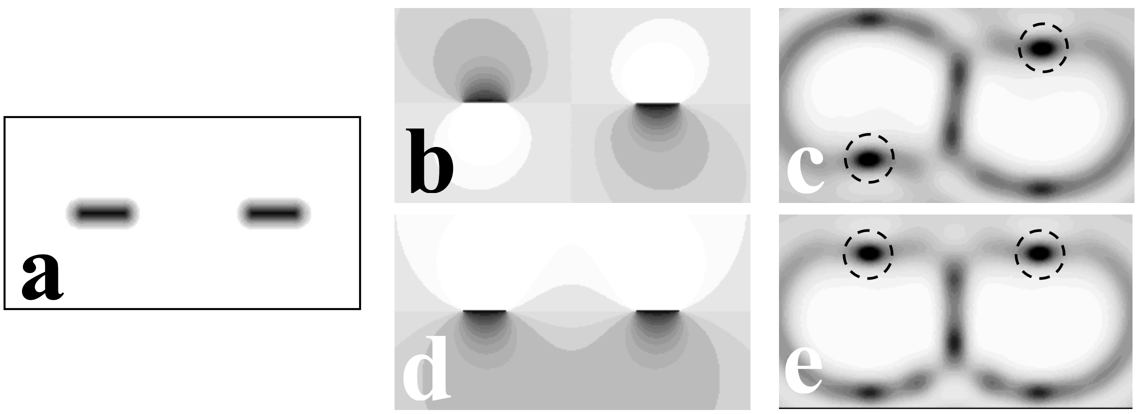

Fig. 11 Positive nonlinearity. Initial amplitude (a) and phases (b,d) of a pair of in-line ODBs with opposite (b) and equal ES phase dislocations (d) and the respective intensity profiles (d) and (e). ODB length-to-width ratio *LtW*=2.2 corresponding to encoded dislocation length 2b=32 pix. Center-to-center dislocation offset δ=4b.

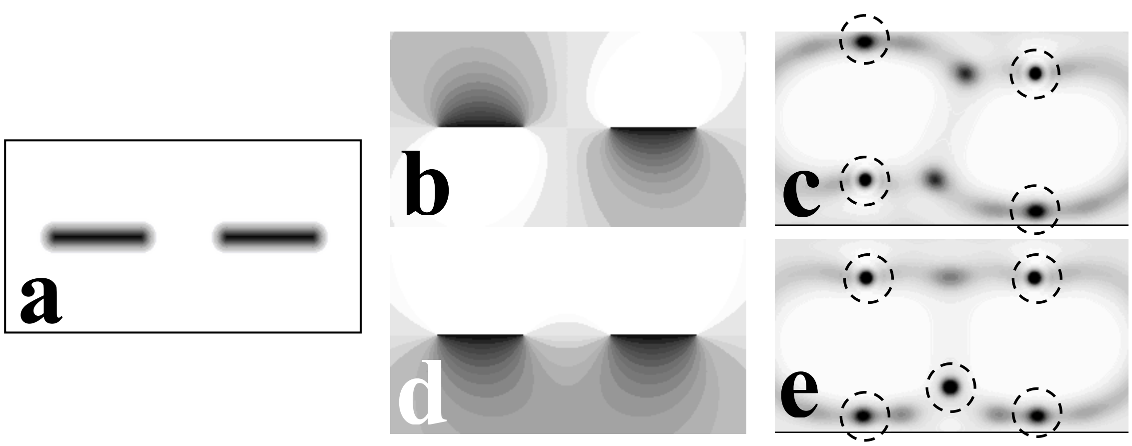

Fig. 12 The same as in Fig. 11 but for twice longer ES phase dislocations 2b=64 pix. Center-to-center offset δ=2b.

5. CONCLUSION

The presence and evolution of odd dark beams of finite length carrying edge-screw phase dislocations (fractional vortex dipoles) are noticeably perturbing the carrying background beam and, in a self-focusing Kerr nonlinear media, can initiate filamentation of the background beam. In the case of a single fractional vortex dipole, geometry-controlled conditions for changing the intensity ratio of the peaks and their offset are found. Depending on their orientation, two parallel or two in-line mixed phase dislocations carried by a common background beam are predicted to perturb it and to initiate filamentation of different number of peaks with different spatial distributions. In view of the above, the control of the positions and distributions of the self-focusing filaments on the background beam seems feasible.

6. ACKNOWLEDGMENTS

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