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Symbiotic Light Pairs Sustained by Self-Phase Modulation and Cross-Phase Modulation

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Abstract

The conditions for propagation of a light-pulse-pair with nearly constant temporal and spatio-temporal parameters in nonlinear media are found in a broad range from quasi-cw to subpicosecond pulses. Because of the crucial dependence of the beam/pulse parameters from those of the complementary beam/pulse, the regime exhibits a symbiotic character. A set of balance solitary conditions are found between the signs and the values of the nonlinear susceptibilities for self-phase modulation, cross-phase modulation and the pump powers. The temporal and spatio-temporal modulation stability of the symbiotic light pair are discussed.

1. Introduction

When two high-power pulses/beams copropagate in a nonlinear medium, the physical picture becomes complicated because of the self-phase modulation (SPM) and the possible cross-phase-modulational (XPM) coupling [1–5]. In the temporal domain, for example, the XPM can reverse the well-known situation and a bright solitary wave can propagate undistorted in the normal dispersion regime, if it is coupled to a dark pulse in an anomalous dispersion regime [6].

Generally, the nonlinear pulse/beam propagation in the slowly varying envelope approximation (SVEA) [7] is described by the Schrödinger equation (SE) with three transverse dimensions (two spatial and time). Recently [7] the possible existence of stable three dimensional optical formation (light bullet) is predicted and analyzed.

The physical idea for symbiotic solitary-wave pairs formation in optical fibers [8] is based on the interference of the SPM and XPM. The collimation and guiding of symbiotic light-beam-pair in a bulk nonlinear medium, resulting in a diffractionless beam propagation, seems also obtainable [9].

In this paper we derive conditions for propagation of a light-pulse/beam -pair with nearly constant temporal and spatio-temporal parameters in nonlinear media in a broad range from quasi-cw to subpicosecond pulses. We show that a proper balance can be found between the signs and the values of the nonlinear susceptibilities for SPM and XPM and the pump powers, minimizing the spectral/spatial broadening of the pulses involved. An improved temporal modulational stability of the symbiotic pair is predicted under certain conditions. The calculated balance peak powers are within the range covered by high repetition-rate Q-switched and mode-locked lasers.

2. Theoretical analysis

Our analysis is based on the SE, derived under the SVEA approximation. It should be noticed however, that the SE may fail in describing adequately the space–time nature of the self-focusing of femtosecond pulses [10]. In order to represent the results in an analytical form, the starting equations are solved using the variational approach [8], reducing them to a system of ordinary differential equations for the respective parameters. The detailed numerical analysis has shown [11] that for energies near the self-focusing critical value and for long propagation distances, the variational approach gives more adequate results than the aberrationless paraxial approximation.

2.1. *Conditions for symbiotic light pulse pair formation*

Let us consider two pulses of wavelengths λ_1 and λ_2, copropagating in a nonlinear medium. The coupled amplitude equations, which describe the pulse evolution, have the form [6]

$$i\left(\frac{\partial}{\partial x} + V_{gi}^{-1}\frac{\partial}{\partial t}\right)\psi_i + \alpha_i\frac{\partial^2\psi_i}{\partial t^2} + k^{\mathrm{SPM}}(\lambda_i)|\psi_i|^2\psi_i + k^{\mathrm{XPM}}(\lambda_i)|\psi_j|^2\psi_i = 0 \quad i, j = 1, 2;\ i \neq j, \tag{1}$$

where $V_{gi} = (\mathrm{d}\omega/\mathrm{d}k_i)_{\omega_i}$ ($i = 1, 2$) is the group velocity of the pulse, $\alpha_i = (1/2)[\lambda^3/(2\pi c^2)][\partial^2 n/\partial\lambda^2]_{\lambda=\lambda_i}$ is the group velocity dispersion (GVD) coefficient and $k^{\mathrm{SPM}}(\lambda_i)$ and $k^{\mathrm{XPM}}(\lambda_i)$ are the nonlinear coefficients for SPM and XPM,

$$k^{\mathrm{SPM}}(\lambda_i) = -\frac{n_2^{\mathrm{SPM}}(\lambda_i)k_i}{2n_{0i}} = -\frac{k_i}{2n_{0i}} \times \frac{3\pi N\chi^{(3)}_{\mathrm{SPM}}(\lambda_i)}{n_{0i}}$$

$$k^{\mathrm{XPM}}(\lambda_i) = -\frac{n_2^{\mathrm{XPM}}(\lambda_i)k_i}{2n_{0i}} = -\frac{k_i}{2n_{0i}} \times \frac{6\pi N\chi^{(3)}_{\mathrm{XPM}}(\lambda_i)}{n_{0i}}.$$

N is the particle density and $\chi^{(3)}$ denotes the corresponding nonlinear susceptibility. For simplicity we assume that $k^{\mathrm{XPM}}(\lambda_1) = k^{\mathrm{XPM}}(\lambda_2) = k^{\mathrm{XPM}}$.

It can be shown, that eqs (1) are Euler–Lagrange equations for the Lagrangian

$$L = L_1 + L_2 + L_{\mathrm{XPM}}, \tag{2a}$$

where

$$L_j = (i/2)\left(1 + \frac{1}{V_{gj}}\right)\left[\psi_j^*\frac{\partial\psi_j}{\partial x} - \psi_j\frac{\partial\psi_j^*}{\partial x}\right] - \alpha_j\left|\frac{\partial\psi_j}{\partial t}\right|^2 + \frac{k^{\mathrm{SPM}}(\lambda_j)}{2}|\psi_j|^4, \quad (j = 1, 2), \tag{2b}$$

and

$$L_{\mathrm{XPM}} = k^{\mathrm{XPM}}|\psi_1|^2|\psi_2|^2. \tag{2c}$$

We assume, that the pulses have a Gaussian form:

$$\psi_1(x, t') = A_1(x) \exp\left\{-\frac{(t' - \tau_D)^2}{2\tau_1^2(x)} + ib_1(x)t'^2\right\}, \quad (3a)$$

$$\psi_2(x, t'') = A_2(x) \exp\left\{-\frac{t''^2}{2\tau_2^2(x)} + ib_2(x)t''^2\right\}, \quad (3b)$$

where $A_i(x)$ denotes the complex, slowly varying amplitude of the i-th pulse, $\tau_i(x)$ is the corresponding pulse duration on 1/e-level' $b_i(x)$ is the generated chirp rate, t' and t'' are the coordinate frames moving at the group velocity of each pulse and τ_D is the eventual initial delay between them. The initial conditions for the variational parameters are $b_{1,2}(x = 0) = 0$ and $\tau_{1,2}(x = 0) = \tau_{1,2}^0$. Following the variational procedure [8], we get the following system of differential equations:

$$\frac{d\tau_i}{dx} = 4\alpha_i b_i \tau_i, \quad i, j = 1, 2;\ i \neq j, \quad (4a)$$

$$\frac{db_i}{dx} = -4\alpha_i b_i^2 + \frac{\alpha_i}{\tau_i^4} - \frac{k^{SPM}(\lambda_i)|A_i^2|}{2\sqrt{2}\tau_i^2} - \frac{k^{XPM}|A_j|^2\tau_j}{(\tau_1^2 + \tau_2^2)^{3/2}} \times \left\{1 - \frac{2(\tau_D + xv_{12})^2}{\tau_1^2 + \tau_2^2}\right\} \exp\left\{-\frac{(\tau_D - xv_{12})^2}{\tau_1^2 + \tau_2^2}\right\}, \quad (4b)$$

where $v_{12} = (1/V_{g1} - 1V_{g2})$ is related to the walk-off length of the pulses $L_w = (\tau_1^2 + \tau_2^2)^{1/2}/|v_{12}|$. The evolution of the pulse amplitudes is governed by the energy conservation of the form $|A_i(x)|^2\tau_i(x) = \text{const}$.

The balance condition for symbioticity in the propagation of the pump pulses can be derived from eqs (4) by setting $db_i/dx = 0$, $i = 1, 2$. With respect to the pump pulse durations, two characteristic regimes can be separated.

If the pulses are cw or quasi-cw (down to the nanosecond region), one can neglect the group velocity mismatch v_{12} and the GVD and, at $\tau_D \ll \tau_{1,2}$ and equal beam radii, the balance condition has the form

$$P_1/P_2 = -\frac{k^{XPM}}{k^{SPM}(\lambda_1)} \times \frac{2\sqrt{2}\tau_1^2\tau_2}{(\tau_1^2 + \tau_2^2)^{3/2}} = -\frac{k^{SPM}(\lambda_2)}{k^{XPM}} \times \frac{(\tau_1^2 + \tau_2^2)^{3/2}}{2\sqrt{2}\tau_2^2\tau_1}. \quad (5)$$

It is evident, that the mode of propagation analyzed is possible only if $k^{SPM}(\lambda_1)$ and $k^{SPM}(\lambda_2)$ have the same sign, the opposite one to those of k^{XPM} {i.e. sgn $[k^{SPM}(\lambda_1)]$ = sgn $[k^{SPM}(\lambda_2)] = -$sgn (k^{XPM})}. The fact, that the balance condition has, the form of a ratio does not mean that the critical power/intensity for medium breakdown can be exceeded. From another point of view, this sign-combination is indicative for compensation of the SPM by the XPM, which can result in an increase in the critical power for (self)induced focusing [9]. Small initial delay between the pulses ($\tau_D < \tau_{1,2}$) will modify the balance conditions, but is unwanted in view of the pulse-overlapping reduction.

If the pulses are in the picosecond range, the balance condition for propagation of pulses with constant temporal and spectral parameters [$b_{1,2}(x) = 0$ and $\tau_{1,2}(x) = \tau_{1,2}^0$] has the form [see eqs (4)]

$$\frac{\alpha_i}{\tau_i^4} = \frac{k^{SPM}(\lambda_i)|A_i^2|}{2\sqrt{2}\tau_i^2} + \frac{k^{XPM}|A_j|^2\tau_j}{(\tau_1^2 + \tau_2^2)^{3/2}} D, \quad i, j = 1, 2;\ i \neq j, \quad (6a)$$

where

$$D = \left\{1 - \frac{2(\tau_D + xv_{12})^2}{\tau_1^2 + \tau_2^2}\right\} \exp\left\{-\frac{(\tau_D + xv_{12})^2}{\tau_1^2 + \tau_2^2}\right\} \quad (6b)$$

accounts for the relative temporal pulse position. Considering single pulse propagation in the nonlinear medium, from eqs (6) one can obtain an expression for the critical power for fundamental "Gaussian" soliton formation [2]. Solutions of eqs (6) are valid if

$$k^{SPM}(\lambda_1)k^{SPM}(\lambda_2) \neq (k^{SPM})^2 \frac{\tau_1\tau_2}{(\tau_1^2 + \tau_2^2)^3} D^2. \quad (7)$$

For synchronous pulses ($\tau_D = 0$, $v_{12} \approx 0$) of equal duration ($\tau_1 = \tau_2 = \tau$), in terms of pump powers, the solution of eqs (6) has the form

$$P_i = \frac{c}{2\sqrt{2} \times 10^7} \left\{\frac{\alpha_i k^{SPM}(\lambda_j) - \alpha_j k^{XPM}}{k^{SPM}(\lambda_1)k^{SPM}(\lambda_2) - (k^{XPM})^2}\right\} \frac{a^2}{\tau^2}, \quad i, j = 1, 2;\ i \neq j. \quad (8)$$

In eqs (8) the powers are given in Watts, the nonlinear susceptibilities in esu, the beam radii a and the pulse durations τ – in centimeters and seconds, respectively, and c is the speed of light. At high powers the SPM and XPM have to counteract between itself {i.e. sgn $[k^{SPM}(\lambda_{1,2})] = -$sgn (k^{XPM}) and sgn $[k^{SPM}(\lambda_1)]$ = sgn $[k^{SPM}(\lambda_2)]$}.

2.2. *Conditions for symbiotic light beam/pulse pair formation*

In a planar nonlinear waveguide the system of coupled time-dependent paraxial amplitude equations describing the spatio-temporal evolution of two laser beams/pulses has the form [7, 12]

$$i\frac{\partial\psi_i}{\partial x} + \left[\alpha_i \frac{\partial^2}{\partial t^2} + \beta_i \frac{\partial^2}{\partial r^2}\right]\psi_i + k^{SPM}(\lambda_i)|\psi_i|^2\psi_i + k^{XPM}(\lambda_j)|\psi_j|^2\psi_i = 0, \quad i, j = 1, 2;\ i \neq j, \quad (9)$$

where r denotes the diffraction nonlimited transverse coordinate. A single comoving coordinate system and equal pulse group velocities are considered. The restrictions posed by this assumption will be discussed later. In eq. (9) $\beta_i = -1/(2k_i)$, where k_i is the wave-number for the i-th wave. The modified Lagrangian could be obtained from eq. (2b) by adding a diffraction-dependent term $(-\beta_i|\partial\psi_i/\partial r|^2$, $i = 1, 2)$. The input waves are assumed to be Gaussian in both time and space

$$\psi_1(r, x, t) = \frac{A_1(x)}{\omega_1(x)} \exp\left\{-\frac{(t - \tau_D)^2}{2\tau_1^2(x)} + ib_1(x)t^2\right\} \times \exp\left\{-\frac{r^2}{a_1^2\omega_1^2(x)} - i\frac{k_1\rho_1(x)r^2}{2}\right\} \quad (10a)$$

$$\psi_2(r, x, t) = \frac{A_2(x)}{\omega_2(x)} \exp\left\{-\frac{t^2}{2\tau_2^2(x)} + ib_2(x)t^2\right\} \times \exp\left\{-\frac{r^2}{a_2^2\omega_2^2(x)} - i\frac{k_2\rho_2(x)r^2}{2}\right\}, \quad (10b)$$

where ω_i are the normalized radii of the beams [$\omega_i(x = 0) = 1$], a_i are their initial physical radii on 1/e-level, and ρ_i are functions of the inverse radii of curvature of the respective wavefronts [$\rho_i(x = 0) = 0$ in a plane-wave approximation]. Under the requirement that the pulse/beam pairs sustain themselves in the presence of SPM and XPM [$\omega_{1,2}(x) = 1$ and $\tau_{1,2}(x) = \tau$], the variational results obtained can be simplified yielding the following system of equations:

$$k^{\mathrm{SPM}}(\lambda_1)|A_1|^2 + k^{\mathrm{XPM}}|A_2|^2 = -4/(k_1 a^2), \tag{11a}$$

$$k^{\mathrm{SPM}}(\lambda_2)|A_2|^2 + k^{\mathrm{XPM}}|A_1|^2 = -4/(k_2 a^2), \tag{11b}$$

$$k^{\mathrm{SPM}}(\lambda_1)|A_1|^2 + k^{\mathrm{XPM}}|A_2|^2 = 4\alpha_1/\tau^2, \tag{11c}$$

$$k^{\mathrm{SPM}}(\lambda_2)|A_2|^2 + k^{\mathrm{XPM}}|A_1|^2 = 4\alpha_2/\tau^2. \tag{11d}$$

For simplicity of presentation $a_1 = a_2 = a$ is assumed, and the unwanted initial temporal delay τ_D is set to zero.

This result requires some preliminary comments. The SPM and XPM should compensate for both diffraction and GVD. Therefore, this regime of propagation could be achieved in anomalous GVD only [i.e. sgn $(\alpha_{1,2}) = -1$]. Second, eqs (11) form an overdetermined system of equations. Hence, the following condition is to be fulfilled:

$$k_i a^2 = \tau^2/|\alpha_i|, \quad i = 1, 2. \tag{12}$$

Physically, the diffraction length $L_{D_i} = k_i a^2/2$ and the dispersion length $L'_{D_i} = \tau^2/2|\alpha_i|$ for each wave should be equal. This result could motivate the symmetric renormalization of the SE [eq. (9)] used in [7, 12]. The condition, given by eq. (12) requires, that $|\alpha|$ increases with increasing the wavelength. For example, this is a real situation in the optical materials in the region of anomalous GVD [at $\lambda > 1.55\,\mu$m for dispersion-shifted materials, where $\alpha = -0.0375\lambda^2(\lambda - 1.550)/(2\pi c)$]. Solutions of eqs (11) are valid if

$$k^{\mathrm{SPM}}(\lambda_1)k^{\mathrm{SPM}}(\lambda_2) \neq (k^{\mathrm{XPM}})^2, \tag{13}$$

and we obtain that the balance powers are $\sqrt{2}$ times higher than those given by eq. (8). The possible sign combination is analogous to that discussed in Section 2.1.

3. Results and discussion

In this section we will show the possibility for symbiotic laser-pulse/beam-pair formation and will analyze it's modulational stability.

As seen from the previous section, the regime proposed depends crucially on the values and the signs of the respective nonlinear susceptibilities. The condition sgn $[k^{\mathrm{SPM}}(\lambda_1)] = \mathrm{sgn}\,[k^{\mathrm{SPM}}(\lambda_2)] = -\mathrm{sgn}\,(k^{\mathrm{XPM}})$ can be fulfilled in a resonant nonlinear medium (e.g. inert gas), where the wavelengths λ_1 and λ_2 are far from single-photon resonances and the allocation of $2\lambda_1$, $2\lambda_2$ and $\lambda_1 + \lambda_2$ with respect to the two-photon resonances determines the values and signs of the nonlinear susceptibilities [9]. In a Kerr nonlinear medium (e.g. CS_2) the signs of the nonlinear coefficients for SPM and XPM are polarization dependent [13]. However, the response time of the medium is limited to several picoseconds.

In view of the spatio-temporal analogy, the conditions for symbiotic light-pulse pair formation are similar to those obtained for the collimation and guiding of a symbiotic light-beam pair [9]. It should be noticed, that the results presented in Section 2.2. could not be obtained treating the problem separately in time and space.

As a first step, we will discuss briefly the formation of a symbiotic pulse-pair. Let us assume that the pump pulses are in the nanosecond range [Case (a)]. Figure 1 shows P_1/P_2 *vs.* pump-pulse-duration ratio τ_1/τ_2 for different

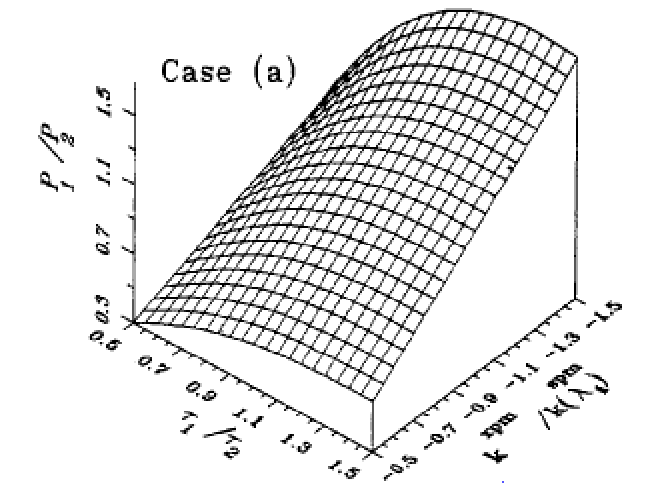

Fig. 1. Balance pump power ratio P_1/P_2 as a function of the pump pulse duration ratio τ_1/τ_2 for different values of $k^{\mathrm{XPM}}/k^{\mathrm{SPM}}(\lambda_1)$. For simplicity $k^{\mathrm{SPM}}(\lambda_1) = k^{\mathrm{SPM}}(\lambda_2)$ is assumed

values of $k^{\mathrm{XPM}}/k^{\mathrm{SPM}}(\lambda_1)$ [see eq. (5)]. For simplicity $k^{\mathrm{SPM}}(\lambda_1) = k^{\mathrm{SPM}}(\lambda_2)$ is assumed. As seen from the figure, P_1/P_2 increases with increasing $k^{\mathrm{XPM}}/k^{\mathrm{SPM}}(\lambda_1)$, while it varies relatively weakly with changing τ_1/τ_2 with a maximum at $\tau_1/\tau_2 \approx \sqrt{2}$. Choosing Xe as a nonlinear medium, a possible pump-wavelength pair is $\lambda_1 = 252.43$ nm and $\lambda_2 = 258.8$ nm. The corresponding nonlinear susceptibilities are calculated to be

$$\chi^{(3)}_{\mathrm{SPM}}(\lambda_1) = 5.1 \times 10^{-33}\,\mathrm{esu}; \quad \chi^{(3)}_{\mathrm{SPM}}(\lambda_2) = 5.5 \times 10^{-34}\,\mathrm{esu}$$

and

$$\chi^{(3)}_{\mathrm{XPM}} = -1.1 \times 10^{-33}\,\mathrm{esu}.$$

If the pulses are in the picosecond time scale one can choose $\lambda_1 = 252.44$ nm and $\lambda_2 = 258.77$ nm (Case (b)) yielding

$$\chi^{(3)}_{\mathrm{SPM}}(\lambda_1) = 2.4 \times 10^{-33}\,\mathrm{esu}; \quad \chi^{(3)}_{\mathrm{SPM}}(\lambda_2) = 1.7 \times 10^{-32}\,\mathrm{esu}$$

and

$$\chi^{(3)}_{\mathrm{XPM}} = -1.1 \times 10^{-33}\,\mathrm{esu},$$

$$\alpha_1 = -9.5 \times 10^{-29}\,\mathrm{s^2/cm}; \quad \alpha_2 = -1.0 \times 10^{-28}\,\mathrm{s^2/cm}.$$

The pump powers, required for symbiotic light pulse-pair formation [see eqs (8)] are

$$P^s_1 = 5.8 \times 10^{-18}(a^2/\tau^2); \quad P^s_2 = 1.5 \times 10^{-18}(a^2/\tau^2),$$

where a is the beam radius in centimeters and τ is the pulse duration in seconds. Choosing $a = 0.1$ cm and $\tau = 10^{-12}$ s, the corresponding pump powers are $P^s_1 = 58$ KW and $P^s_2 = 15$ KW.

The temporal synchronization of the pump pulses is important since the possible initial delay τ_D between the pulses will reduce the pulse overlapping resulting in a (self)induced spectral broadening. Figure 2 plots the spectral broadening of the pump pulses $\Delta\omega$ *vs.* τ_D/τ_1 at $\lambda_1 = 252.44$ nm (dashed line) and $\lambda_2 = 258.77$ nm (solid line), normalized to the spectral width of the pulse in the symbiotic regime $\Delta\omega^s$, at the exit of a 10 cm long xenon-filled gas cell. The estimation shows that, by equal other conditions, the efficiency of the four-wave frequency mixing processes increases with decreasing the rms of the pump laser linewidths δ as $1/\delta^2$. Because of the relatively low efficiency of

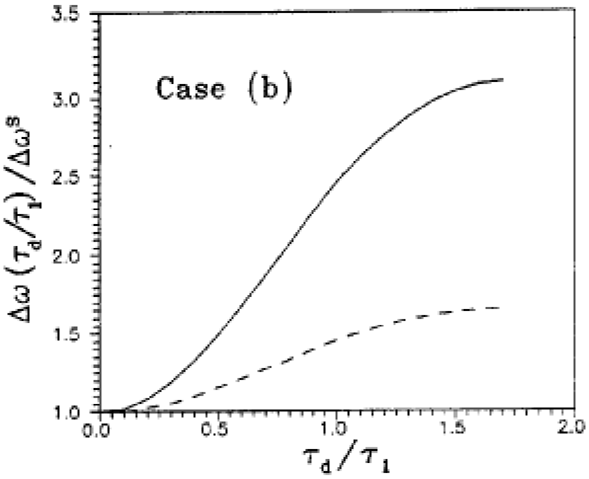

Fig. 2. Spectral broadening of the pump pulse $\Delta\omega(\tau_D/\tau_1)$ normalized to the spectral broadening in the symbiotic case $\Delta\omega^s$ *vs.* initial delay τ_D between the pulses

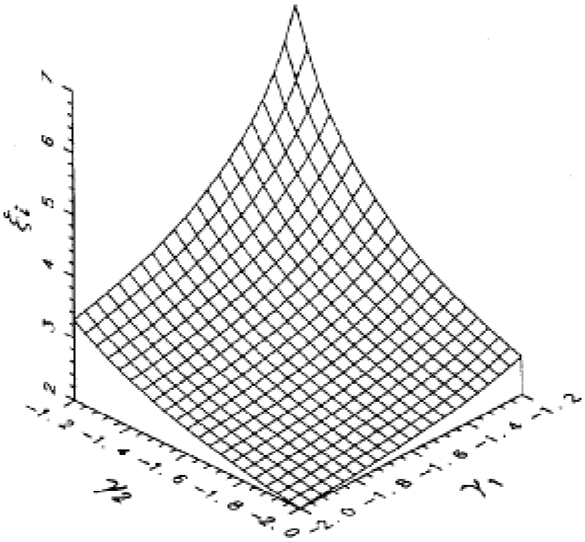

Fig. 3. Critical power enhancement factor $\xi_i = P_i/P^{SF}_{crit}(\lambda_i)$ as a function of $\gamma_i = k^{SPM}(\lambda_i)/k^{XPM}$, $i = 1, 2$. The wavelength ratio λ_2/λ_1 is chosen to be 1.2

the parametric processes in the VUV and XUV [1, 14–16] (typically 0.1%) it is reasonable to neglect the pump depletion and the induced phase modulation on the pump, originating from the generated signal. Therefore, this regime of propagation could lead to an enhanced efficiency of the four-wave mixing conversion.

Further, we will discuss the possibilities for symbiotic beam/pulse pair formation. Three cases could be distinguished:

Case (A): Self- and induced-focusing nonlinear medium. The XPM enhances the SPM (in both time and space) and the input powers should be kept below the powers required for beam self-trapping (fundamental soliton formation, respectively). This regime requires $k^{SPM}(\lambda_1)k^{SPM}(k_2) \neq (k^{XPM})^2$.

Case (B): Self-focusing and induced-defocusing medium. This regime of propagation is possible only if $k^{SPM}(\lambda_1)k^{SPM}(\lambda_2) > (k^{XPM})^2$. Since the XPM compensates partially for the SPM, the pump powers should exceed the corresponding critical powers P_{crit} for beam self-focusing (for fundamental soliton formation, respectively). Figure 3 plots the critical power enhancement factor $\xi_i = P_i/P^{SF}_{crit}(\lambda_i)$ *vs.* $\gamma_i = k^{SPM}(\lambda_i)/k^{SPM}$, $i = 1, 2$, where $\lambda_2/\lambda_1 = 1.2$ is assumed.

Case (C): Self-defocusing and induced-focusing medium. This mode of propagation is possible at $k^{SPM}(\lambda_1)k^{SPM}(\lambda_2) < (k^{XPM})^2$ only. The beam/pulse spreading and self-action is compensated by their mutual XPM coupling, which dominates over the SPM.

The temporal and spatial instabilities are distinguished depending on whether the pulses/beams are modulated after passing the nonlinear medium. It was shown recently [17], that the spatio-temporal instability is of fundamental importance, since time and space are coupled through diffraction and dispersion. An interesting feature of this instability is that it does not necessarily require anomalous GVD.

Let us now analyze the stability of the steady-state by considering how weak perturbations $\delta\psi_j(r, x, t)$ ($|\delta\psi_j| \ll |\psi_j|$) of the form

$$\delta\psi_j(r, x, t) = \delta A_j \exp\{-\mathrm{i}\Gamma_j x\} \times \exp\{\mathrm{i}[K^{(j)}_{\perp} r + \Omega^{(j)} t - h_j x]\} + \mathrm{c.c.} \quad (14)$$

evolve along the nonlinear medium [18, 19]. In eq. (14) $K^{(j)}_{\perp}$, $\Omega^{(j)}$ and h_j, $i = 1, 2$, are the spatial ant temporal perturbation frequencies and the perturbation wave-number, respectively, and Γ_i accounts for the nonlinear corrections to the wave-number due to the SPM and XPM. Substituting eq. (14) in eq. (9), neglecting the quadratic and higher order terms in δA_i, we obtained conditions for spatio-temporal stability of the symbiotic beam/pulse pair in the form

$$\alpha_i \Omega^{(i)2} + \beta_i K^{(i)2}_{\perp} \geqslant \Lambda^{(i)}_{crit}, \quad i = 1, 2, \quad (15a)$$

where

$$\Lambda^{(i)}_{crit} = 2\{k^{SPM}(\lambda_i)|A_i|^2 + k^{XPM}\sqrt{|A_1|^2|A_2|^2}\}. \quad (15b)$$

Equation (15b) agrees with the results of Agrawal and coworkers [18] on the temporal modulation instability, with our recent results on the spatial stability under SPM and XPM [9] as well as with the recent results of Liou *et al.* [17]. The present analysis, however, is more general in considering the spatio-temporal stability of a symbiotic light pair sustained by SPM and XPM. The result obtained is of particular importance, since by opposite signs of $k^{SPM}(\lambda_{1,2})$ and k^{XPM} (i.e. under a counteraction of SPM and XPM), the spatio-temporal stability conditions can be improved significantly.

In this work equal group-velocities of the two copropagating pulses/beams are assumed, which restricts the wavelength of operation. It has been shown [20] however, that an induced frequency shift could just cancel the walk-off effect. The XPM gives a rise to an attractive force, which is strong enough to bind the copropagating pulses to a pair of bright temporal solitons. In the opposite case, the validity of the present results is limited to nonlinear media of lengths, shorter than the pulse walk-off length L_w.

4. Conclusion

We derived analytical conditions, under which two copropagating laser beams/pulses can form a symbiotic light-pulse pair and can propagate with nearly constant temporal and spatio-temporal parameters in planar nonlinear media.

The conditions for an enhanced temporal (respectively, spatio-temporal) modulational stability of the symbiotic formation are found. This regime of propagation can prove to be very useful in the four-wave mixing experiments leading to a reduction in the spectral broadening enhancement of the pump pulses and, therefore, to higher (up to one order of magnitude) conversion efficiencies.

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