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Odd and even two-dimensional dark spatial solitons

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Abstract. The formation of two-Dimensional (2D) odd dark spatial solutions is analyzed numerically at an initial helical dark-beam phase distribution. Experimental results are presented for the first time confirming the existence of two-dimensional optical even dark solitons (ring dark solitons). Several aspects of the evolution of input 1D and 2D odd/even dark beams are compared qualitatively.

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The theory of optical solitons has been extensively studied since 1971 when Hasegawa and Tappert [1] pointed out the existence of temporal solitons in Kerr media as a result of a balance between Group-Velocity Dispersion (GVD) and Self-Phase Modulation (SPM). The bright temporal solitons are applicable to optical communication systems and results have been published in [2, 3]. Generally, the evolution of a beam/pulse is described by the (3+1)-dimensional NonLinear Schrödinger Equation (NLSE)

$$\mathrm{i}\frac{\partial E}{\partial z} + \alpha\frac{\partial^2 E}{\partial t^2} + \beta\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E + kn_2 \mid E \mid^2 E = 0\ , \quad (1)$$

where z is the longitudinal coordinate, x, y and t are the two transverse and the temporal coordinate, respectively, $k = 2\pi/\lambda$ is the wave number corresponding to the pulse wavelength λ, α is a constant reflecting the GVD, and the term comprizing $\beta = (2k)^{-1}$ accounts for the spatial beam diffraction in both transverse dimensions. The nonlinear term in (1) involves the intensity-dependent medium refractive-index correction $n_2 \mid E \mid^2$, leading to SPM. The behaviour of the solitons depends on the relative sign of the nonlinearity and diffraction/dispersion. The diffraction coefficient β is always a positive one (i.e., the diffraction tends to broaden the beam only), whereas the GVD coefficient α, depending on the refractive-index wavelength-dispersion $n_0(\lambda)$, can change its sign. As a consequence, self-defocusing media could support dark spatial solitons only, while bright and dark temporal solitons could both exist in the case of $n_2 < 0$.

The one-Dimensional (1D) fundamental dark spatial/temporal soliton is an anti-symmetric function of space/time with an abrupt phase shift (phase step) of π, and zero intensity at its center [4]. This soliton is denoted as 'dark odd soliton', in contrast to the 'dark even soliton', which do not have an initial phase shift. As a consequence, independent from the background intensities, the even input formations split at least into two odd dark solitons with a reduced contrast, each one with its own phase shift (less than π) [5, 6]. 1D Dark Spatial Solitons (DSS) were experimentally observed in the form of dark soliton stripes [7]. The influence of backgrounds of finite extent on their evolution characteristics was studied in [8].

For the first time, the existence of stable dark 2D self-supported beams was described in [9] on the conjecture that solitons are analogs of linear guided waves. Optical Vortex Solitons (OVS) were observed [11] and analyzed [12] by Swartzlander and Law at $n_2 < 0$. The OVS is characterized as a dark cylindrical beam with a 2π helical phase ramp. The formation of a dark vortex core is known from the pioneering analyses of superfluidity [13, 14]. Experimentally, OVS are generated by employing the modulational instability of 1D DSS against a long-period transverse modulation (discrete transverse phase retardation [11]).

Self-defocusing nonlinear media can support also dark solitary waves with a ring symmetry. The existence of such formation was predicted recently by Kivshar and Yang [15]. A general formula describing the internal dynamics of the ring dark solitons was also derived.

The goal of the present analysis is to demonstrate additional evidences that two-dimensional DSS (2D DSS) do exist. We present numerical results on the generation of 2D Odd Dark Spatial Solitons (2D ODSS). The initial conditions considered (hyperbolic-tangent intensity distribution of a rotational symmetry and a 2π helical phase ramp) enable us to study the 2D ODSS formation in the absence of an initial 2D modulational instability. We present, up to our best knowledge, the first experimental results confirming the existence of 2D Even Dark Spatial Solitons (2D EDSS also called ring dark solitons). The terms '2D ODSS' and '2D EDSS' are preferred instead of 'optical vortex soliton' and 'optical ring soliton', since:

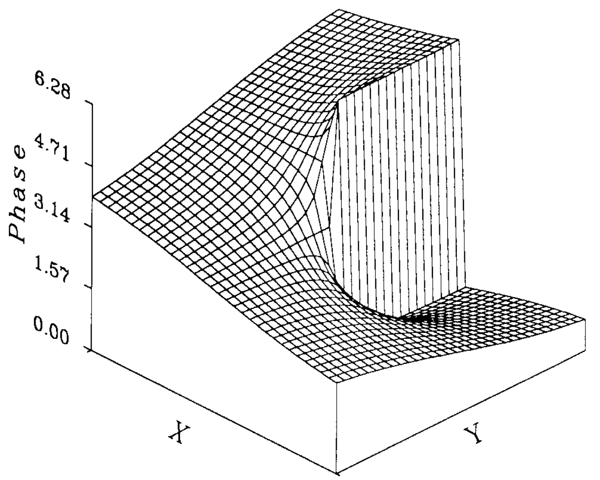

Fig. 1. Helical phase distribution for obtaining a π phase jump in an arbitrary radial direction

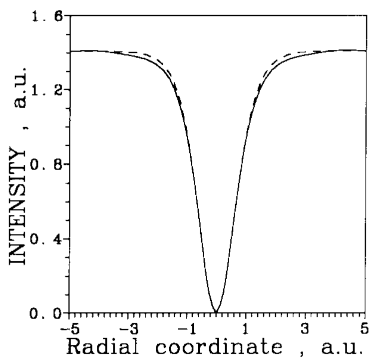

Fig. 2. Cross sections of the 2D input odd dark formation (*dashed line*) and the self-supported 2D ODSS at $z = 5L_{NL}$ (*solid line*)

(*i*) The qualitative similarity in the evolution of the 1D and 2D dark self-supported formations should be accompanied with a terminological similarity;

(*ii*) OVS and 2D ODSS seem to differ significantly only in the early stages of their formation. The 2D DSS should be stable solutions of the (2+1)-dimensional NLSE [16]

$$i\frac{\partial E}{\partial z} + \beta\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E + kn_2\,|E|^2\,E = 0\,, \tag{2}$$

which can be solved numerically.

Our modelling of the nonlinear evolution of the 1D and the 2D input dark formations is based on solving (2) for continuous-wave (CW) or quasi-CW beams. The numerical procedure used is a 2D generalization of the split-step Fourier method [17]. The 1D version of this method is frequently used for modelling of nonlinear optical processes in optical fibers.

In order to ensure adequate initial conditions for the generation of a 2D ODSS [11, 12], a 2D spatial phase distribution, shown in Fig. 1, should be imposed on a plane wave front. Characteristic of this phase distribution is the π phase shift in each radial direction (localized at the soliton center). This distribution could be predicted intuitively by adding a rotational symmetry to the transverse phase distribution

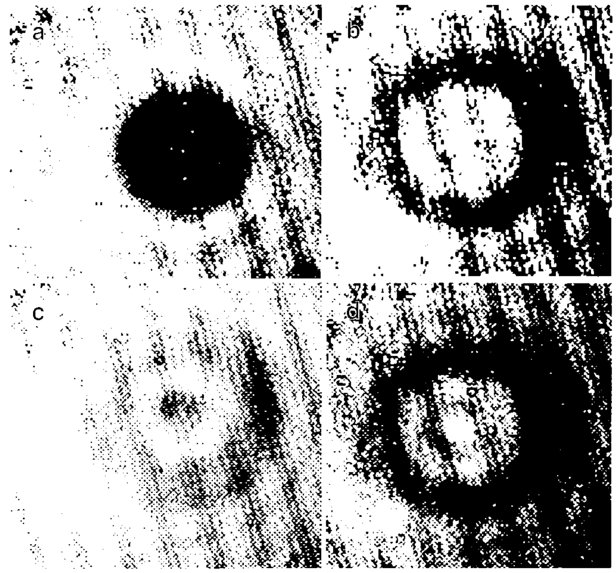

Fig. 3a–d. Characteristic evolution stages of a 2D EDSS obtained experimentally: (**a**) diffraction compensated by the medium nonlinearity; (**b**) first soliton ring; (**c**) initial stages of a second soliton ring; (**d**) two soliton rings

of a 1D odd dark soliton [18]. On the basis of numerical simulations, we proved that it is adequate in two spatial dimensions. We analyzed the evolution of an initial intensity dip of the form (written in cylindrical coordinates):

$$E(r,\varphi,z=0) = A_0 B(r)\tanh(r/r_0) \times \exp[i\Phi(r,\varphi)]\,, \quad r = (x^2+y^2)^{1/2}\,, \tag{3}$$

with a phase distribution

$$\Phi(r,\varphi) = m\varphi\,, \quad \varphi \in (0, 2\pi)\,, \tag{4}$$

and a super-Gaussian background profile

$$B(r) = \exp\left[-(r/15r_0)^{16}\right] \tag{5}$$

along a self-defocusing ($n_2 < 0$) nonlinear medium. In (3–5), r and φ are the radial and azimuthal coordinate, respectively, and m is an odd number. The known initial condition for the 1D case could be reproduced by omission of the coordinate y in (3–5). In order to avoid a dip-to-background interaction, the half width at 1/e level of the background beam is chosen to be 15 times larger than the width of the dark formation.

Figure 2 shows a radial cross section of the input ($z = 0$) 2D dark odd formation (dashed line). The solid curve demonstrates the 2D dark beam shape at $Z = 5L_{NL}$, where $L_{NL} = (kn_2A_0^2)^{-1}$ is the characteristic length of beam self-action (nonlinear length). As usual for dark solitons, L_{NL} depends on the background intensity. The shape of the resulting 2D dark formation as well as its phase portrait closely reproduce the initial distributions given by (3–5). The smoother wings (Fig. 2) indicate that the input 2D hyperbolic-tangent profile (assumed in 2D) slightly differs from the exact one for the 1D case. Nevertheless, this approximation [14] seems reasonable [11]. It is interesting to note that the background intensity (A_0^2) required for obtaining a 2D ODSS is found to be $\sqrt{2}$ times higher than the respective value in the 1D

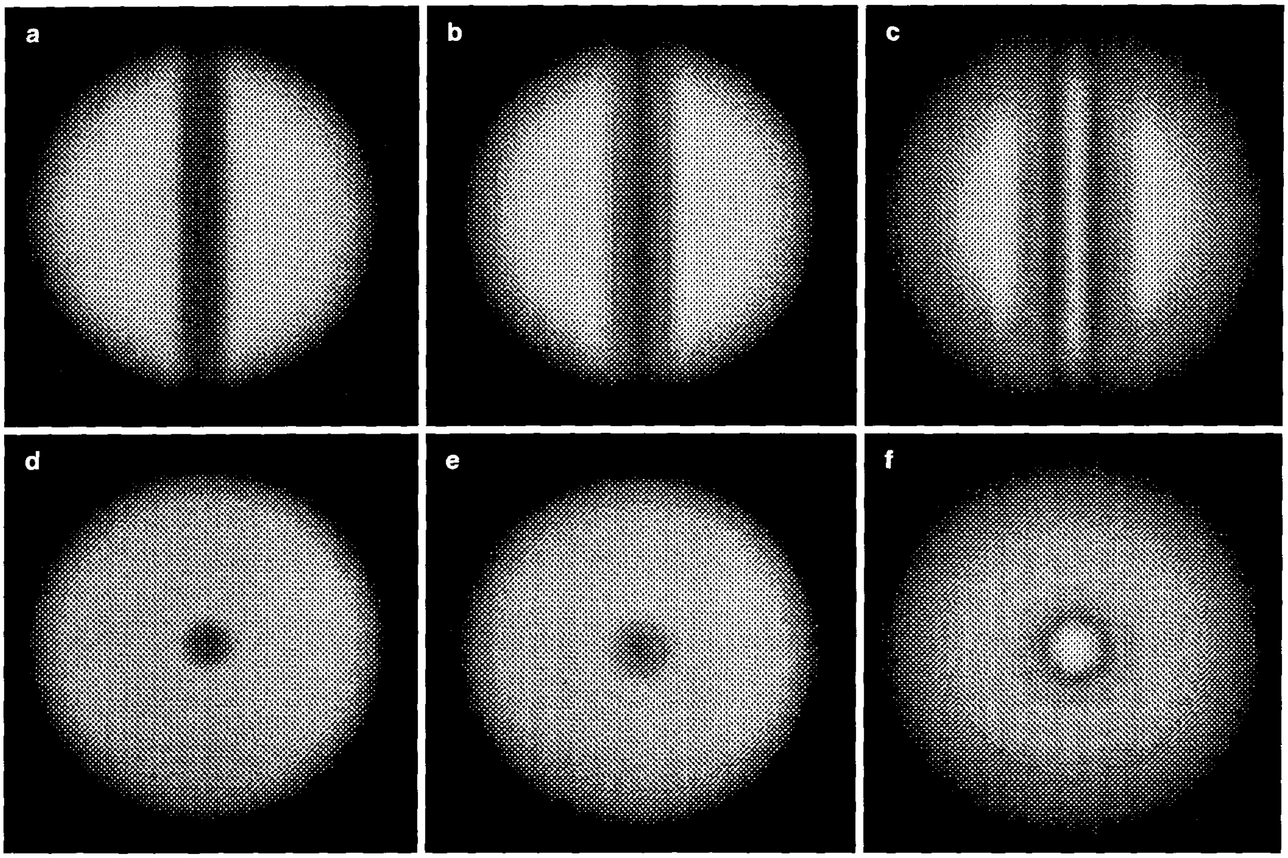

Fig. 4a–f. Comparative numerical results on the evolution of 1D (**a**, **b**, **c**) and 2D (**d**, **e**, **f**) even input dark soliton formation. The characteristic lengths for diffraction compensation (**b**, **e**) and first soliton pair/ring formation (**c**, **f**) are mentioned in the text

case. This was mentioned for the first time in [11] [comment after (2)]. Physically, these numerical results could be attributed to the higher diffraction in two spatial dimensions as compared to the diffraction in one dimension.

Exceeding more than twice the background intensity for the fundamental 2D ODSS results in an evolution of the input single odd dark formation into an on-axis fundamental 2D ODSS and a diverging "gray" fringe. This behaviour, known for the 1D odd dark solitons [18], strongly supports the statement that 2D ODSS should really exist. Mutual 2D dark soliton attraction and repulsion, known for 1D bright solitons, was observed too during our numerical simulations and will be discussed elsewhere.

As a second step, we modelled numerically and observed experimentally the evolution of 2D Even Dark formations (2D EDSS). Similarly to the 1D EDSS, the absence of an initial spatial phase modulation results in a splitting of the input dark formation into a diverging gray ring. In the first analysis [15] these formations are called 'ring dark solitons'. The initial conditions we described by (3, 5) at Φ = const. 2D EDSS are easily produced experimentally by imposing 2D amplitude masks in front of the nonlinear medium (ethanol slightly dyed in red). For the purpose of a comparative experimental analysis, the masks consisted of dots (2D) and stripes (1D) of equal diameters/widths ranging from 50 to 250 μm. A copper vapour laser source (P = 4W) was used to produce the background signal, required for a background self-defocusing in the thermal nonlinear medium. This technique is routinely used for generating of 1D dark spatial solitons (dark soliton stripes) [7, 19, 20]. The evolution of the input dark formations is recorded by a CCD video camera and a frame-grabber.

Four characteristic pictures are observed experimentally by increasing the intensity – 2D dark formation with a diffraction compensated by the nonlinearity (Fig. 3a), dark ('gray') ring with a center intensity higher than the background level (Fig. 3b), dark ring with a central dark dot (Fig. 3c) and two coaxial dark rings (Fig. 3d). In order to avoid the CCD camera saturation, Fig. 3d was recorded with a neutral density filter placed directly in front of the CCD array. Our numerical simulations confirm that, after the first ring formation, the center intensity should be expected to be higher than that of the background. The interference lines resulted from cuvette input and output faces and were difficult to avoid. Nevertheless, the results form Fig. 3 are indicative for the modulation stability of the 2D ODSS.

The comparative numerical simulations in one and two spatial dimensions (Fig. 4) show that, in qualitative agreement with experiment, the input even dark formations (Figs. 4a,d: Z = 0) do split along the nonlinear medium, forming a pair of stripes (1D ODSS): Fig. 4c (at $Z = 7.66L_{NL}$) or a 2D dark ring formation (2D ODSS): Fig. 4f (at $Z = 7.0L_{NL}$).

It should be pointed out that the initial diffraction compensation (Figs. 4b,e) and the subsequent spatial splitting (Figs. 4c,f) take place earlier (i.e., at shorter distances in the nonlinear medium) for the 2D even input dark formations (at $Z = 2.33L_{\rm NL}$) as compared to the 1D dark ones (at $Z = 2.67L_{\rm NL}$). In consistence with the 1D results of other work [18], our numerical results showed the existence of phase jumps (less than π) across 1D dark pairs. In the numerical results, we observed similar phase jumps across the 2D dark ring formation. Opposite phase changes across the 2D EDSS ring are experimentally observed and will be discussed elsewhere. The transverse velocity of the 2D EDSS exhibits a similar behaviour but it was found to be higher than that of the 1D EDSS. These results are well-pronounced indications for the existence of the 2D EDSS.

In conclusion, we have shown first experimental results confirming the existence of 2D even dark spatial solitons (ring dark solitons [15]). The generation of 2D ODSS is analyzed numerically. Several aspects of a qualitatively similar nonlinear evolution of input 1D and 2D even/odd dark beams are discussed. From a practical point of view, 2D ODSS could be used to form 2D single-mode optical waveguides with nonlinear claddings [9, 11]. Two dimensional EDSS may appear to be useful for all optical manipulation of light, directing, switching and multiplexing/demultiplexing of channels for transmission of optical information in bulk media. The intensity dependence in the evolution of both types of 2D formations discussed may allow real-time reconfiguration of these nonlinear devices to be achieved.

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