Appl. Phys. B 62, 139–142 (1996)

Applied Physics B Lasers and Optics

Phase measurements of ring dark solitons

A. Dreischuh[1], W. Fließer[2], I. Velchev[1], S. Dinev[1], L. Windholz[2]

[1] Department of Quantum Electronics, Sofia University, 5, J. Bourchier Blvd., BG-1126 Sofia, Bulgaria
(Fax: + 359-2/463 589, E-mail: ALDREI@phys.uni-sofia.bg)
[2] Institut für Experimentalphysik, Technische Universität Graz, Petersgasse 16, A-8010 Graz, Austria
(Fax: + 43-316/812658, E-mail: WINDHOLZ@fexphds01.tu-graz.ac.at)

Received: 13 February 1995/Accepted: 4 May 1995

Abstract. A four-frame technique for interferogram analysis is used to measure the transverse phase distribution of nonlinearly split 2D dark beams. Pairs of diametrical phase shifts within each ring are retrieved from the experiment. This result is one of the tests required to denote the formations observed as ring dark solitons.

PACS: 42.65

In view of their potential applications in all-optical guiding and switching devices, the dark optical solitons are a subject of extensive theoretical and experimental analyses [1]. One of the tests [2] used to identify the dark spatial soliton is the characteristic phase profile of the electric field near the irradiance minimum. The existence of a phase step (of π) localized at the dark-beam centre can lead to the generation of a fundamental odd dark soliton [2, 3]. In the absence of such phase step, the 1D dark beam splits into two diverging "gray" odd beams, each one with its own phase shift less than π [1, 4]. Recently, the generation of a periodic array of 1D dark spatial solitons from two plane waves with an internal angular offset is reported [5]. The presence of an adiabatic amplification in the medium of negative third-order nonlinearity is shown to be important for clean soliton generation.

The physical picture becomes more complicated in two transverse dimensions. The Optical Vortex Solitons (OVS) generated [6, 7] could be characterized as dark cylindrical beams with an on-axis 2π helical phase ramp [8]. In the first analyses [6, 7], OVS are generated utilizing the modulation instability of soliton stripes against long-period modulation. Pairs of OVS are obtained by computer-generated holograms [9]. In recent analyses it is proposed that self-defocusing nonlinear media can support also dark solitary waves with ring symmetry [10, 11].

In this work we show experimental evidences on the existence of pairs of opposite diametrical phase shifts across nonlinearly split 2D even dark beams. The numerical simulations carried out showed a good qualitative agreement to the experiment.

1 Numerical simulations

The longitudinal nonlinear 2D evolution of the input dark beam is described by the (2 + 1)-dimensional nonlinear Schrödinger equation (NLSE) [10, 11]:

$$\mathrm{i}\frac{\partial E}{\partial z} + \beta\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E + kn_2\,|E|^2 E = 0, \tag{1}$$

where the term comprising $\beta = (2k)^{-1}$ accounts for the beam diffraction and $n_2|E|^2$ is the intensity-dependent medium refractive-index correction leading to a background-beam self-defocusing at $n_2 < 0$. We analysed the propagation of an input intensity dip of the form [4]

$$E(r, \varphi, z = 0) = A_0 B(r)\,[1 - m\,\mathrm{sech}(r/r_0)]\exp(\mathrm{i}\Phi) \tag{2a}$$

superimposed onto a background beam of a super-Gaussian profile

$$B(r) = \exp[-(r/15r_0)^{16}], \tag{2b}$$

where m is the depth of the modulation.

As a first step let us consider what kind of a nonlinear evolution of the input 2D even dark formation should be expected in an experiment. The numerical procedure used for solving (1) is a 2D generalization of the beam-propagation method in a transverse grid of 256×256 points.

The phase front is assumed to be plane $[\Phi(r, \varphi, z = 0) = 0]$ at the entrance of the nonlinear medium. Fig. 1 plots the transverse intensity (Fig. 1a) and phase (Fig. 1b) distribution of the dark beam at $z = 5L_{\mathrm{NL}}$. The nonlinear length L_{NL} is related to the background intensity I_0 via $L_{\mathrm{NL}} = (k|n_2|I_0)$. The input beam (2a) evolves in a diverging ring of a reducing contrast. Further (along the z-axis) an undesired ring-to-background interaction was observed. The interaction is a consequence of the finite transverse dimensions of the background beam (due to the finite numerical discretization) which prevents two or more dark rings to be achieved at

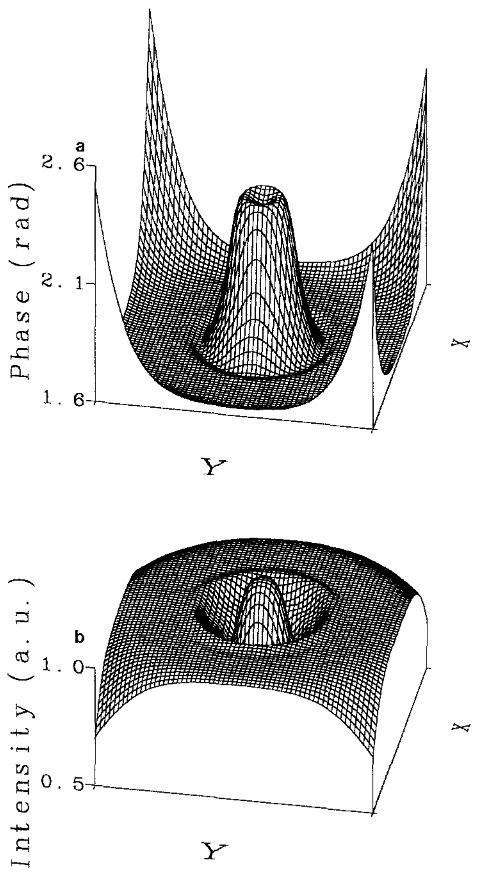

Fig. 1a, b. Transverse intensity (**b**) and phase (**a**) distribution of a nonlinearly evolved 2D even dark beam at $z = 5L_{\mathrm{NL}}$

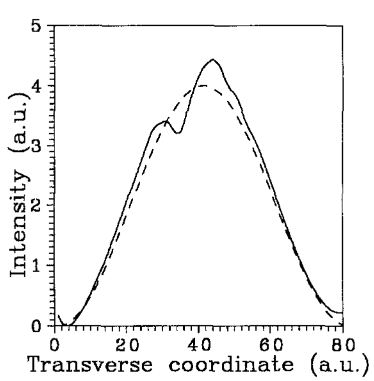

Fig. 2. Comparison between the transverse interference pattern of two 2D plane waves (*dashed line*) and the pattern obtainable with one of the waves replaced by the nonlinearly evolved ($z = 5L_{\mathrm{NL}}$) wave from Fig. 1a, b

higher intensities. For this reason we restricted our calculations to $z = 5L_{\mathrm{NL}}$. The phase portrait (Fig. 1b) of the dark ring (Fig. 1a) is a consequence of a nonlinear evolution in a medium with $n_2 < 0$. A reduced refractive index (and, therefore, a decreased phase) corresponds to the high intensity parts of the background. Pairs of diametrical phase shifts can be seen across each diametrical slice of the dark ring. As mentioned, this 2D behaviour has an 1D analogue in the evolution of the 1D even dark spatial solitons into a diverging pair of 1D odd solitons [1, 4].

When two 2D plane waves with unit amplitudes and suitable chosen phase differences interfere, they could form a pattern of parallel interference lines. A cross section of such an interference line is plotted on Fig. 2 (dashed line). Let us assume that one of the waves involved in the interference has the intensity and phase distribution, shown on Fig. 1 (b and a, respectively; $z = 5L_{\mathrm{NL}}$). As a result, the interference pattern should be expected to change its shape, and the numerically simulated result is plotted on Fig. 2 with a solid line. The peak and the valley near the maximum of the unperturbed pattern result from the phase shifts across the ring dark soliton. Qualitatively, the interference line reshapes and shifts slightly.

2 Experimental results

The experimental arrangement for measuring the transverse phase shifts of nonlinearly split 2D even dark beams is shown on Fig. 3. Interference was obtained with a He–Ne laser beam (50 mW) entering a Mach-Zehnder interferometer (M1, M3, M4, M6, M7) with a Michelson interferometer (M2, M5, M7) build-in in its reference arm. The input 2D even dark formation was achieved by illuminating an Amplitude Mask (AM), consisted of reflecting dots (diameters ranging from 50 to 250 μm, photolitografically produced). The nonlinear splitting of the dark beam was achieved by focusing onto the entrance of the NonLinear Medium (NLM). The last one is 5 cm quvette containing ethanol, dyed to absorb ≈14% of the input power (i.e. $|\Delta n_{\mathrm{NL}}| = |n_2| I_0 \leq 10^{-3}$). This technique is routinely used for generation of 1D dark spatial solitons in thermal (Kerr-like) self-defocusing nonlinear media [2–4, 6, 7]. Variable phase delay between the object and reference beams of the Mach-Zehnder interferometer is introduced by moving the mirror M7 and sensed by the

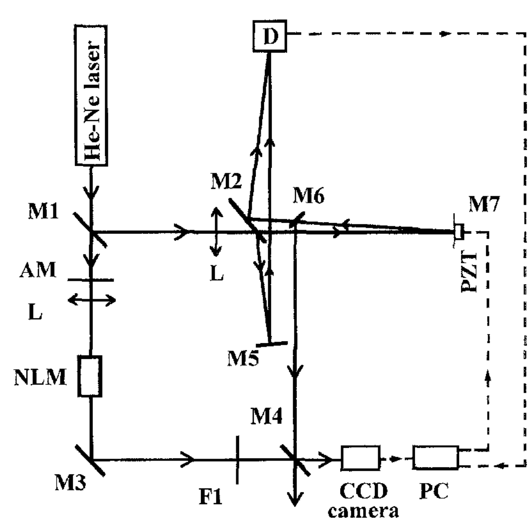

Fig. 3. Experimental setup (M1, M2, M4: beam splitters; M3, M5, M6: mirrors; M7: mounted mirror; (PZT): piezoceramic transducer; AM: amplitude mask; L: focusing lens; NLM: nonlinear medium; F1: linear-gradient variable filter; D: detector; PC-AT: compatible computer)

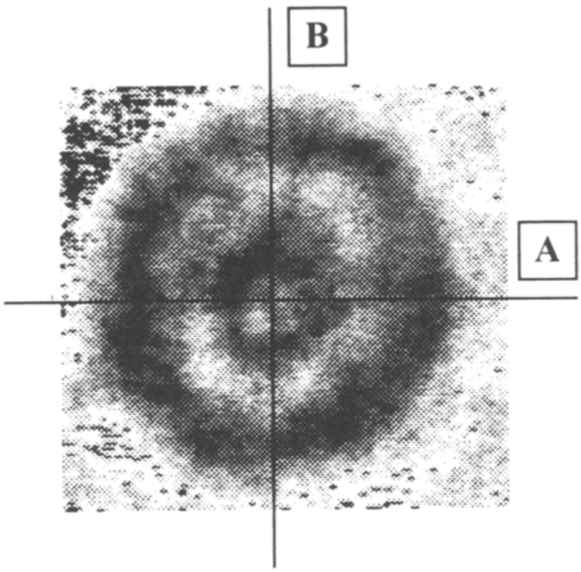

Fig. 4. Grayscale image of the nonlinearly splitted 2D input dark formation from the object arm of the Mach-Zehnder interferometer (Fig. 3). *Lines A* and *B* indicate the directions at which the diametrical phase distributions shown in Figs. 6 and 7 are evaluated

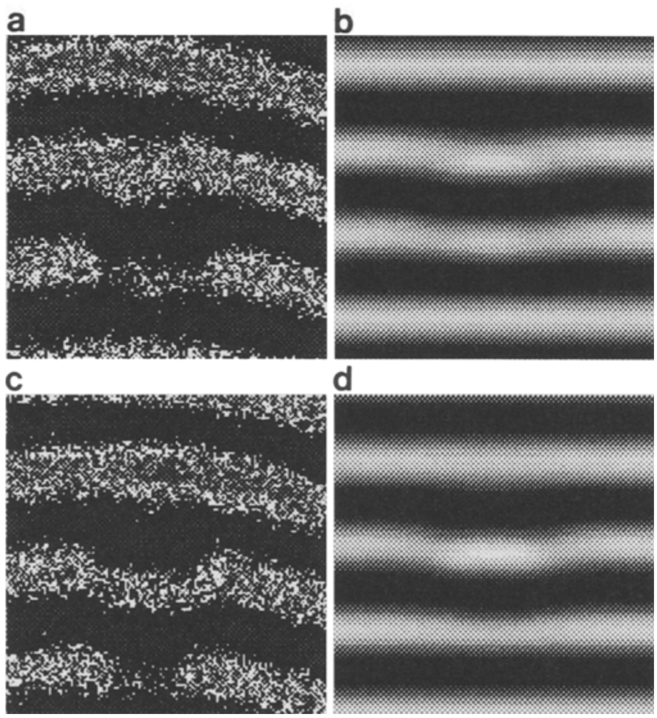

Fig. 5a–d. Two of the four interference pictures [at phase shifts $\pi/2$ (**a**) and $3\pi/2$ (**c**)] used or reconstructing the object 2D phase portrait and the corresponding numerical simulations (**b**) and (**d**) (Fig. 1)

detector *D* (namely, the pattern in the Michelson interferometer). This arrangement , known as four-frame technique, allows to record a set of four interference pictures at known phase shifts (0, $\pi/2$, π, and $3\pi/2$) per exposure and therefore to reconstruct the wavefront in the object arm [12].

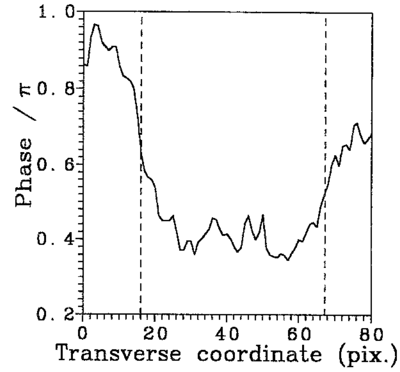

Fig. 6. Diametrical phase distribution of the ring formation from Fig. 4, retrieved from the set of closely spaced interference patterns (Fig. 5a, c)

Figure 4 presents a grayscale image of the nonlinearly split 2D input even dark beam. Two dark rings are clearly seen, in contrast to the numerical simulations on Fig. 1a, restricted to five nonlinear lengths because of the abovementioned reasons, where a single ring is present. Nevertheless, qualitatively similar behaviour of the interference line-structure recorded by a CCD-camera (Figs. 5a, c) and the numerically modelled (Figs. 5b, d) pattern could be observed. The interference lines are distorted, which is an indication (Figs. 2) for the existence of phase shifts in the object arm of the interferometer. It was proven experimentally that those phase changes have their origin in the nonlinear evolution of the input 2D even dark beam.

Applying this four-frame technique for interferogram analysis [12], we reconstructed the phase portrait of the ring formation from Fig. 4. Most easily and correctly the phase distribution is evaluated along a diametrical slice of the rings, parallel to the interference lines (i.e., at a nearly constant phase background). Figure 6 plots this phase distribution (in units of π) expanded over approximately 64 pixels of the CCD array used (512×512 pixels). These shifts of about 0.4π are clearly seen across the outer ring (marked by vertical dashed lines). Phase changes with a reduced contrast could be observed across the inner ring, but it was difficult to determine their position and magnitude. In our opinion, this problem arises primarily due to the curvature of the interference-line set used (Figs. 5a, c). We repeated the procedure with other sets of interference lines. The most representative result (Figs. 7a, b) was obtained by shifting a single broad (about three times the outer-ring diameter) interference line. The transverse phase portrait of this line and the phase changes, due to the presence of the dark-ring structure are visualized on Fig. 7a. Changing the CCD-camera position, the transverse resolution was enhanced about twice as compared to that on Fig. 7a. Phase shifts of about $\pi/2$ are reconstructed (Fig. 7b) within the outer ring (marked by long-dashed lines). Within the inner ring (marked by short-dashed lines) the phase was found to change itself in $\pi/5$ of magnitude. These values agree qualitatively to the diametrical π-jump at the OVS's centre. It should be mentioned, that the 1D even dark formation splits into pairs of 1D odd dark beams (each one with its own phase shift) and the soliton constant (the square of the product of

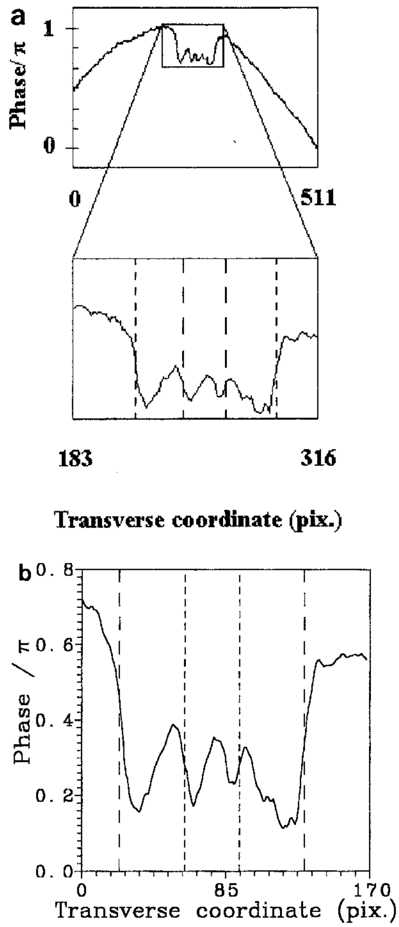

Fig. 7a, b. Diametrical phase distribution of the ring formation, retrieved from a transverse slice of a single broad phase-delaying interference line (see text). *Vertical dashed lines* indicate the positions of the outer and inner dark ring intensity minima

the width and the peak dark irradiance) conserves for each beam of the pair. In the 2D case, however, the splitting of the even dark formation results in a diverging "gray" ring of a decreasing soliton contrast [10, 11]. Therefore, a complicated longitudinal evolution of the 2D phase distribution could be expected.

3 Conclusion

In conclusion, we would like to note, that the observed phase shifts across nonlinearly split 2D even dark beams should be regarded as a confirmation of the soliton nature of the ring dark solitons. In order to retain a terminological similarity to the 1D case, these formations could be called also 2D even dark spatial solitons.

Acknowledgements. The authors would like to thank to U. Domiaty and D. Gruber for the useful collaboration. One of us (A.D.) thanks the members of the Institut für Experimentalphysik, Technische Universität Graz, where the experiment was done, for their warm hospitality. This work was supported in part by the National Science Fund, Bulgaria, under contract # F-206.

References

1. See, for example, Yu. Kivshar: IEEE J. QE-**29**, 250 (1993) and references therein
2. G. Allan, S. Skinner, D. Andersen, A. Smirl: Opt. Lett. **16**, 156 (1991)
3. G. Swartzlander Jr., D. Andersen, J. Regan, H. Yin, A. Kaplan: Phys. Rev. Lett. **66**, 1583 (1991)
   D. Andersen, D. Hooton, G. Swartzlander Jr., A. Kaplan: Opt. Lett. **15**, 783 (1990)
4. W. Tomlinson, R. Hawkins, A. Weiner, J. Heritage, R. Thurston: J. Opt. Soc. Am. B **6**, 329 (1989)
5. P. Mamyshev, Ch. Bosshard, G. Stegeman: J. Opt. Soc. Am. B **11**, 1254 (1994)
6. G. Swartzlander Jr., C. Law: Phys. Rev. Lett. **69**, 2506 (1992)
7. C. Law, G. Swartzlander Jr.: Opt. Lett. **18**, 586 (1993)
8. P. Coullet, L. Gil, F. Rocca: Opt. Commun. **73**, 403 (1989)
9. B. Luther-Davies, R. Powles, V. Tikhonenko: Opt. Lett. (1995) (in press)
10. Yu. Kivshar, X. Yang: Phys. Rev. E **50**, R40 (1994)
11. Yu. Kivshar, X. Yang: Chaos, Solitons, Fractals **4** (1994)
12. C. Creath: In *Temporal Phase Measurement Methods in Interferogram Analysis*, ed. by D. Robinson, G. Reid (Institute of Physics, Bristol 1993) pp. 94–140