Pulse Shaping and Shortening by Spatial Filtering of an Induced-Phase-Modulated Probe Wave

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Abstract—All-optical deflection and spatial filtering have been used to theoretically model pulse shortening and shaping. Good agreement is obtained with the experimental results of other authors. Pulses with a shortening coefficient of the order of 10, special forms as super-Gaussian, triangular-like, and pairs of pulses with an adjustable peak-to-peak ratio are obtained.

I. INTRODUCTION

In many fields of modern optics (e.g., high-speed optical nonlinear fiber optics, time-resolved spectroscopy) specially shaped optical pulses are desired [1], [2]. In recent years, a variety of techniques for pulse shaping have been developed. One of these techniques includes programmable pulse shaping of femtosecond optical pulses using a spatial filter mask [3]–[6] or a multielement liquid crystal modulator [7] to manipulate the spatially dispersed optical frequency components. Alternative approaches employ intracavity self-phase modulation and temporal pulse shaping in a grating pair compressor [8]–[10], Bragg selectivity of volume holograms [11], or cross-phase-modulation-induced compression of probe pulses by pump pulses [12]. The approach of Kobayashi et al. [13], [14] involves a high-speed electrooptic element to deflect and phase-modulate the light passed through, followed by a spatial filter.

The spatial effects of the cross-phase modulation in an optical Kerr-like medium (induced focusing [15]–[17], self- and induced deflection [18]–[20]) are studied extensively, both theoretically and experimentally. These phenomena originate in the induced refractive index change along and across the nonlinear medium and beam cross section, respectively. Recently, the two-beam interference technique has been used for the observation of beams' self-deflection in a three-dimensional Kerr medium [21]. Intensity-dependent self-deflection, combined with far-field spatial filtering, is used for picosecond laser-pulse shortening. This result has stimulated the present analysis.

In this paper we analyze theoretically the spatio-temporal evolution of a probe beam/pulse, when a pump and probe wave copropagate in a nonlinear medium with an initial angular deviation and/or off-axial separation. With this base, we show that a simple spatial filtering of the deflected probe wave can lead to the generation of optical pulses with a special shape and to a reduction of their duration. Good agreement is found with the experimental results of Barthelemy et al. [21].

II. THEORETICAL ANALYSIS

Let us consider the following arrangement (Fig. 1). Two beams/pulses, the pump at wavelength \( \lambda_p \) and the probe at \( \lambda_p \), enter a planar nonlinear medium of length \( L_{NL} \) with an initial off-axis separation \( r_0 \) and/or initial angular deviation \( \theta \). Without loss of generality [19], the nonlinear medium is considered to be resonant and self-focusing. A slit placed at a distance \( L_{o} \) behind the exit of the nonlinear medium performs a spatial filtering of the deflected phase-modulated probe beam.

Generally, the pump and probe beam/pulse evolution is described by the nonlinear equations

\[
\begin{align*}
\frac{\partial}{\partial x} \left[ \frac{\partial}{\partial x} + \frac{1}{V_p} \frac{\partial}{\partial t} \right] \psi_p + \frac{1}{2} \beta_{2p} \frac{\partial^2 \psi_p}{\partial t^2} + \alpha_p \frac{\partial^2 \psi_p}{\partial t^2} + k_{pm}^p |\psi_p|^2 \psi_p &= 0, \\
\frac{\partial}{\partial x} \left[ \frac{1}{V_p} \frac{\partial}{\partial t} \right] \psi_s + \frac{1}{2} \beta_{2s} \frac{\partial^2 \psi_s}{\partial t^2} + \alpha_s \frac{\partial^2 \psi_s}{\partial t^2} + k_{pm}^s |\psi_p|^2 \psi_s &= 0,
\end{align*}
\]

where \( V_p \) and \( \beta_{2p} \) are the group velocities and the group-velocity dispersion coefficients for the respective waves, \( \alpha_p = (2k_p b) \), \( \alpha_s \) are the corresponding wave numbers, and \( k_{pm}^p \) and \( k_{pm}^s \) are the nonlinear coefficients for self- and induced-phase modulation (SPM and IPM, respectively), related to the nonlinear refractive indexes \( n_{2pm}^p \) and \( n_{2pm}^s \)

\[
k_{pm}^p = n_{2pm}^p k_p/(2n_{0p}), \quad k_{pm}^s = n_{2pm}^s k_p/(2n_{0s}).
\]

The four-wave frequency mixing terms in (1) could be neglected, since they are not phase matched in the interaction geometry considered [19].

Because of the limited computing resources available, we solved (1) using a modification of the split-step Fourier method, assuming a planar nonlinear medium with a diffraction-nonlimited coordinate \( r \) and neglecting the group-velocity mismatch and the group-velocity disper-
sion. The latter two assumptions enable us to treat time as a parameter at the expense of failing to adequately describe the physical picture of input pulses in the femtosecond time domain. More precisely, \( L_{\text{NL}} \) and \( L_p \) should be much smaller than the dispersive length \( L_p = \frac{\tau_p^2}{4} \mid \beta_{2p} \mid \) and the walk-off length \( L_w = \left( \frac{\tau_p}{2} + \frac{\tau_0}{2} \right)^{1/2} \mid V_{g1} \mid \). In view of the above restrictions, the femtosecond response of the media with an electronic nonlinearity (e.g., inert gases) could be regarded as instantaneous. Each pulse was divided into \( 4N \) slices, each having duration \( \tau_{\text{slice}} = \tau_p/N \). The minimum slice number required \((N = 50)\) was obtained from both energy conservation and the reproducibility of the results against increasing \( N \) twice. As a consequence, the initial temporal delay \( \tau_0 \) could be involved adequately if \( \tau_0 = M \tau_{\text{slice}} \), where \( M \) is integer. For simplicity \( \tau = \tau_p = \tau_0 \) is assumed.

### A. Qualitative Picture of the Process

The input waves are assumed to be Gaussian in both time and space:

\[
\psi_p(r, x = 0, t) = A_{p0} \exp \left\{ -\frac{r^2}{\alpha_p^2} \right\} \exp \left\{ -\frac{t^2}{2\tau_p^2} \right\} \tag{2a}
\]

\[
\psi_1(r, x = 0, t) = A_{s0} \exp \left\{ -\frac{(r - r_0)^2}{\alpha_t^2} \right\} \cdot \exp \left\{ -\frac{t - \tau_0}{2\tau_0^2} \right\}, \tag{2b}
\]

where \( A_{p0} \) and \( A_{s0} \) are the slowly varying field amplitudes, \( \alpha_t \) and \( \alpha_r \) are the beam's radii, and the spatio-temporal coordinate system is taken to be connected with the pump. Fig. 2 shows the probe-wave intensity distribution in both time and space at the exit of the nonlinear medium. The parameters considered are \( L_{\text{NL}} = 6 \text{ cm}, L_0 = 94 \text{ cm}, \alpha_p = 0.1 \text{ cm}, \alpha_r = 0.4 \), \( r_0 = 0.4 \), \( \tau_0 = 0 \), \( \Delta n = 1/(2\alpha_p^2) \mid A_{p0} \mid^2 = 5.5 \times 10^{-5} \), wavelengths \( \lambda_p = 248 \text{ nm} \) and \( \lambda_s = 264.4 \text{ nm} \) and the nonlinear medium is Xe at 1 atm.

In a resonant medium if \( \lambda_p + \lambda_s \) is near a two-photon resonance, \( \lambda_p \) and \( \lambda_s \) being far from single- and two-photon resonances, one can ignore the pump beam/pulse self-action. Accounting for the pump beam diffraction only simplifies considerably the calculations and reduces the computing time required. Generally, the nonlinear absorption and energy flow, enhanced by a two-photon resonance, could lead to distortions of the interacting wave profiles. In the starting equations (1), the four-wave-mixing terms are omitted, since they are not phase matched. In addition, the nonlinear absorption (proportional to \( |A_{p0}|^2 |A_{s0}|^2 \)) is expected to be low in a pump/probe approximation.

Our attempt to precisely model the experimental results of Barthel et al. [21] in CS\(_2\) revealed input intensities several times higher than those causing self-focusing instabilities. The two-beam interference pattern used in [21] experimentally allows stabilization of this effect, but is very difficult to model. The parameters of our model, in spite of the resonant interaction, allow a nearly quantitative comparison with the experimental data.

The interplay between diffraction and probe-beam deflection inside the nonlinear medium does not significantly change the intensity distribution straight at the exit of the medium (Fig. 2). Only a small spatial asymmetry near the pulse center can be observed. The picture changes completely when the probe wave passes a distance \( L_0 = 94 \text{ cm} \) after the exit of the nonlinear medium (Fig. 3). The spatial oscillatory behavior of the beam at a fixed local time is a manifestation of the spatial analog of the optical wave breaking recently observed [19]. At synchronous pump- and probe- pulse propagation, it is natural to expect a maximum probe-beam deflection near the common pulse center. As seen from the figure, the spatial oscillations are also most pronounced near the pulse maximum. This is to be expected, since the maximum pump intensity in this area induces the highest phase shift on the probe wave. The eventual temporal delay can modify the spatio-temporal symmetry of the probe wave. Intensity dependent probe-beam deflection and optical wave breaking can also be expected at coincident but angularly deviated beams at the nonlinear entrance face \( (r_0(x = 0)) = 0, \theta \neq 0 \). Fig.
Fig. 3. Intensity distribution of the probe beam/pulse propagated 94 cm from the exit of the nonlinear medium. Note the formation of spatial wave breaking near the pulse center.

Fig. 4. Transverse intensity distribution of the deflected beam at \( t = 0 \), initially coincident pump and probe beam centers \( (x_0 = 0) \) and different angular deviation \( \theta \): 1—\( \theta = 0^\circ \); 2—\( \theta = 0.5^\circ \); 3—\( \theta = 1^\circ \); 4—\( \theta = 1.5^\circ \); 5—\( \theta = 2^\circ \).

Fig. 5. Signal pulses obtained by the spatial filtering for zero \( r_{0}(x = 0)/a_{p} = 0.4 \) (1) and nonzero \( r_{0}/a_{p} = 0.4 \) (2) offset between the centers. A 100-\( \mu \)m slit is centered at the maximum deviated peak at local time \( t = 0 \). Curves 3, 4—the same, centering the slit at the maximum probe beam deviation for local time \( t = t/4 \). Insert: intensity distribution of the deflected probe beam (local time \( t = 0 \)) in front of the slit, corresponding to pulses 1 and 2.

Gaussian \( m = 2 \). The signal obtained at \( \theta = 0^\circ \) and \( r_{0}(x = 0)/a_{p} = 0.4 \) (curve 2) is approximately 4.7 times shorter than the input pulse probe. This result is in a good agreement with the experimental value of 5 of Barthelemy and co-workers [21], achieved via spatial filtering of a self-deflected beam in a nonresonant Kerr medium by 30 ps input pulses. As seen from Fig. 5, an initial angular deviation \( \theta (\sim 1.5^\circ) \) instead of an initial off-axis distance, may lead to a significant (2.25 times) increase of the signal peak power. The inset in Fig. 5 shows the transverse signal intensity distribution of the probe pulse.

B. Numerical Analysis

As a first step, we have modeled the probe pulse, resulting from a spatial filtering of the deflected probe beam (later called “signal”). Synchronous pulse propagation is assumed and the model parameters are \( a_{p} = 0.1 \) cm, \( a_{p}/a_{p} = 0.4 \), \( \Delta_{\text{ion}} = 1.1 \cdot 10^{-4} \), \( L_{\text{NL}} = 6 \) cm, and free-space propagation length \( L_{o} = 94 \) cm. Under these conditions, the slit could be regarded as being placed in the far field. The particular values of \( L_{o} \) and the slit width are necessary to determine the conditions under which the energy efficiency (compression coefficients, respectively) are calculated. It should be pointed out that the center of gravity of the deflected probe wave and the peak of the beam/pulse do not coincide [22]. In our view, a simple imaging in the focal plane of a lens is not suitable for achieving the actual far-field probe beam profile in front of the slit.

Fig. 5 plots the signal power versus \( t/\tau_{b} \) at \( \theta = 0 \) and \( r_{0}(x = 0)/a_{p} = 0.4 \) (dashed curves), as well as at \( \theta = 1.5^\circ \) and \( r_{0}(x = 0) = 0 \) (solid curve). Centering a 100-\( \mu \)m slit at the maximum deviated peak of the probe wave (local time \( t = 0 \)), the signal transmitted (curve 1) has a super-Gaussian (SG) form (power of the super-
central part (local time \( t = 0 \)) just in front of the slit. The dashed lines indicate the corresponding probe beam axis in the linear regime of propagation. The comparison is in agreement with the observation in [21] that the greater the deflection, the narrower the transmitted pulse, and that the beam deviation reaches its maximum with the pump pulse intensity and then returns to its initial value (see also, Fig. 3). It should be pointed out once more that the detailed comparison with [21] is difficult due to the complicated highly elliptical pattern of the beam used there to form a spatial soliton. The intensity in the interference fringes along the great axis of the ellipse could only be approximately determined. Nevertheless, the nonlinear deflection angle in our model is 4 mrad, in reasonable agreement with the result of Barthelemy et al. of \( \Delta \theta = \theta_0 \alpha t / \alpha = 4.2 \) mrad [21].

From our point of view, the approach introduced in [21] has a large potential for pulse shaping and shortening. Several possible schemes will be analyzed in the following.

At \( \tau_d = 0 \), the leading and the trailing edges (local times \( t \neq 0 \)) are deflected symmetrically, but less than the central part of the probe pulse. Placing the same slit closer to the probe beam axis in the linear regime, one can select two identical pulses from the less-deflected probe beam parts. Such pulse pairs are plotted in Fig. 5 with a dashed curve \( r_0(x = 0) / a_p = 0.4, \vartheta = 0 \) and a solid line \( r_0(x = 0) = 0, \vartheta = 1.5^\circ \). The 100-\( \mu \)m slit is centered at the maximum deviation for a local time \( |t| = \tau_d / 4 \). Again, the regime with an initial angular deviation seems more attractive for achieving higher signal intensities. All curves in Fig. 5 are normalized to the SG at \( \vartheta = 0^\circ \). The energy efficiency of this scheme at the 100-\( \mu \)m slit is 1.8 percent (respectively, 4.8 percent for \( \vartheta = 1.5^\circ \)) for the SG pulses and 1.6 percent (respectively, 4.0 percent for \( \vartheta = 1.5^\circ \)) for the twin pulses. Our calculations have shown that for the SG pulses, the signal power energy increases and the signal rise and fall times reduce with increasing the slit width up to 1 mm (energy efficiency of approximately 13.2 percent). These values are comparable to the 15 percent energy efficiency reported in [1], [4], but the square optical pulses generated have rise and fall times below 100 fs and a duration below 1 ps. An advantage of the technique analyzed in the present work is that no initial femtosecond pulses are needed. Shortening starting from longer pulses, an adjustable short pulse, or pulse pair formation could be obtained in the picosecond and subpicosecond ranges. Opening the slit wider results in approaching the initial Gaussian pulse shape. (Note that this is probably due to the neglect of the group-velocity dispersion and requires further analysis at initial pulse durations of less than 10 ps). In the generation of twin pulses, the slit should be kept reasonably narrow in order to achieve higher values of the peak-to-valley contrast. In the case considered, at a 500-\( \mu \)m slit, the two-peak pattern practically transforms in a smooth, more or less rectangular pulse.

At a nonzero initial delay \( \tau_d \), the physical picture becomes more complicated. The probe pulse symmetry (see Fig. 3) at a local time \( t = 0 \) is lost. Nevertheless, this offers another possibility for probe pulse shaping and shortening via spatial filtering of the deflected probe wave. Fig. 6 shows the pulses, potentially obtainable at \( \tau_d = 0.5 \tau_r \) (curve b) and \( \tau_d = \tau_r \) (curve c) by fixing a 100-\( \mu \)m slit at the transverse position of the maximum deviated probe peak at \( \tau_d = 0 \). For comparison, the signal pulse shapes at \( \tau_d = 0 \) are also presented (curve a). The respective dashed lines represent the case \( r_0(x = 0) / a_p = 0.4, \vartheta = 0 \), the solid lines correspond to \( r_0(x = 0) = 0, \vartheta = 1.5^\circ \). All curves are normalized to the peak power of the SG pulse at \( \vartheta = 0 \) and \( r_0(x = 0) / a_p = 0.4 \). The comparison shows that an initial angular deviation is preferable in order to achieve larger peak powers (respectively, energies). With increasing \( \tau_d \), the pulses become asymmetric, approaching triangular form. However, as expected, increasing \( \tau_d \), the peak energy tends to decrease. An initial delay \( \tau_d \) will also change the double-peaked symmetric pulse formation from Fig. 5. Fig. 7 shows the signal pulse shapes, obtained numerically with \( r_0(x = 0) / a_p = 0.4, \vartheta = 0^\circ \) and a 100-\( \mu \)m slit placed at the maximum deviation for local time \( |t| = \tau_d / 4 \). Curve a corresponds to \( \tau_d = 0 \) (see Fig. 5, the dashed curve), curve b corresponds to \( \tau_d = \tau_d / 4 \), and curve c corresponds to \( \tau_d = \tau_r \). This figure indicates the possibility of achieving pairs of pulses with durations considerably shorter than the initial one, and with an adjustable ratio of their peak powers. Such pairs of pulses may be potentially applicable in studying soliton–soliton interactions and in pump and probe schemes. Qualitatively similar behavior was found in the case of initially centered but angularly deviated pump and probe beams.

Furthermore, we tried to numerically find the maximum probe pulse shortening, e.g., the minimum achievable signal pulse duration after the spatial filtering of the deflected probe wave. Our analysis shows that at \( \tau_r = \tau_p \), the optimum initial delay is \( \tau_d = \tau_r \). This may be attrib-
Fig. 7. Evolution of the twin pulses into signal pulse pairs with adjustable peak power ratio. Collinear beams $\theta = 0$ and offset $r_0/a_0 = 0.4$; slit = 100 $\mu$m. Curve $a = r_p = 0$; curve $b = r_p = r/4$, curve $c = r_p = r_p$.

Fig. 8. Signal pulse narrowing at $r_p = r_p$. Solid curves—$\theta = 1.5^\circ$, $r_p = 0$; dashed curves—$\theta = 0$, $r_0/a_0 = 0.4$: 1—slit 300 $\mu$m, compression 11.8 times, energy conversion 5.1 percent; 2—slit 300 $\mu$m, compression 20, energy conversion 2.1 percent 3—slit 100 $\mu$m, compression 15, energy conversion 1.7 percent 4—slit 100 $\mu$m, compression 22, energy conversion 0.7 percent.

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