Induced waveguiding in a medium with cubic nonlinearity

S. G. Dinev, A. A. Dreischuh, and A. M. Naidenov

Department of Physics, Sofia University, 5 A. Ivanov Boulevard, 1126-Sofia, Bulgaria

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The variational approach and the potential-curve method are used to study the effect of waveguiding of a probe pulse by an external pulse in a nonlinear medium. The initial delay and the group-velocity mismatch are involved in the analyses without an aberrationless approximation. An analytical expression is obtained for the critical power for induced waveguiding and focusing by an increase in the pump-beam radius with respect to that of the probe beam.

INTRODUCTION

When a weak probe pulse copropagates with a pump, the spectral, temporal, and spatial properties of the probe pulse can be affected by the time variation of the nonlinear refractive index originating from the pump pulse.[1,2] The process is defined to be induced phase modulation (IPM) and is of fundamental importance, since it can be involved in a number of nonlinear interactions.[3-7] Induced focusing (IF) occurs on account of the radial change of the refractive index, induced by an intense pump beam.[5] The effect has been studied[8] in a weak-signal-pulse approximation that neglects the group-velocity dispersion, self-steepening, and the group-velocity mismatch. The induced focusing in self-defocusing nonlinear media has been discussed[9] as a result of cross-phase modulation. Planar waveguides and off-axis beams are required for focusing one of the beams onto a particular point.

In this paper we study the induced waveguiding (IW) and the IF in the case of comparable pulse durations (i.e., when the mutual disposition becomes important) and different ratios γ betweeen the pump- and signal-beam radii. Making use of the variational approach for solving the Schrödinger equation[10] and the analogy with a particle in a potential well, we derive an analytical expression for the critical power/intensity for IF as a function of γ.

THEORETICAL MODEL

The equations describing the influence of the IPM on the spatial characteristics of signal and pump pulses, according to the slowly varying envelope approximation, are[8]

$$\nabla_T^2 A_P - 2ik_P(\partial A_P/\partial z) + (k_P^2/n_{0P}) \times [n_2^{\mathrm{SPM}}(\nu_P)|A_P|^2 A_P + n_2^{\mathrm{IPM}}(\nu_P)|A_S|^2 A_P] = 0, \quad (1a)$$

$$\nabla_T^2 A_S - 2ik_S(\partial A_S/\partial z) + (k_S^2/n_{0S}) \times [n_2^{\mathrm{SPM}}(\nu_S)|A_S|^2 A_S + n_2^{\mathrm{IPM}}(\nu_S)|A_P|^2 A_S] = 0, \quad (1b)$$

where A_S and A_P are the amplitudes of the signal beam and the pump beam, respectively, ∇_T^2 is the transverse coordinate of the Laplacian, and SPM denotes self-phase modulation.

In a probe-pulse approximation $|A_S|^2 \ll |A_P|^2$, when the sum of the pump frequency ν_P and the signal frequency ν_S is near a two-photon resonance in a nonlinear medium (e.g., inert gas), the relation $n_2^{\mathrm{SPM}} \ll n_2^{\mathrm{IPM}}$ holds. It is convenient to introduce the notation

$$\alpha = -1/2k_S, \qquad k^{\mathrm{IPM}}(\nu_S) = -n_2^{\mathrm{IPM}}(\nu_S)k_S/2n_{0S}.$$

Under these assumptions Eqs. 1(a) and 1(b) are transformed into

$$i(\partial A_P/\partial z) - (1/2k_P)\nabla_T^2 A_P = 0, \quad (2a)$$

$$i(\partial A_S/\partial z) + \alpha\nabla_T^2 A_S + k^{\mathrm{IPM}}(\nu_S)|A_P|^2 A_S = 0. \quad (2b)$$

The analyses are made in a coordinate system connected with the signal that uses the trial functions

$$A_P(r, z, \tau) = \frac{A_{0P}}{\omega_P(z)}\exp\left[-\frac{(\tau - \tau_D + z\nu_{SP})^2}{2\tau_P^2}\right] \times \exp\left[-\frac{r^2}{a_P^2\omega_P^2(z)} - i\frac{k_P\rho_P(z)r^2}{2}\right], \quad (3a)$$

$$A_S(r, z, \tau) = \frac{A_{0S}}{\omega_S(z)}\exp\left(-\frac{\tau^2}{2\tau_S^2}\right) \times \exp\left[-\frac{r^2}{a_S^2\omega_S^2(z)} - i\frac{k_S\rho_S(z)r^2}{2}\right]. \quad (3b)$$

The initial conditions are $\omega_{S,P}(z = 0) = 1$ and $\rho_{S,P}(z = 0) = 0$, where ω_I ($I = S, P$) is the normalized radius of the beam; ρ_I is a function of the inverse value of the radius of curvature, τ_D is the initial delay between the pulses, $\nu_{SP} = |1/V_{gr}^S - 1/V_{gr}^P|$ is the group-velocity mismatch, and τ_I is the respective pulse duration measured at the $1/e$ level. Under the assumptions considered, there is no self-action and induced action on the pump. Thus, following Ref. 10 and taking into account the initial conditions, we obtain

$$\omega_P(z) = \left\{\frac{z^2}{L_P^2} + \left[1 + \frac{z}{a_{0P}}\left(\frac{da_P}{dz}\right)_{z=0}\right]^2\right\}^{1/2}, \quad (4a)$$

$$\rho_P(z) = \frac{1}{\omega_P(z)}\left(\frac{d\omega_P}{dz}\right). \quad (4b)$$

In Eq. (4a), $L_P = k_P a_{0P}^2/2$ is the Rayleigh diffraction length of the pump beam.

Equation (2b) above describes the spatial evolution of the probe pulse in the nonlinear medium. It can be re-

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duced to a system of ordinary differential equations for ω_S and ρ_S by the use of the variational approach.[11] It is easy to prove that Eq. (2b) is an Euler–Lagrange equation for the Lagrangian

$$L = \left(\frac{i}{2}\right)\left(A_S\frac{\partial A_S^*}{\partial z} - A_S^*\frac{\partial A_S}{\partial z}\right) + \alpha\left|\frac{\partial A_S}{\partial r}\right|^2 - k^{\mathrm{IPM}}(\nu_S)|A_S|^2|A_P|^2. \quad (5)$$

The reduced variational problem has the form $\delta\int\langle L\rangle \mathrm{d}z = 0$, where $\langle L\rangle = \int_{-\infty}^{+\infty} L_G\, \mathrm{d}r$ and L_G denotes the result of inserting the trial functions into the Lagrangian. The assumption that the maximum nonlinear effect is induced from the peak of the pump pulse is justified,[3] and we can separate, for convenience, the time dependence in the trial functions:

$$\tilde{A}_{0S} = A_{0S}\exp\left(-\frac{\tau^2}{2\tau_S^2}\right),$$
$$\tilde{A}_{0P} = A_{0P}\exp[-(\tau_D - z\nu_{SP})^2/2\tau_P^2]. \quad (6)$$

Following the variational procedure,[11] we obtain a system of equations for the normalized radius of the signal beam, ω_S, and the inverse radius of curvature of the wavefront, ρ_S:

$$\frac{\mathrm{d}\omega_S}{\mathrm{d}z} = \omega_S\rho_S, \quad (7a)$$

$$\frac{\mathrm{d}\rho_S}{\mathrm{d}z} = -\rho_S^2 + \frac{4}{k_S^2 a_S^4\omega_S^4} - \frac{2n_2^{\mathrm{IPM}}(\nu_S)|\tilde{A}_{P0}|^2 a_P}{n_{0S}\omega_P(a_S^2\omega_S^2 + a_P^2\omega_P^2)^{3/2}}. \quad (7b)$$

Equations (7) are equivalent to the equation

$$\frac{\mathrm{d}^2\omega_S}{\mathrm{d}z^2} = \frac{4}{k_S^2 a_S^4\omega_S^3} - \frac{2n_2^{\mathrm{IPM}}(\nu_S)|\tilde{A}_{P0}|^2 a_P\omega_S}{n_{0S}\omega_P(a_S^2\omega_S^2 + a_P^2\omega_P^2)^{3/2}}. \quad (8)$$

The obtained result takes into account the possibility different pump- and probe-pulse durations τ_S and τ_P, respectively, the eventual initial delay τ_D, and the group-velocity mismatch. This improvement on Manassah's solution[8] does not depend on the approximation used. Quantitatively, at equal pump- and probe-beam radii the nonlinear term in Eq. (8) is $\sqrt{2}$ times lower than that in Eq. (9) of Ref. 8. Therefore an enhancement of ≈40% in the critical power for IF, compared with the aberrationless approximation, is to be expected.

An analogy can be found in the equations describing the evolution of the spatial and temporal characteristics of the probe pulse due to the IPM. Let us interpret α in Eq. (2b) as a parameter that depends on the group-velocity dispersion, i.e.,

$$\alpha \rightarrow \alpha_S = -\frac{1}{2}\left(\frac{\lambda_S^3}{2\pi c^2}\right)\left(\frac{\partial^2 n}{\partial\lambda^2}\right)_{\lambda=\lambda_S},$$

and replace the derivative of the transverse coordinate by a time derivative. The result is an equation describing the time history of the probe pulse.[12] Using the energy-conservation law $|\tilde{A}_{P0}|^2 a_P\omega_P = \mathrm{const}$, we can integrate Eq. (8) once. The expression obtained for $\omega_S(z)$ is analogous to that describing a particle in a potential well:

$$\frac{1}{2}\left(\frac{\mathrm{d}\omega_S}{\mathrm{d}z}\right)^2 + \Pi(\omega_S) = 0, \quad (9a)$$

where

$$\Pi(\omega_S) = \frac{1}{2L_S^2}\left(\frac{1}{\omega_S^2} - 1\right) - \frac{n_2^{\mathrm{IPM}}(\nu_S)k_S|\tilde{A}_{P0}|^2 a_P}{n_{0S}L_S\omega_P} \times \left[\frac{1}{(a_S^2\omega_S^2 + a_P^2\omega_P^2)^{1/2}} - \frac{1}{(a_S^2 + a_P^2\omega_P^2)^{1/2}}\right]. \quad (9b)$$

Assuming that $a_S(z = 0) = a_P(z = 0) = a$ and with the notation

$$A = \frac{1}{2L_S^2}, \qquad B = -\frac{n_2^{\mathrm{IPM}}(\nu_S)k_S|\tilde{A}_{P0}|^2}{n_{0S}L_S\omega_P},$$

the potential function takes the form

$$\Pi(\omega_S) = \frac{A}{\omega_S^2} + \frac{B}{(\omega_S^2 + \omega_P^2)^{1/2}} - \left[A + \frac{B}{(1 + \omega_P^2)^{1/2}}\right]. \quad (10)$$

Under the conditions $\tau_D = 0$ and $\nu_{SP} \approx 0$ and neglecting the pump divergence $[\omega_P(z) \approx 1]$, we can isolate three characteristic regions with respect to B/A:

(1) Linear case (B = 0).
(2) Subcritical nonlinearity $(0 > B/A > -4\sqrt{2})$. The condition $B/A = -4\sqrt{2}$ corresponds to a constant signal-beam radius, i.e., IW.
(3) Region of IF $(B/A < -4\sqrt{2})$.

The long-dashed curve in Fig. 1 indicates the linear potential, the solid curve indicates the potential corresponding to a subcritical nonlinearity, and the short-dashed curve indicates the region of IF. The second region predicts oscillations of the probe-beam radius over values higher than the initial one, while the oscillations in the IF

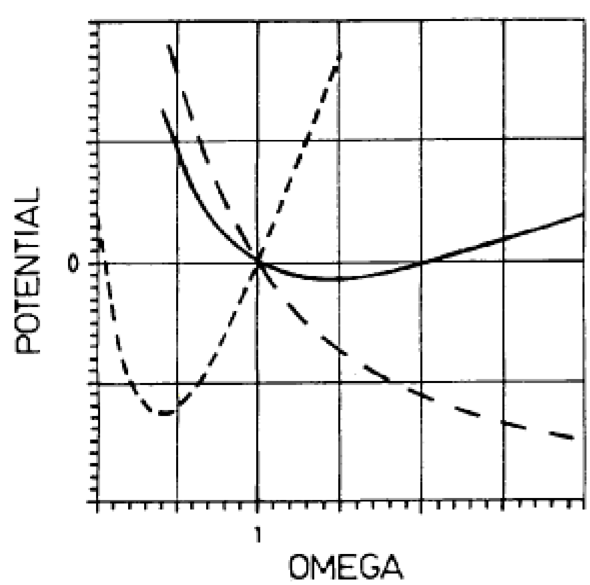

Fig. 1. Qualitative plot of the potential function $\Pi(\omega_S)$ given by Eq. (10) in the linear [$(B = 0)$; long-dashed curve] and subcritical [$(0 > B/A > -4\sqrt{2})$; solid curve] cases and in the region of IF [$(B/A < -4\sqrt{2})$; short-dashed curve].

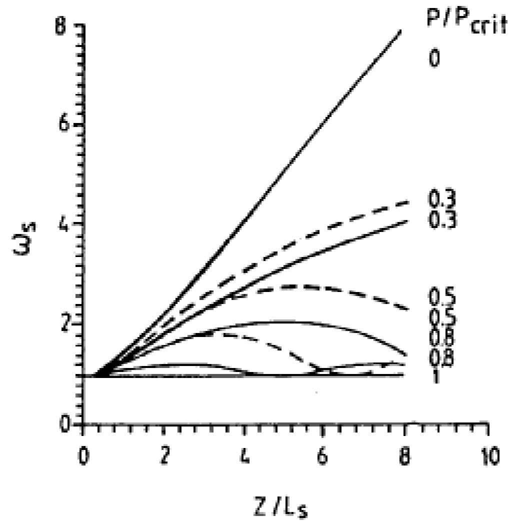

Fig. 2. Normalized signal-beam radius evolution given by Eq. (8) for different values of the parameter $P^{IF}/P^{IF}_{crit}(\gamma = 1)$. The solid curves are for $\gamma = 1$, and the dashed curves are for $\gamma = 2$.

region are for radius values smaller than the initial value. In the latter case an optical breakdown could occur in the medium prior to compensation of the IF by the diffraction divergence. Therefore the signal power should be kept reasonably low compared with the breakdown critical value. Certainly, the division into regions is rather symbolic, since the walk-off effect between the pulses gradually deforms the potential to a linear one.

An expression for the critical power is obtained from the condition $\Pi'(\omega_S = 1) = 0$, expressing the degeneration of the potential into a single point, namely,

$$P^{IF}_{crit} = \frac{\sqrt{2}\, c\lambda_S'^2}{8\pi^2 n_2^{IPM}(\nu_S)}, \tag{11}$$

where $\lambda_S' = \lambda_S n_{0S}$ is the wavelength of the probe pulse in the medium. The value obtained is a factor of $\sqrt{2}$ higher compared with the aberrationless approximation.[10] It has been shown[13] that the values of the critical field for (self-)focusing under that approximation are ≈40% lower than the calculated values from the numerical analyses. Thus the potential model provides a good qualitative, as well as a better quantitative, description of the IF compared with the aberrationless approximation.

If the pump-beam radius is higher than the signal one, the latter will be less influenced by the change in the refractive index over its cross section. Therefore the critical power/intensity for IF will be higher. In the IPM of a picosecond probe pulse from a pump pulse with comparable duration the length of the nonlinear medium is limited by the pulse walk-off. The use of a pump beam with a larger diameter would permit higher pump powers to be used, and consequently a larger chirp bandwidth and eventually a higher compression ratio could be obtained. It should be noted that the effect is not a simple generalization of the self-focusing effect. From $\gamma = a_P(z = 0)/a_S(z = 0) > 1$ the potential $\Pi(\omega_S)$ from Eq. (9b) can be presented in the form

$$\Pi(\omega_S) = \frac{A}{\omega_S^2} + \frac{B\gamma}{(\omega_S^2 + \gamma^2\omega_P^2)^{1/2}} - \left[A + \frac{B\gamma}{(1 + \gamma^2\omega_P^2)^{1/2}}\right]. \tag{12}$$

Assuming again a well-collimated pump beam (which is easy to obtain, since $a_P > a_S$), from the condition for potential degeneration into a point we obtain

$$P^{IW}_{crit} = \frac{(1 + \gamma^2)^{3/2}}{\gamma} \frac{c\lambda_S'^2}{16\pi^2 n_2^{IPM}(\nu_S)}; \tag{13}$$

i.e.,

$$P^{IF} > P^{IW}_{crit} = f(\gamma) P^{IW}_{crit}(\gamma = 1), \tag{14}$$

where

$$f(\gamma) = (1 + \gamma^2)^{3/2}/2\sqrt{2}\,\gamma. \tag{15}$$

Equation (14) shows the increase in the critical power for IF by increasing a_P with respect to a_S. At higher values of the ratio $\gamma = a_P(z = 0)/a_S(z = 0)$ the dependence tends to a quadratic one, and hence a strong increase in P^{IW}_{crit} is expected.

Figure 2 shows the dependence of the normalized radius of the signal beam on medium length (normalized to the Rayleigh length), calculated from Eq. (8). The solid curves are for $\gamma = 1$, and the dashed curves are for $\gamma = 2$, for different values of the parameter $P/P^{IW}_{crit}(\gamma = 1)$. As the figure shows, the IW effect for $\gamma = 2$ is less pronounced for the same pump power. In the particular case in which, simultaneously with the beam collimation, an induced linear chirp and consequently pulse compression are aimed, the increase in the pump-beam diameter is desirable. If the beam is to be transported to larger distances, a larger pump beam will be less influenced by the

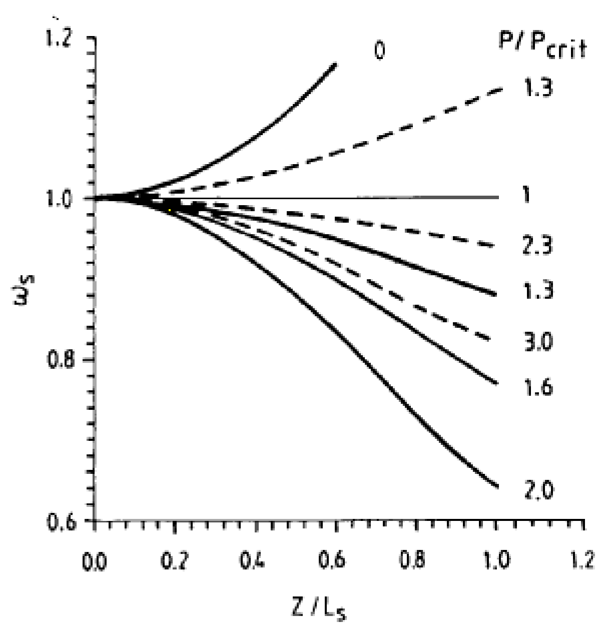

Fig. 3. Normalized signal-beam radius evolution given by Eq. (8) for different values of the parameter $P^{IF}/P^{IF}_{crit}(\gamma = 1)$ in the region of IF for $\gamma = 1$. The solid curves are for $\gamma = 1$, and the dashed curves are for $\gamma = 2$.

diffraction spreading, while longer pump pulses will increase the walk-off distance and hence the length of induced collimation. Figure 3 plots the similar dependence $\omega_S(z/L_S)$ in the region of IF. Again the IW/IF effect is smaller for $\gamma = 2$ (for example, at $P/P_{crit} = 1.3$ in Fig. 3), corresponding to a change in the critical value for IF [see relation (14)]. Equations (7) and (8) above are evaluated by means of fourth-order Runge–Kutta formulas. The accuracy was tested by a comparison of the results of the procedure with single and double increments.

CONCLUSION

We have shown that a focusing, i.e., waveguiding, effect can be induced in a cubic nonlinear medium by an external light field. The variational approach and the potential-curve method are adequate for a description of the induced focusing (IF) and the induced waveguiding (IW) of a probe pulse by a pump pulse. The description is straightforward without the aberrationless approximation. We obtained an analytical expression for the increase in the critical power for IF by increasing the pump-beam radius with respect to that of the probe beam. The effect of IW seems to be useful, especially for pulse compression in the short-wavelength region, for fast switching and guiding devices, for the transporting of laser beams, etc.

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