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Femtosecond optical vortices *

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Abstract.

In this work we review recent results demonstrating the possibility to create spatial phase dislocations in broadband coherent optical fields. We consider optical system utilizing a computer generated hologram combined with spatial dispersion compensation. Analytical results describing three possible arrangements (dispersionless $4f$-setup, $2f-2f$-setup, and double-pass grating compressor scheme) are presented. Experimental data confirming the theoretical predictions are discussed.

1 Introduction

Singularities are points or lines where mathematical quantities become infinite, or change abruptly. Interest in physical singularities can be traced back to the first half of the 19th century to the studies of Hamilton, Whewel and Airy on conical refraction, tides in the ocean, and the rainbow phenomenon [1]. The connection between these early singularity effects and phase dislocations in optics

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was established in 1974, with the pioneering work of Nye and Berry [2]. Nowadays, the studies of phenomena associated with phase singularities are gradually developing as a new branch of optics, called *singular optics* [3].

Phase singularities occur very generally whenever the phase of a complex function depends continuously on two or more variables. The simplest function with a phase singularity is the a from the Cartesian space to the complex plane $\psi(x, y) = x + iy = R\exp(i\varphi)$. Interpreting the radial and azimuthal coordinates R and φ as amplitude and phase of the electric field, the phase is defined everywhere except the beam axis ($x = 0, y = 0$). A very intuitive example is the closed loop east-to-west journey around the world. The international date line introduces an one-dimensional discontinuity of a day's jump along a line joining the two poles, whereas at the poles there are point discontinuities. The presence of phase singularities in the wavefront of a light beam determines its phase and intensity structure. The wave amplitude vanishes at the singularity owing to total destructive interference. Each one-dimensional π-phase dislocation is coupled with a line of zero intensity. An isolated point singularity with a screw-type phase distribution is associated with an optical vortex (OV). The characteristic helical phase profile of an OV is described by $\exp(im\varphi)$ multipliers, where φ is the azimuthal coordinate and the integer number m is its topological charge.

Nonlinear interaction of light with matter is a key for light control and processing in all-optical technologies. To access the material nonlinearities, high light intensities are necessary and this in turn, inevitably requires the use of ultra-short laser pulses. The width of the pulse is inversely proportional to its spectral bandwidth, so that the ultrashort pulses are naturally polychromatic. The challenge in creating spatial phase dislocations in ultrashort pulses is to impose the desired dislocation structure onto all spectral components while keeping the pulse width and shape undistorted [4]. The known methods for generating phase singularities applicable in the cw and quasi-cw regime are not suited for femtosecond lasers. Astigmatic transverse mode converters [5, 6] can not be used directly since they require transverse modes higher than the fundamental TEM_{00}. The suggested approach [7] to prepare a Hermite-Gaussian-like (HG_{01}) mode at the entrance of the converter by splitting a HG_{00} mode and spatially offsetting its two out-of-phase halves is feasible but it requires an additional interferometrically-controlled delay line. Intracavity phase elements [8] and beam rotators [9] are not applicable in the femtosecond regime because of the emitted transverse mode. Transparent spiral wave plates [10, 11] and structured liquid-crystal modulators [12, 13] preserve the beam path, however, the magnitude of the phase jump of the dislocation will deviate from π for the different spectral components of the short pulse and topological dispersion will be present [14]. Glass platelets of a varying thickness providing linear phase retardation on one half of a (cw) laser beam are able to produce optical vortices [15]. However, because of the space-dependent dispersion and time delays this technique can not be applied to ultrashort pulses, either.

2 General approach

A well known and widely used method to generate spatial phase dislocations is the reconstruction of computer-generated holograms (CGHs) [16]. This method can be used to realize screw [17, 18], step [19] and mixed type singularities [20, 21] as well as arrays of such dislocations [22] in the first-order diffracted beams. In order to impose the encoded phase dislocation onto all spectral components of the ultrashort pulse while keeping the pulse undistorted, the CGH has to be aligned as a part of an optical system with compensated spatial dispersion. In the following we describe recent results demonstrating the possibility to create phase singularities (e.g. OVs) in femtosecond optical beams/pulses at the stages of pulse shaping, stretching, and compression.

Without loss of generality, we normalize the electric field amplitude to unity and assume that the spatial profile of the optical field is Gaussian $E \sim \exp[-(x_0^2 + y_0^2)/\sigma_0^2]$, where σ_0 is the beam width at $1/e$-level and the aperture of the CGH is large enough not to cause edge diffraction. The field evolution after passing the CGH is analysed by using the Fresnel integral

$$E(x, y, z = s) = \frac{\exp(iks)}{i\lambda s} \int\int E(x_0, y_0, 0) \exp(\frac{i\pi r^2}{\lambda s}) dx_0 dy_0, \quad (1)$$

and directly behind the diffraction grating/CGH the field has the form

$$E(x_0, y_0, 0) = T(x_0, y_0) \exp[-(x_0^2 + y_0^2)/\sigma_0^2], \quad (2)$$

where $r^2 = (x - x_0)^2 + (y - y_0)^2$, λ is a particular wavelength within the generated spectral bandwidth, $k = 2\pi/\lambda$, and $T(x_0, y_0)$ is the grating transmission function containing the phase profile $\varphi(x_0, y_0)$ of the desired singularity.

3 Results

3.1 Creation of OVs within a dispersionless 4f setup

In order to model the evolution of the electric field inside the 4f-setup (Fig. 1) we use the integral relation between the field distributions in the front and back focal planes of a thin lens which is obtained from the diffraction integral (Eq. 1) accounting for the transmission of the lens of an optical thickness nd and focal length f. Following the evolution of the first-order diffracted wave (from the first grating G; see Fig. 1) to the back focal plane (Fourier plane) of the first lens and, further, to the CGH (output grating of the $4f$ setup) we derived an analytical expression for the electric field amplitude $E'(x, y)$ at the exit of the $4f$-system [23]:

$$E'(x, y) = \frac{C_1 A_1}{(\pi\lambda f)^2} \exp\left[-\frac{x^2 + y^2}{(\beta\sigma_0)^2}\right] \exp[i\varphi(x, y)] \exp\left[i\frac{2\pi}{d}(1 + \frac{1}{\beta})x\right]. \quad (3)$$

The last multiplier in Eq. 3 accounts for the net spatial dispersion at the exit. For a perfect alignment $\beta = -1$ and, the $4f$-system is dispersion-free. Therefore, arbitrary oriented dark beams with phase dislocations generated in each individual spectral component are recombined spatially and temporally to overlap at the exit without any chirp.

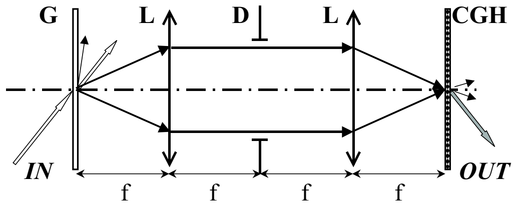

Figure 1. Illustration of the $4f$ setup analyzed theoretically. G, diffraction grating; CGH, computer-generated hologram with an encoded phase singularity; L, lenses of focal length f; D, iris diaphragm.

In our first experiment we used a Ti:sapphire laser emitting nearly transform-limited 20-fs pulses at a central wavelength of $797nm$ and binary CGH produced photolithographically with a stripe period of $d = 30\mu m$. The $4f$-setup used was folded in the Fourier plane by a silver-coated mirror and a single quartz lens with a focal length $f = 20cm$ was aligned carefully to minimize aberrations. The CGH of an optical vortex was positioned in a way to reconstruct the encoded point phase dislocation in the center of the background beam. In the peripheral part of this grating the stripes are parallel. This region was used as an effective second grating to recombine the spectral components at the exit. The graph

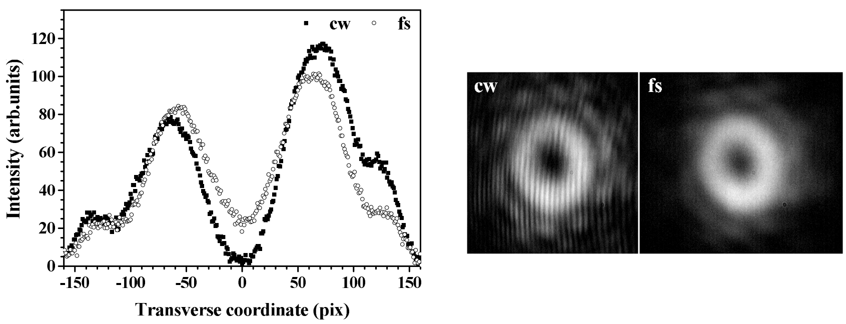

Figure 2. Vertical cross-sections of the OV beams extracted from the recorded images (right frames; $35cm$ after the $4f$ setup) in both cw and femtosecond regime.

in Fig. 2 shows vertical cross-section of the cw- and femtosecond OV beams (solid squares and open circles, respectively) extracted from the experimental greyscale images presented in the right-hand-side of Fig. 2. The contrast of the femtosecond OV beam is improved as compared to the case when no $4f$-system is used and approaches the contrast of the cw OV beam. Further improvement can be achieved by filtering out all parasitic reflections from the CGH substrate.

3.2 Embedding of phase singularities in a 2f-2f setup

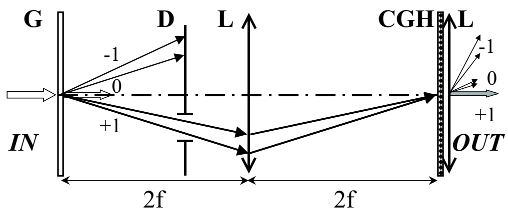

Figure 3. Scheme of the 2f-2f setup used in [24]. The notations are the same as in Fig. 1. Note that the second lens is positioned just behind the CGH (outside the 2f-2f scheme). The CGH is assumed to reproduc e the encoded phase singularity at the exit of the pulse stretcher.

In this case the two gratings are arranged in a setup as shown in Fig. 3. The incoming optical beam/pulse first passes through the first diffraction grating. An off-centre aperture transmits the beam diffracted in $+1$st order only. A converging lens then focuses the optical wave on the exit grating, namely the CGH. This lens with a focal length f is positioned at a distance $2f$ from both gratings. A second lens, identical to the first one, has to be placed immediately behind the CGH in order to ensure a linear magnification of -1 and to preserve the beam (de)collimation. The key result from the developed mathematical model is the expression for the electric field amplitude at the exit of the $2f - 2f$ setup

$$E'(x,y) =\propto \exp\left[-\frac{x^2+y^2}{(\beta\sigma_0)^2}\right] \exp\{-iKx\} \exp\{iN[Kx+\varphi(x,y)]\}, \qquad (4)$$

where $K = 2\pi/d$ is inversely proportional to the grating constant d. This equation shows that the amplitude of the field diffracted in the 'desired' ($N = +1$) diffraction order is free of angular dispersion. In contrast, the spatial chirp of the wave propagating in the 'idler' ($N = -1$) diffraction order is doubled instead of being eliminated. The authors of [24] used pulses of nearly the same duration and central wavelength and CGHs with a stripe period of some $d \approx 50\mu m$. It should be mentioned that they achieved a very good contrast of the experimental data and succeeded to record an interferogram of a femtosecond optical vortex. The 2f-2f system, however, acts as pulse stretcher and therefore further analyses are required to tests its applicability to ultrashort pulses.

3.3 Creation of OVs within a double-pass grating compressor

In this case one of the gratings (either the first or the last one) has to be replaced by a CGH of a period d equal to that of the other grating(s). The corresponding optical system is shown schematically in Fig. 4. Theoretical analysis of this system amounts to evaluation of the diffraction integral between the planes in which the diffraction gratings are located. Plus or minus sign in the phase multipliers in the transmission functions $T(x_1, y_1) = C_1 \exp[i(2\pi/d)x_1]$, $T(x_j, y_j) = C_1 \exp[-i(2\pi/d)x_j]$ (for $j = 2, 3$), and $T(x, y) = C_1 \exp[i(2\pi/d)x] \exp[\varphi(x, y)]$ correspond to the beam propagation in the $+1$ or in the -1 diffraction order behind the respective grating. In this way we derive an analytical expression for the output amplitude of the electric field. It can be written in a compact form [25]:

$$E'(x, y) = C_1^4 E_{diff} \exp\{ik[s - l(\lambda/d)^2]\} \exp[\varphi(x, y)]. \quad (5)$$

Here E_{diff} is the electric field amplitude diffracted in the course of the optical beam propagation in the compressor [accurate to accumulated linear phase $\exp(iks)$; see Eq. 1]. The first phase term in Eq. 5 accounts for the different propagation path lengths (and transit times) of the different spectral components, i.e. for the negative group-velocity dispersion of the grating compressor. The last term contains the phase profile encoded in the CGH.

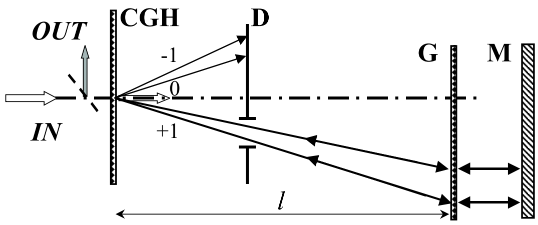

Figure 4. Illustration of the double-pass grating compressor used in [25]. G, diffraction grating; CGH, computer-generated hologram; l, compressor length; M mirror reflecting the beam for the second pass. The CGH is assumed to reproduce the encoded phase singularity at the entrance or at the exit of the compressor.

The behaviour of the phase dislocations carried by broad bandwidth of femtosecond laser pulses is imitated by sets of measurements conducted with a cw laser tuned at different wavelengths. A cw tunable Ti:sapphire laser (Coherent 899-21) and $CGHs$ with a period of $80\mu m$ are used. Taking a real spectrum of an amplified ultrashort pulse and integrating a set of laser beam power density distributions recorded experimentally at discrete wavelengths, we simulated the encoding of phase dislocations in a real double-pass grating compressor. In that sense, this experiment serves as a proof-of-principle. The dotted and solid curves in Fig. 5 represent profiles of the compensated and "idler" output beams simulated in this way. The clearly higher contrast of the optical vortex generated in the “compressor” as compared to the contrast of the “idler” vortex strongly supports the conclusion that OVs can be generated in chirped femtosecond laser beams by using $CGHs$ aligned as a part of double-pass grating compressors.

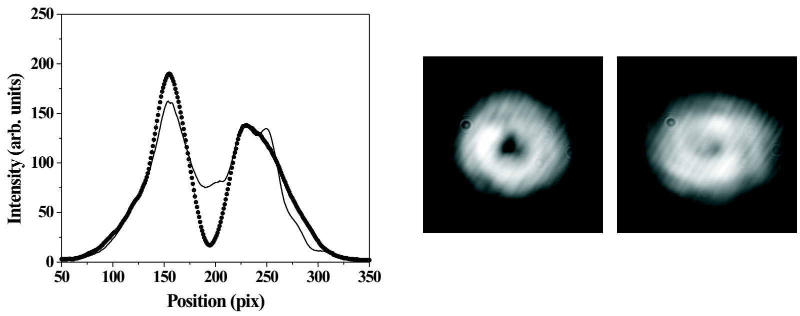

Figure 5. Proof-of-principle experiment and emulation of an optical vortex encoded at the exit of the compressor. Graph: Transverse cross-sections of the uncompensated “idler” (solid) and desired OV (dotted curve). Images: Spectrally integrated experimental frames (compensated OV - left; “idler” OV - right).

4 Conclusions

In this work we have reviewed the recent theoretical and experimental results demonstrating the possibility to create spatial phase dislocations in broadband (e.g. femtosecond) optical fields by using computer-generated holograms. In order to cancel the spatial dispersion introduced the vortices should be generated in either $4f$-setups [23], in $2f - 2f$-setups [24], or in a double-pass grating compressors [25]. The first approach does not affect the width of the ultrashort pulses, while the others will cause pulse stretching/compression. These results open the way to the unexplored field of the interaction of optical pulses carrying orbital angular momentum with matter [26].

Acknowledgments

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