Light Bullets Formation in a Bulk Media

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Abstract—The conditions for the formation of "light bullet" pulse, propagating with nearly constant spatial and temporal parameters in a bulk nonlinear medium, are obtained for the first time. A pump and probe pulse configuration is employed using an induced phase modulation. An analytical balance condition is obtained for the spatial and temporal soliton-like formation.

I. Introduction

When pump and probe pulses copropagate in a nonlinear medium, the refractive index on the probe wavelength can be changed by the powerful pump beam. The phase of the probe beam undergoes spatial and temporal modulation, which alters its temporal and spectral characteristics. The process is known as induced phase modulation (IPM) and is studied intensively in respect to pulse tailoring [1]–[5]. Quite recently, the focusing effect of the IPM on a probe beam [6], focusing in an off-axis geometry in a planar waveguide and defocusing medium [7] as well as induced waveguiding (IW) and in a bulk media [8], have been considered. The self-induced effect of phase modulation (SPM) has lead to the recently proposed "light bullets" pulses that propagate without changing their temporal and spatial profiles [9]. However, due to the inherent instability of the self-focusing effect in a three-dimensional medium, these self-trapped light formations are considered to be realizable in a single space-axis structure, i.e., planar waveguide.

In this paper we analyze what we believe to be the first model of light bullets in a bulk medium. The IPM effect provides a well controllable spatial and temporal trapping of the probe pulse. We derive analytical expressions for the critical parameters of the system, incorporating input power, pulse duration, pump and probe beam radii, medium parameters, etc. The system parameters analyzed are rather general and include varying pump and probe pulse durations and delay. The constant parameters of the light formation are shown to be limited by the walk-off effect.

II. Theoretical Model

The mathematical description of the IPM-induced spatial and temporal soliton is made under the following assumptions:

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i) negligible self-phase modulation;

ii) pump and probe pulse approximation ($I_s << I_p$), i.e., the phase modulation induced by the signal on the pump is negligible;

iii) low efficiency of the accompanying parametric processes, i.e., the pump depletion is small;

iv) the time response of the medium is fast compared to the pulse duration.

The time dependent paraxial wave equation in the presence of group velocity dispersion (GVD) is [10]

$$i\frac{\partial \psi_S}{\partial x} + \alpha \nabla_T^2 \psi_S + \alpha_S \frac{\partial^2 \psi_S}{\partial \tau^2} + k^{\text{IPM}}(\lambda_S)\,|\psi_P|^2 \psi_S = 0 \tag{1}$$

where $\alpha_S = (-1/2)\,[\lambda_S^3/(2\pi c^2)]\,[\partial^2 n/\partial \lambda^2]_{\lambda=\lambda_S}$ is the GVD coefficient, $k^{\text{IPM}}(\lambda_s) = 3\pi^2 N \chi^{(3)}_{\text{IPM}}(\lambda_S)/[\lambda_S n_0^2(\lambda_s)]$ is the nonlinear coefficient for IPM at the signal wavelength λ_S. N is the density of the medium, $\alpha = 1/(2k_S)$ and k_S are the wave vector of the probe pulse. The λ_S and λ_P pair is near a two-photon resonance on the high-frequency side of the atomic transition. Therefore, the sign of the nonlinear coefficient $k^{\text{IPM}}(\lambda_S)$ is positive, thus inducing a focusing effect on the probe beam. Due to the resonant structure of the nonlinear medium considered (inert gas), the nonlinear effects of the pump pulse itself can be neglected when λ_P is far from single- and two-photon resonances.

The coordinate system is fixed to the probe pulse. Equation (1) is solved using the variational method [11], reducing it to a system of coupled ordinary differential equations for the respective variational parameters. The analytical representation of the results is the main advantage of this method. A convenient choice of the trial functions is the commonly used Gaussian form both in time and space.

$$\psi_S(r, x, \tau) = \frac{A_{OS}}{\omega_S(x)} \exp\left\{-\frac{\tau^2}{2\tau_S^2} + ib_S(x)\tau^2\right\} \cdot \exp\left\{-\frac{r^2}{a_S^2\omega_S^2(x)} - i\frac{k_S\rho_S(x)r^2}{2}\right\} \tag{2a}$$

$$\psi_P(r, x, \tau) = \frac{A_{OP}}{\omega_P(x)} \exp\left\{-\frac{(\tau - \tau_D + x\nu_{SP})^2}{2\tau_P^2}\right\} \cdot \exp\left\{-\frac{r^2}{a_P^2\omega_P^2(x)} - i\frac{k_P\rho_P(x)r^2}{2}\right\} \tag{2b}$$

with initial conditions $b_S(x = 0) = 0$, $\tau_{S,P}(x = 0) = \tau_{S,P_0}$, $\omega_{S,P}(x = 0) = 1$ and $\rho_{S,P}(x = 0) = 0$. In (2) A_S and A_P are the respective complex slowly varying amplitudes of the probe and pump pulses, τ_S and τ_P are the half-pulse duration at $1/e$ level, $b_S(x)$ is the induced chirp-rate, τ_D is the eventual initial delay between the pulses, $\nu_{SP} = |v_{GS}^{-1} - v_{GP}^{-1}|$ is the group velocity mismatch, ω_S and ω_P are the normalized radii, a_S and a_P are the physical radii at $1/e$ level. Finally, ρ_S and ρ_P are functions of the inverse radii of curvature of the respective wavefronts. We can see that the system parameters are rather general and within reasonable physical limits there are no restrictions on the initial pulse duration and synchronization as well as on the wavelength pairs. As discussed elsewhere [8], the pump beam radius should be kept equal to or larger than the probe one.

It can be shown that (1) is an Euler–Lagrange equation for the Lagrangian:

$$L = (i/2)\left[\psi_S \frac{\partial \psi_S^*}{\partial x} - \psi_S^* \frac{\partial \psi_S}{\partial x}\right] + \alpha \left|\frac{\partial \psi_S}{\partial r}\right|^2 + \alpha_S \left|\frac{\partial \psi_S}{\partial \tau}\right|^2 - k^{\mathrm{IPM}}(\lambda_S)\, |\psi_S|^2\, |\psi_P|^2. \qquad (3)$$

The parameters of interest $\tau_S(x)$ and $\omega_S(x)$ are expressed through a set of ordinary differential equations obtained via the variational method from (1), (2a) and (2b):

$$\frac{d^2\tau_S}{dx^2} = \frac{4\alpha_S^2}{\tau_S^3} - \frac{4\alpha_S k^{\mathrm{IPM}}(\lambda_S)\, |A_P|^2 \tau_S \tau_P}{(\tau_S^2 + \tau_P^2)^{3/2}} \cdot \exp\left\{-\frac{(\tau_D - x\nu_{SP})^2}{\tau_S^2 + \tau_P^2}\right\} \cdot \left\{1 - \frac{2(\tau_D - x\nu_{SP})^2}{\tau_S^2 + \tau_P^2}\right\} \frac{a_P}{\omega_P(a_S^2\omega_S^2 + a_P^2\omega_P^2)^{1/2}} \qquad (4)$$

$$\frac{d^2\omega_S}{dx^2} = \frac{4}{k_S^2 a_S^4 \omega_S^3} - \frac{4k^{\mathrm{IPM}}(\lambda_S)\, |A_P|^2 a_P \omega_S}{\omega_P k_S (a_S^2\omega_S^2 + a_P^2\omega_P^2)^{3/2}} \cdot \exp\left\{-\frac{(\tau_D - x\nu_{SP})^2}{\tau_S^2 + \tau_P^2}\right\} \frac{\tau_P}{(\tau_S^2 + \tau_P^2)^{1/2}}. \qquad (5)$$

This form of the equations is particularly convenient. Owing to the energy conservation law in the form $|A_P|^2 \tau_P a_P \omega_P = \mathrm{const}$, (4) and (5) can be integrated once and can be reduced to equations, characteristic for a particle in a potential well [12], [13]:

$$\frac{1}{2}\left(\frac{dy}{dx}\right)^2 + \Pi(y) = 0. \qquad (6)$$

A. Description of the IW and IF by the Potential Well Model

In the case of IF $y \equiv \omega_S(x)$ and

$$\Pi^{\mathrm{IF}}(\omega_S) = \frac{1}{2L_S^2}\left(\frac{1}{\omega_S^2} - 1\right) - \frac{2k^{\mathrm{IPM}}(\lambda_S)\, |A_P|^2 a_P}{L_S \omega_P} \cdot \exp\left\{-\frac{(\tau_D - x\nu_{SP})^2}{\tau_S^2 + \tau_P^2}\right\} \cdot \left\{\frac{1}{(a_S^2\omega_S^2 + a_P^2\omega_P^2)^{1/2}} - \frac{1}{(a_S^2 + a_P^2)^{1/2}}\right\} \cdot \frac{\tau_P}{(\tau_S^2 + \tau_P^2)^{1/2}}. \qquad (7)$$

It is useful to introduce the notations

$$A = \frac{1}{2L_S^2},$$

$$B = -\frac{2k^{\mathrm{IPM}}(\lambda_S)\, |A_P|^2}{L_S \omega_P} \cdot \exp\left\{-\frac{(\tau_D - x\nu_{SP})^2}{\tau_S^2 + \tau_P^2}\right\} \frac{\tau_P}{(\tau_S^2 + \tau_P^2)^{1/2}}. \qquad (8)$$

The parameter L_S in (7), (8) has the physical meaning of the Rayleigh diffraction length of the probe beam. The counteraction between the diffraction spreading of the beam and the induced nonlinearity modifies the parameters of the potential well, depending on the ratio $\boldsymbol{B/A}$. By synchronous pulses $\tau_D = 0$ and in the absence of group velocity mismatch ($\nu_{SP} \cong 0$), we can separate several ranges of B/A parameter, corresponding to a specific behavior of the potential, similarly to Anderson [11] in the case of SPM:

1) Linear case ($\boldsymbol{B} = 0$);
2) subcritical nonlinearity ($-4\sqrt{2} < \boldsymbol{B/A} \leq 0$);
3) region of induced focusing ($\boldsymbol{B/A} < -4\sqrt{2}$).

The condition $\boldsymbol{B/A} = -4\sqrt{2}$ corresponds to a full compensation of the two counteracting effects. The signal beam radius does not change alongside the medium. Therefore, it is denoted as induced waveguiding.

Certainly, this division to ranges is conditional since in the course of their copropagation the pump and probe pulses can go out of synchronization, which gradually deforms the potential to a linear one. Fig. 1 shows the development of the potential as a function of length in the medium with subcritical nonlinearity by $\tau_d = 0$. In the beginning the normalized radius $\omega_S(z = 0) = 1$, so the physical signal beam radius will start to oscillate within the potential well formed. As the pulses propagate alongside the medium, they are gradually desynchronized, causing a deformation of the potential curve to a repulsive

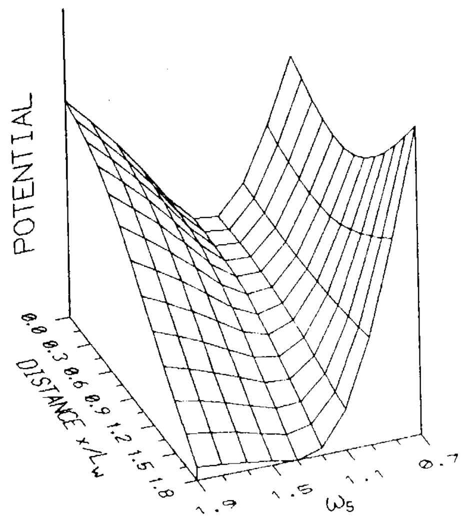

Fig. 1. Qualitative plot of the development of the potential $\Pi^{IF}(\omega_S)$ given by (7) in the case of subcritical nonlinearity ($-4\sqrt{2} < B/A < 0$).

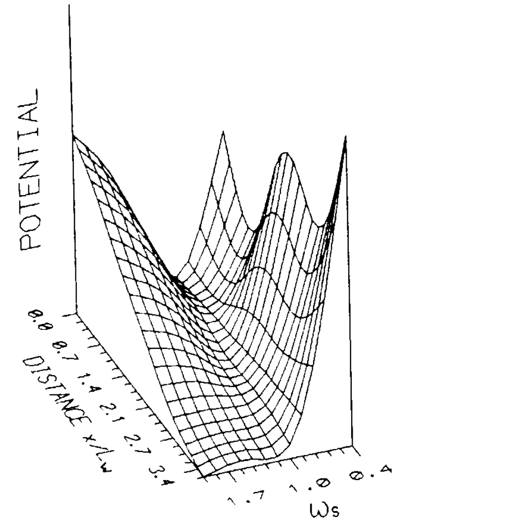

Fig. 2. Qualitative plot of the development of the potential $\Pi^{IF}(\omega_S)$ given by (7) in the region of induced focusing ($B/A < -4\sqrt{2}$).

one. In physical terms, the diffraction divergence will dominate over the signal beam.

Fig. 2 plots the analytical dependence in the range of overcritical nonlinearity, i.e., IW and IF. Starting again with a radius $\omega_S(z = 0) = 1$, we have an oscillation trend within the radius values lower than or equal to the initial value. In order to avoid optical breakdown, the pump and signal power should be kept within reasonable limits. Under the pump and probe beam approximation, the probe pulse intensity can be chosen orders of magnitude below the breakdown intensity. The desynchronization of pulses at a certain distance develops a potential, corresponding to a radius oscillation with an amplitude higher than the initial value. Further on the potential continues to modify into a repulsive one. In order to separate these three characteristic ranges, the length of the medium in Fig. 2 is chosen twice higher than in the previous figure.

Let us introduce the dimensionless parameter $\gamma = a_P(z = 0)/a_S(z = 0) \geq 1$ describing how much the pump radius is higher than the probe one. The critical power for IF, with γ as a parameter, by synchronous pulses with equal durations, can be obtained from the condition for potential curve degeneration into a point $\Pi^{IF}(\omega_S = 1)' = 0$:

$$P_{CRIT}^{IW} = \frac{(1 + \gamma^2)^{3/2}}{2\gamma} \times \frac{\sqrt{2}c\lambda_S'^2}{8\pi^2 n_2^{IPM}(\lambda_S)} \quad (9)$$

where $\lambda_S' = \lambda_S n_{0S}$ is the wavelength of the probe pulse in the medium. The second factor on the right-hand side is the IF critical power for equal beam diameters ($\gamma = 1$). The first term takes into account pump beams larger than probe one, which results in a decrease of grad $|n(\lambda_S)|$ over the probe beam cross-section. The expression for P_{CRIT}^{IW} is in a good agreement with the numerical results reported in [6], [14].

B. Description of IPM by the Potential Well Model

In the case of IPM the variable y has a meaning of pulse duration $y \equiv \tau_S(x)$ and for synchronous pulses

$$\Pi^{IPM}(\tau_S) = \frac{2\alpha_S^2}{\tau_S^2(x)} - \frac{4\alpha_S k^{IPM}(\lambda_S)\,|A_P|^2 \tau_P}{[\tau_S^2(x) + \tau_P^2(x)]^{1/2}} \cdot \frac{a_P}{\omega_P(a_S^2\omega_S^2 + a_P^2\omega_P^2)^{1/2}} + C' \quad (10)$$

where C' is an integration constant determined from the initial conditions. For brevity and convenient normalization we assume equal pump and probe pulses $\tau_S(x) = \tau_P(x) = \tau(x)$, $\tau(x = 0) = \tau_0$, and $\kappa(x) = \tau(x)/\tau_0$. The $\boldsymbol{A}$, $\boldsymbol{B}$, and the new $\boldsymbol{C}$ parameters have the form

$$A = \frac{2\alpha_S^2}{\tau_0^2} > 0$$

$$B = -\frac{4\alpha_S k^{IPM}(\lambda_S)\,|A_P|^2 \tau}{\sqrt{2}\tau_0^3} \frac{a_P}{\omega_P(a_S^2\omega_S^2 + a_P^2\omega_P^2)^{1/2}}$$

$$C = C'/\tau_0^2 \quad (11a)$$

and the potential function is transformed to

$$\Pi^{IPM}(\kappa) = \frac{A}{\kappa^2} + \frac{B}{\kappa} - [A + B]. \quad (11b)$$

In this temporal analysis the following ranges can be determined:

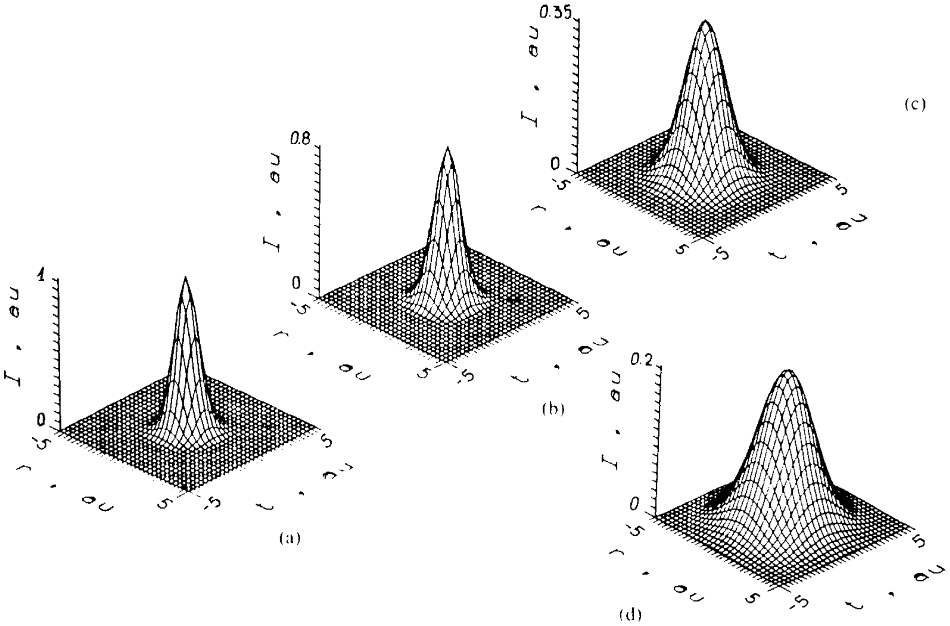

Fig. 3. Evolution of the initial probe pulse (a) in the light bullet region $L = 0.2L_W$ (b), in the case of complete desynchronization $L = L_W$ (c) for $L_{DS} = L_S$, and probe-pulse-shape at $L = L_W$ without an influence from the pump (d).

1) $B/A > 0$, i.e., $\alpha_S/k^{\mathrm{IPM}}(\lambda_S) < 0$, corresponding to a nonlinearity, accelerating the dispersion spreading of the probe pulse.

By $B/A < 0$ the sign of the ratio of the GVD coefficient to the IPM nonlinear coefficient changes to positive ($\alpha_S/k^{\mathrm{IPM}}(\lambda_S) > 0$). Physically, the nonlinear effect increases the trend to compensate for the dispersion broadening of the pulse.

2) $-1 < B/A < 0$ (weak nonlinearity). The induced nonlinearity counteracts the dispersion spreading of the probe pulse, however, it is not strong enough to compensate for it.

3) $-2 < B/A - 1$ (intermediate nonlinearity). Corresponds to a potential well forming and oscillating behavior of the probe pulse duration until the pulses are desynchronized.

4) $B/A < -2$ (strong nonlinearity). A potential well is formed for probe pulse duration not exceeding the inital value.

The B/A parameter, obtained from the condition for degeneration of the potential curve into point $\mathrm{II}^{\mathrm{IPM}}(\kappa = \gamma' = 0$, is equivalent to the critical power, which should be exceeded to observe a soliton-like propagation of the probe pulse.

$$P_{CRIT}^{\mathrm{SOL}} = \frac{\sqrt{2}k_2 n_0^2(\lambda_S)c}{4\pi k_S n_2^{\mathrm{IPM}}(\lambda_S)\tau_0^3} S_{\mathrm{eff}} \frac{(1+\gamma^2)^{1/2}}{\gamma} \qquad (12)$$

where $k_2 = 2\alpha_S$ and $S_{\mathrm{eff}} = \pi r_S^2$. Following the notation in the previous Section II-A, $r_S \equiv a_S\omega_S$. The comparison of this expression with the result, given by Akhmanov *et al.* [15], is good.

C. A Condition for "Light Bullet" Formation

In the case of self-phase modulation it has been shown [9] that in the presence of dispersion, the spatial-temporal shape of a pulse undergoes what could be considered as a four-dimensional self-focusing. The condition for light bullet formation due to IPM can be derived from (9) and (12) in the form

$$\tau_0 = a_S\omega_S\left\{\frac{2\pi\alpha_S}{n_o(\lambda_S)\,\lambda_S(1+\gamma^2)}\right\}^{1/2}. \qquad (13)$$

This expression relates the probe pulse duration, the physical radius of the signal $a_S\omega_S$, the GVD parameter α_S, the wavelength λ_S, the refractive index of the probe beam $n_o(\lambda_S)$, and the ratio γ between the pump and probe beam radii.

Fig. 3(a)–(c) plots the temporal and spatial evolution of the probe pulse copropagating with the pump pulse ($\tau_P = \tau_S$). For clarity of presentation we assume that the Rayleigh diffraction length L_S and the dispersion length L_{D_S} are equal and the walk-off distance is twice higher $L_W = 2L_S$. Compared to the initial probe pulse [Fig. 3(a)], the time/space dimensions are kept nearly constant until the pulse is synchronous with the pump pulse. Until the overlapping of the pulses is 80%, $L = 0.2L_W$ [Fig. 3(b)], the

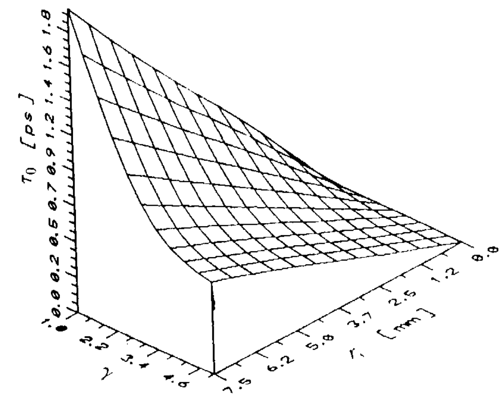

Fig. 4. Plot of the simultaneous conditions for soliton like formation for λ_S = 207 nm and λ_P = 248 nm, P_P being in the range (10^5–10^6 W).

temporal and spatial dimensions are only 10% increased. The pump-induced formation in this part of the bulk medium is called a "light bullet," similarly to the planar waveguide considered by Silberberg [9.] In our opinion, it would be stable both in the space and time since the conditions for its propagation are controlled via the pump beam parameters.

In the particular case, we consider a pulse duration of the order of several hundreds of femtoseconds and the main limitation is from the time walk-off distance L_W. When the pulses are fully desynchronized [$L = L_W$, Fig. 3(c)], the pulse spreading in time and space is 70% over the initial values at $1/e$ level. The balance condition (16) is fulfilled in Fig. 3(a)–(c). In order to compare the effect of the pump, Fig. 3(d) plots the probe pulse at $L = L_W$ when the pump is off. The spreading in this case is 130% compared to the initial pulse.

Evidently, the length of the efficient interaction in the case considered is limited by the ultrashort pulse walk-off. If the pump pulse is chosen with a much higher duration ($\tau_P >> \tau_S$), the interaction length will be limited by the pump diffraction divergence, i.e., pump Rayleigh length L_P. Since the parameter γ can be chosen to be greater than unity, the Rayleigh diffraction length of the pump L_P can be increased to a reasonable value.

As an illustration, we consider a XeI gas as a nonlinear medium, signal wavelength λ_S = 207 nm and pump wavelength λ_P = 248 nm, close to two-photon resonance, Fig. 4. The diagram plots simultaneously the conditions for soliton-like formation [see (12)] and induced waveguiding [see (9)], which is actually the balance condition (13). As seen from Fig. 4 shorter signal pulses require smaller signal beam diameters as well as larger pump beams. In the case considered, signal pulses with a duration of 0.5–1 ps would be formed into a light bullet by a signal beam diameter 1–8 mm and pump beam diameter two to four times larger. The range of critical powers required seems reasonable (10^5–10^6 W). The walk-off distance at $N_{Xe} = 2.5 \cdot 10^{19}$ cm^{-3} is L_W = 40 cm and the light bullet region has a length of about 8 cm for τ_0 = 1 ps. These parameters are to be compared with the recent experimental results of Aitchison *et al.* [16] on spatial solitons. Using 75 fs pulses they have observed spatial selfcollimation of the beam over a distance of 5 mm in a planar waveguide with a thickness 3–4 μm and input beam elipticity of 10 : 1.

III. Conclusions

Using the variational method for solving the Schrödinger equation and the analogy with a particle in a potential well, we have analyzed the influence of the induced phase modulation on the spatial and temporal characteristics of a probe pulse, copropagating with a pump pulse. We have shown for the first time, that pulses with nearly constant spatial and temporal parameters can propagate in a bulk media. A good agreement is observed between the critical power for induced focusing P^{IF}_{CRIT} and soliton-like propagation P^{SOL}_{CRIT} and the results, calculated by other authors for SPM in similar cases. An analytical expression is derived for light bullet formation by IPM. The proposed method seem to be especially promising in the shortwavelength region, using inert gas as a nonlinear medium.

The inherent pump beam divergence and group velocity mismatch can be compensated for and, hence, the light bullet parameters made fully reproducible, by methods discussed in the literature [17], [18], including periodic amplification and reshaping of the pump, symbiotic light pairs, etc.

The spatial instabilities connected with filament formation is a problem to be addressed separately and will be discussed elsewhere.

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