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Manipulation of the Transverse Dynamics of Ring Dark Solitary Waves

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Abstract

The spatial dynamics of ring dark solitary waves is studied numerically in order to determine the appropriate initial conditions for an effective control of their transverse velocity. We show that the evolution of the solitary-wave parameters radius, ring width and contrast can be controlled by increasing the ring radius, by an initial phase modulation of the background beam, and as a result of a nonlinear interaction with a second dark formation.

1. Introduction

Spatial optical solitons form a specific class of beams for which the diffraction is compensated by the beam's spatial self-action. Dark spatial solitons could be characterized as localized intensity-dips superimposed upon intense background beams and propagating in self-defocusing nonlinear media. The existence of ring dark solitary waves (RDSW), namely dark solitary waves of circular symmetry, is predicted in [1, 2]. Each diametrical slice of the RDSW represents two intensity-dips of hyperbolic-tangent profiles spaced at twice the dark ring radius. Characteristic for the phase-distribution of the RDSWs is the presence of a pair of opposite abrupt π-phase shifts localized at the zero-intensity positions across the ring. This two-dimensional dark wave will be denoted further as "black" RDSW in contrast to the "gray" one, for which the modulation depth is less than unity and the phase-shift is less than π, respectively. Similarly to the one-dimensional and quasi-2D experiments described in the Refs. [3, 4], 2D gray RDSWs are generated experimentally [5] at a pure amplitude modulation at the entrance of the nonlinear media. Their characteristic phase-singularities are recorded experimentally [6] at the exit of the nonlinear medium.

As shown in [1, 2, 5], the RDSW does obey its inherent dynamics changing slowly its radius, width and contrast. Probably because of that reason the interest of the researchers is directed mainly toward the optical vortex solitons [7]. In this paper we study the spatial evolution of RDSWs in respect to a minimum change of their parameters. The proposed experimental conditions for a controllable manipulation of the transverse velocity makes them attractive as short-distance guiding/switching structures for multiple parallel optical information channels in bulk nonlinear media.

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2. Numerical analysis

The longitudinal nonlinear 2D evolution of the dark beams is simulated numerically by solving the (2 + 1) dimensional nonlinear Schrödinger equation [1, 2]:

$$\mathrm{i}\,\frac{\partial E}{\partial z} + \beta\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E + kn_2\,|E|^2E = 0, \tag{1}$$

where the term comprising $\beta = (2k)^{-1}$ describes the beam diffraction and $n_2\,|E|^2$ is the intensity-dependent medium refractive-index correction leading to a background-beam self-defocusing at $n_2 < 0$. Finite-extent background beams of super-Gaussian profiles

$$B(r) = \exp\{-[r/15r_0]^{16}\}; \quad r = (x^2 + y^2)^{1/2} \tag{2}$$

and widths 15 times larger than this of the dark formations superimposed (r_0), and of amplitudes A_0 are considered. The pure amplitude 2D modulation, evolving in a gray RDSW along the nonlinear medium, is described [6] by

$$E_G(r, \phi, z = 0) = A_0\,B(r)\tanh(r/r_0). \tag{3}$$

The initial condition corresponding to a black RDSW imposed on a background-beam $B(r)$ has the form

$$E_B(r, \phi, z = 0) = A_0\,B(r)\tanh[(r - R_0)/r_0]\exp(\mathrm{i}\Phi), \tag{4a}$$

where R_0 is the dark ring radius. The initial phase distribution Φ of the black RDSW is modelled [5] as followed:

$$\begin{aligned}\Phi(z = 0, \phi) &= 0 \quad \text{at } r \leqslant R_0,\\ \Phi(z = 0, \phi) &= \pi \quad \text{at } r > R_0.\end{aligned} \tag{4b}$$

Grayscale images of the intensity and phase distribution of a black RDSW formed on a super-Gaussian background-beam are shown in Fig. 1a and Fig. 1b, respectively. A transverse grid consisting of 512 × 512 points was used in the simulations. The numerical procedure used for solving eq. (1) is a 2D generalization of the beam-propagation method. Further, the nonlinear propagation path-length will be expressed in units of nonlinear lengths $L_{NL} = (k\,|n_2|\,A_0^2)^{-1}$. During the calculation $A_0 = 1$ and $r_0 = 1$ are

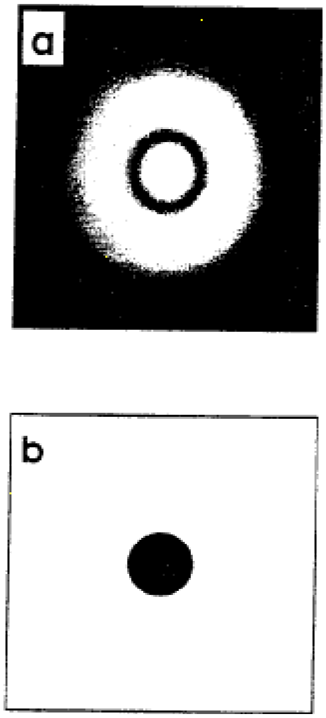

Fig. 1. Grayscale plots for the intensity (a) and phase (b) distribution of a black RDSW (White denotes background intensity equal to unity and phase equal to π).

assumed, i.e. the Rayleigh diffraction length $L_{\mathrm{Diff}} = L_{\mathrm{NL}}(L_{\mathrm{Diff}} = kr_0^2)$.

2.1. *Choice of a suitable RDSW radius and width*

Figure 2 shows the initial (at $z = 0$) diametrical intensity distribution of a black RDSW (solid curve) and at $z = 4L_{\mathrm{NL}}$ (dashed curve). It is evident, that the dark ring radius $R_0(z)$ increases along the nonlinear propagation path. This increase is accompanied with a 2D redistribution of the "lack of energy" and results in a decreasing contrast of the RDSW. The nonzero transverse velocity of the black RDSW could be attributed, in principle, to an interaction between opposite-lying arcs of the ring. This assumption, however, appeared inconsistent with our numerical results. No interaction was observed between one-dimensional black odd dark solitons of tanh-shape at the same spatial offset. One physical reason for the nonzero transverse velocity $W \equiv \mathrm{d}R_0(z)/\mathrm{d}z$ of the black RDSW was found in the finite value of the dark ring radius of curvature. To demonstrate this we consider the situation, at which elliptical tanh-shaped black formations with initial π-phase jumps are imposed on intense background-beams. Except the radii of curvature near the cross-sections, identical initial condition for two slices of the dark elliptical formations vs. background are ensured, when the shorter axis of the first ellipse is chosen to appear as a longer axis of the second ellipse in a separate simulation. The numerical analyses carried out showed, that the reduction of the radius of curvature leads to an enhanced transverse velocity of the axial slice and to a corresponding contrast reduction.

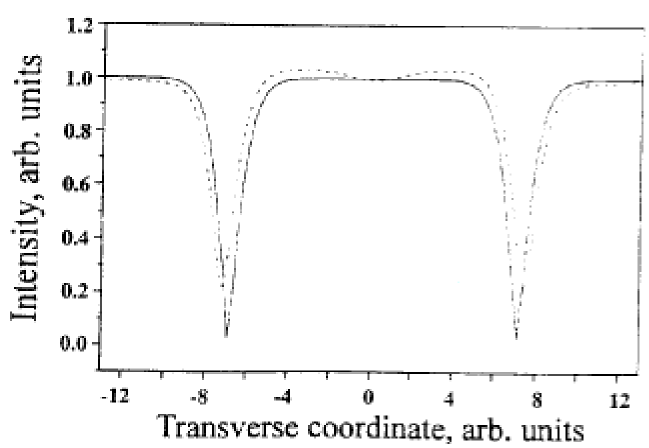

Fig. 2. Diametrical intensity distribution of a RDSW at $z = 0$ (solid curve; $A_0 = 1$, $r_0 = 1$ amd $R_0 = 7$) and $z = 4L_{\mathrm{NL}}$ (dashed curve).

The guiding properties of optical vortex solitons are already demonstrated experimentally [7]. The nonzero transverse velocity and the associated contrast reduction seem undesired for a practically applicable all-optical guiding device based on RDSWs. We tried to estimate numerically conditions, under which the transverse velocity W can be reduced. As a first step we compared the black and gray RDSWs (the last one generated from even initial conditions; see eq. (3)). In order to ensure equal ring radii R_0 and widths r_0 first the formation of a gray RDSW from a pure amplitude modulation is simulated (Fig. 3a, dash-dotted curve) and, thereafter, the initial profile of the black RDSW (Fig. 3a, solid curve) was generated. The dependence of the transverse velocity $W \equiv \mathrm{d}R_0(z)/\mathrm{d}z$ on the nonlinear propagation path z, in units of L_{NL}, is plotted on Fig. 3b. The dash-dotted curve refers to the gray RDSW generated from a pure amplitude modulation at the entrance of the nonlinear medium. In view of the large values of W and the reduced contrast of this formation (see Fig. 3a), gray RDSWs does not seem attractive for practical guiding devices. The dashed line represents the result from a linear evolution ($A_0 \ll 1$) of the odd ring formation (eq. (4)). The transverse velocity W remains relatively small, but the background spreading tends to wash-out the dark formation. The upper solid curve in Fig. 3b refers to the evolution of the black RDSW, whereas the lower one represents the analytical result from the adiabatic approximation of the perturbation theory [1, 2] (namely eq. (6) from Ref. [1]). Physically, this approximation holds at larger values of $R_0(z = 0)$, which was confirmed numerically during the simulations. As a first intermediate result it is worth to note, that a reduced transverse spreading of RDSWs should be expected for initially black RDSW at larger dark ring radii R_0 and smaller widths r_0. Physical limitations on r_0 are posed by the maximum laser intensity available and by the intensity required $I \sim 1/r_0^2$ (at $R \to \infty$ the quantity $Ir_0^2 = 1$ is the soliton constant for 1D odd dark soliton stripe [3, 4]). The increase of R_0 is limited by the necessity to avoid the undesired ring-to-background interaction.

2.2. *Initial phase modulation of the background beam*

The first indication, that a suitable initial phase modulation could force the dark ring first to collapse and, thereafter, to start diverging, could be found in [1, 2]. An optical element able to ensure the phase modulation considered in this work

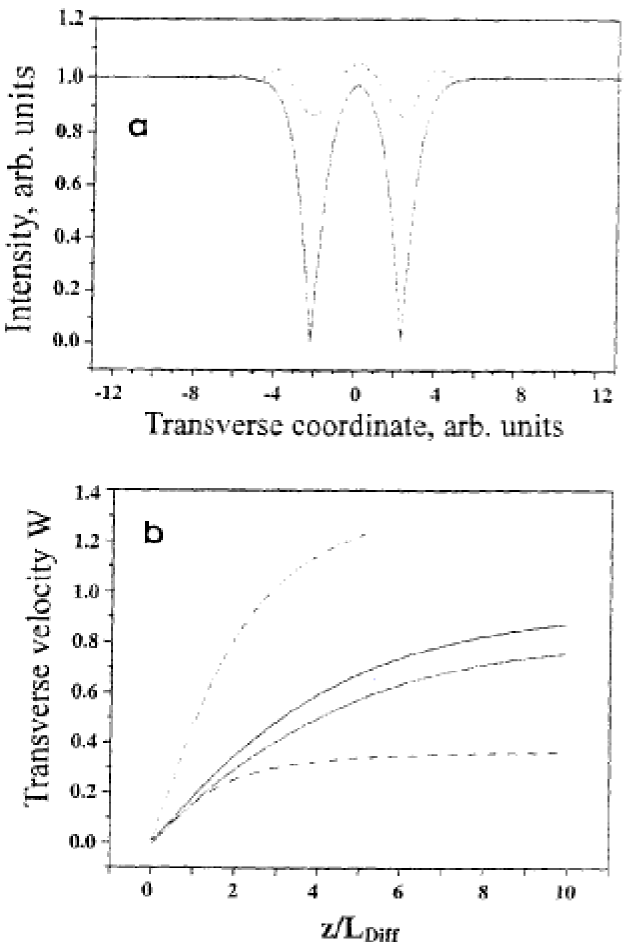

Fig. 3. (a) Radial intensity-profiles of the gray (dash-dotted curve) and of the black RDSW (solid curve) used as initial intensity distributions for evaluating the transverse velocity W (see (b)). (b) RDSW transverse velocity W vs. normalized propagation distance z/L_{NL} (dashed curve-linear mode of propagation, dash-dotted curve – gray RDSW, upper solid curve – present results on the evolution of a black RDSW and lower solid curve – analytical results from Ref. [1])

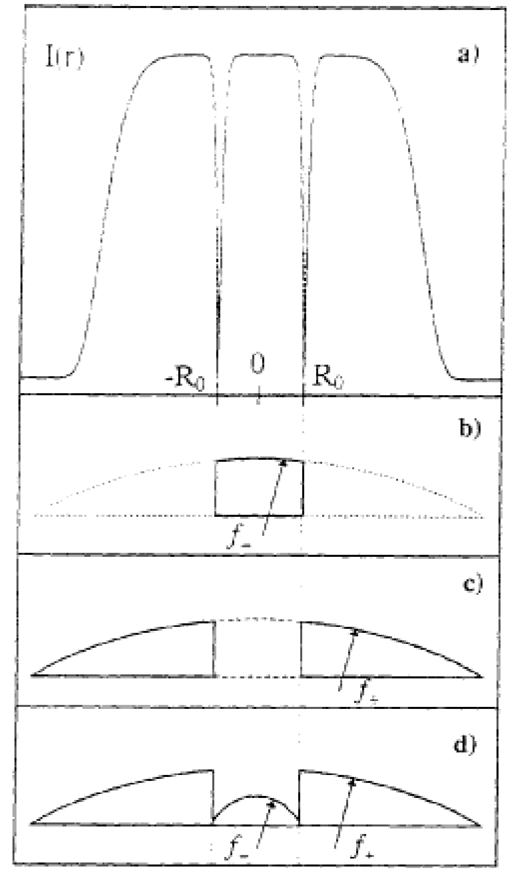

Fig. 4. One-dimensional plot of a RDSW imposed on a finite-extent background (a). The schemes of spherical lenses should illustrate qualitatively the initial phase modulation inside (b) and outside (c) the RDSW, as well as the independent phase modulation on both sides (d).

is the axicone of an aperture wider than the background beam. Much wider possibilities however does exist. The phase-jumps at the intensity dips of the RDSW allows, in principle, to phase modulate independently the background in- and/or outside of the ring. Figure 4 is intended to provide a qualitative idea on the phase modulation out- and inside (Fig. 4b,c) of the dark ring (Fig. 4a). Figure 4d illustrates a simultaneous initial phase modulation on both sides of the RDSW. The spherical lenses plotted should transform a part of the incoming plane wavefront into the concave one desired to manipulate the RDSW transverse velocity. Of course it does not seem reasonable to use such lenses prepared from an optical material. Much easier seems to reproduce suitably designed computer-generated holograms as done for optical vortex solitons [8, 9, 10]. Although the evolution of the dark ring is due to an interplay between 2D diffraction and self-phase modulation under self-defocusing conditions, a simplified paraxial analysis based on the matrix-optics formalism was found to be useful in estimating the approximate focal lengths of the effective lenses shown in Fig. 4 (i.e. the radii of curvature of the background-beam phase distribution). The compensation for the background spreading was found to require an initial spatial phase modulation outside the ring, equivalent to that produced by a hollow lens of a focal length

$$f_+ = R_{BG} \bigg/ \left(\frac{dR_{BG}}{dz}\right), \tag{5a}$$

where R_{BG} is the background-beam radius (twice larger than R_0 and exceeding 15 times r_0 in the cases considered). If the same is to be done by modulating the background inside the dark ring only,

$$f_- = R_0/[1 + \cos^2(\phi(z=0))]^{1/2} \tag{5b}$$

is required, were ϕ is the initial RDSW divergence angle associated with its contrast [1]. Equation (5b) is based on the analytical result of Kivshar and Yang (eq. (6) in [1]) derived under an adiabatic approximation of the perturbation theory for dark solitons. The results from eqs. (5a, b) were used in the first step of the analysis based on solving eq. (1).

Figure 5 shows the evolution of the dark ring radius R_0 normalized to its initial value at the entrance of the nonlinear medium up to $10L_{NL}$. The background is phase modulated inside the ring only and f_-/R_0 is used as a parameter. The upper curve refers to an unperturbed RDSW. Let us assume, that 5% relative change in R_0 is reasonable for

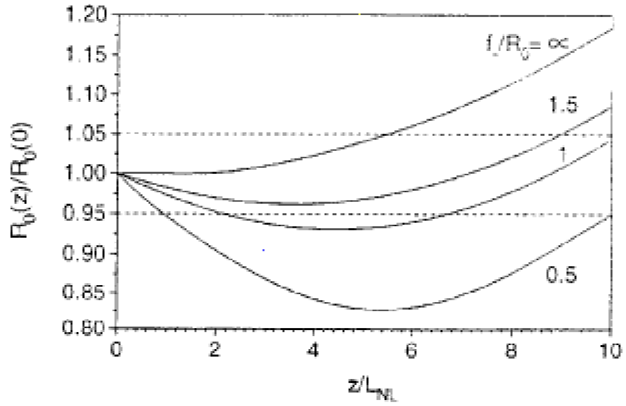

Fig. 5. Phase modulation inside the dark ring: Relative change of the RDSW radius R_0 along the nonlinear propagation path in units of nonlinear lengths. Dashed lines – $\pm 5\%$ limitation. The ratio f_-/R_0 is used as a parameter.

practical reasons and 10% is the highest limit acceptable. One can see that the nonmodulated RDSW reaches the 5%-limit approximately at an 1.6 times shorter propagation distance as compared to the ring at $f_-/R_0 = 1.5$. By breaking the 5% limit in the ring collapse the RDSW of $f_-/R_0 = 1$ broadens with 5% at a distance twice larger than this corresponding to the nonperturbed formation. Qualitatively similar results were obtained for the situation, at which the background outside the ring is modulated only (Fig. 6). Phase modulations of effective focal lengths, $f_+ < 2R_0$ are unacceptable because of the 5% limit posed in this analysis. Nevertheless, within this limit the propagation distance with a reduced transverse spreading could be extended 2.1 times at $f_+/R_0 = 2$ as compared to the unmodulated case (Fig. 6, upper curve). Qualitatively, the improvement of the RDSW contrast associated with the controllable influenced dynamics of the dark ring radius is not different. From a practical point of view, however, the modulation outside the ring seems preferable, since the limited resolution of the eventual computer-synthesized hologram will allow a limited number of coaxial interference fringes to be nested in the ring structure.

It could appear reasonable to modulate the phase in- and outside the RDSW in such a way (see Fig. 4d) forcing first the ring to collapse mainly due to the initial phase modulation inside. The increasing of R_0 after this dark-beam "ring waist" could be suppressed by the comparable weaker background phase modulation outside the RDSW thus resulting in a region of a propagation distances, at which R_0 remains practically constant. The results from this approach are presented on Fig. 7. In the both cases considered the extension of the region of propagation distances, for which the 5% limits is kept, is approximately 1.8 times the corresponding distance for the nonperturbed RDSW. After the initial decrease in R_0 one can clearly observe an extended dark ring "beam waist". Relative changes of less than 2% in the initial value of R_0 were estimated over $4.5L_{NL}$ for $f_-/R_0 = 3.3$ and $f_-/R_0 = 6.6$ (dashed line) and over $5.2L_{NL}$ (solid) for $f_-/R_0 = 2.5$ and $f_+/R_0 = 6.6$. The changes in the RDSW width and contrast are found to be correspondingly negligible. In our view this approach could be a possible one for writing 2D RD solitary guiding structures in bulk photorefractive materials [11]. The estimation of the optimal initial phase variation of the background (linear, quadratic or other type) will be a subject of further analyses.

2.3. *Interaction with coaxial 2D dark structures*

We were motivated to analyze numerically this configuration by the fact, that the interaction potential between 1D dark optical solitons is a repulsive one [12]. Therefore, two coaxial dark rings, each one of them separately representing a RDSW (see eq. (4a)), should repel each other when nested in a common background beam. As a consequence the inner dark ring should be forced to collapse initially, whereas the outer one should be expected to propagate with an enhanced tranverse velocity. This configuration could not be regarded as a specific case of an interaction-suppression [13], since the outer ring dark wave serves as an "idler" one. The initial phase π-jumps at the intensity minima of the rings could be only opposite ones (see eq. (4b)).

Figure 8 plots the relative change of the radius R_0 of the inner RDSW vs. normalized propagation path at an initial outer ring radius $R'_0 = R_0 + \Delta$ (see eq. (4a)). As expected, the decrease of Δ leads to an increased overlapping of the formations and, therefore, to an enhanced tránsverse dynamics of the inner dark ring. For comparison, the upper curve represents the evolution of the nonperturbed RDSW. Interpolating the numerical data it was found, that within a $\pm 2.5\%$ limit of deviation in $R_0(z)/R_0(z = 0)$ the inner dark ring propagates over more than $9.6L_{NL}$. The same quantity estimated within the $+5\%$ and -10% limit is $13L_{NL}$. In

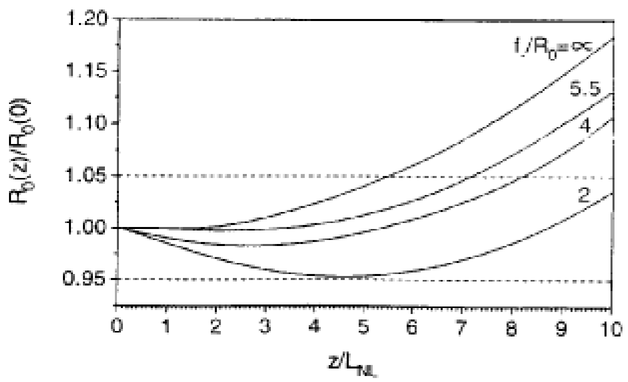

Fig. 6. The same as in Fig. 5 for a phase modulation outside the ring with f_+/R_0 as a parameter

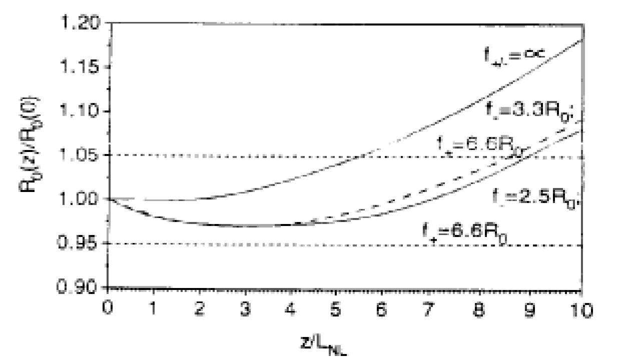

Fig. 7. Evolution of the RDSW radius influenced by a simultaneous phase modulation in- and outside the RDSW. Both effective focal lengths are denoted. Dashed line – $\pm 5\%$ deviation interval.

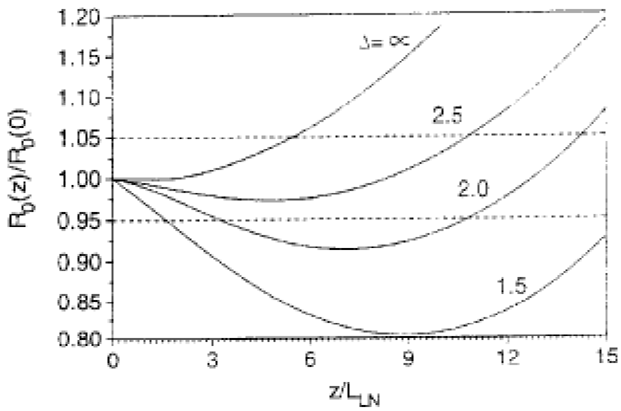

Fig. 8. Reduction of the RDSW transverse dynamics by an outer on-axis dark ring (Δ – radial ring-separation; dashed lines – ±5% limit considered).

view of the above results this all-optically induced suppression of the transverse dynamics of the RDSW seems promising and we will revisit this point again.

The second scheme considered was not related to a suppression of the RDSW transverse velocity W. In contrary, we imposed an optical vortex soliton (OVS) as the center of the dark ring. In this case repulsive interaction was found to enhance W. Because of the geometry considered the radial offset Δ in this case was equal to R_0. Since we considered a fundamental OVS upon a background of an unit intensity, r_0 was set to 1.19 (corresponding to the OVS constant $Ir_0^2 = \sqrt{2}$). At $\Delta = 1.5$ we generated numerically the upper dashed curve plotted in Fig. 9. As seen, the transverse velocity is enhanced as compared to the nonperturbed case (Fig. 9, solid line). It should be pointed out, that this nonperturbed curve differs significantly from the similar curves plotted in Figs. 5–8, since it corresponds to a 6 times smaller value of R_0 and, thus, the ring transverse velocity is higher. Propagation at $10L_{NL}$ already ensures an induced deflection of the RDSW of more than one ring width r_0. Therefore, these two spatial positions of the ring are clearly resolved in space. The lower dashed curve represents the RDSW evolution when influenced by the coproragation of an outlying dark odd ring. The particular curve corresponds to $\Delta = 2$ (see Fig. 8). Once again, the distance of $10L_{NL}$ seems long enough to allow spatial resolution between the nonperturbed and the inner ring in the two-ring mode of propagation. No dependence on the topological charge was found as in [14]. In view of the above one can conclude, that the manipulation of the transverse dynamics of a RDSW could allow to construct a radial switch. Potentially, if multiple information channels are guided by the RDSW at a short distance, a radial switching of parallel optical channels seems realizable. The time needed to rewrite the guiding structure in a photoreactive material (in a quasi steady-state regime) is of the order of 10 ms or less [15]. Therefore, such a device could appear applicable if streams of parallel information are to be redirected relatively rarely. The high transmission capacity of the device due to the parallel switching mode, however, seems attractive. The switching time needed allows to use a programmable liquid-crystal modulator, the diffraction from which ensuring the desired initial profiles of both the OVS and idler outer dark ring [16].

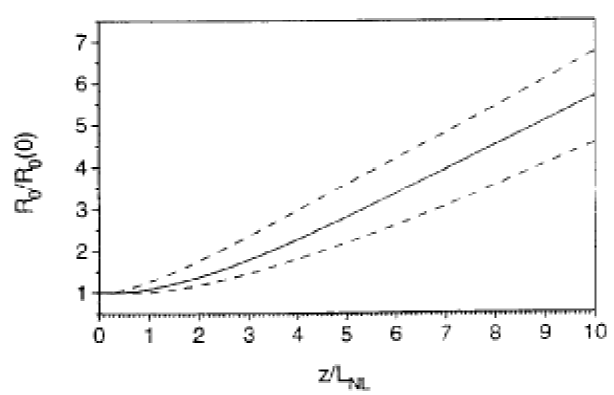

Fig. 9. Induced switching of a RDSW (solid line) by an optical vortex soliton (upper dashed line) and by a second outlying dark ring formation (lower dashed curve). See the text for details.

3. Conclusion

In this work we analyzed numerically different possible schemes for a controllable manipulation of the transverse dynamics of ring dark solitary waves. Phase modulation of the background beam in- and outside the dark ring seems preferable when the changes in the ring radius should be kept at minimum. The interaction with a coaxial OVS or with a second dark ring of a larger radius could allow to construct a practically feasible parallel all-optical radial switch. Phase measurments on the generation and reconstruction of suitably designed computer-generated holograms are in progress.

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