Pulse-front tilt created in misaligned dispersionless optical systems and correct interferometric autocorrelation

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Abstract

In this work we present a matrix analysis of the sensitivity of otherwise dispersionless 4\textit{f} and 2\textit{f}-2\textit{f} optical systems to misalignments resulting in spatio-temporal distortions of ultrashort laser pulses. Special attention is given to the possible creation of a pulse-front tilt (PFT). The exact analytical expression for the second-harmonic interferometric autocorrelation signal of an inverted-field autocorrelator is calculated. It confirms the effective broadening of the ultrashort pulses in the presence of an arbitrary PFT.

Key words: spatio-temporal distortion, dispersionless optical system, pulse-front tilt, inverted-field autocorrelator, second-harmonic autocorrelation

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1. Introduction

The pulse front tilt (PFT) is a specific spatio-temporal distortion of (ultra)fast optical pulses - the pulse front is tilted with respect to the direction of beam/pulse propagation, while its phase front remains perpendicular to

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The PFT is one of the major issues in chirped pulse amplification systems [1, 2, 3]. Most commonly this aberration is caused by misaligned pulse stretchers and/or compressors. PFT can also occur when femtosecond pulses are focused [4, 5] or passed through birefringent crystals [6]. Generally, PFT leads to an undesired increase of the effective pulse duration. Even the overlap of femtosecond pulses is not trivial anymore [7]. However, it should also be mentioned that in some cases the PFT is in fact useful. When the lifetime of an amplifying medium is shorter than the driving laser pulse, pump pulses with tilted pulse fronts offer the possibility to progressively deposit the pump energy along the gain medium at a speed, equal to the transient speed of the amplified wave ( [8, 9, 10, 11, 12], just to mention a few). Another example is the efficient phase-matched terahertz radiation generation by optical rectification of femtosecond laser pulses [13, 14] resulting in near-single-cycle terahertz pulses recently reported in the literature [15, 16].

Specific diagnostic techniques for detecting and measuring PFT are available: tilted pulse front autocorrelation [17, 18, 19], spectrally resolved interferometry [20], and Grating-Eliminated No-nonsense Observation of Ultra-short Incident Laser Light E-fields (GRENOUILLE) [21]. The usual interferometric second-harmonic autocorrelators based on Michelson- or Mach-Zehnder-type schemes are not able to detect PFT unless one of the beams/pulses is inverted in space [22, 23]. In such inverted-field autocorrelators the delay between the pulses depends also on the particular transverse coordinate across the beam and, hence, the recorded autocorrelation trace contains information on the effective broadening of the ultrashort pulse due to the PFT [23, 24].

In this work we first present a matrix analysis of dispersionless optical systems, in particular 4f- and 2f-2f-systems, which are frequently used for various pulse shaping applications including creation of spatial phase singularities in broadband femtosecond beams [25, 26, 27]. The analytical results show that small displacements of a lens and/or a grating inevitably introduce pulse front tilt. In the second part of our analysis we theoretically calculate the exact PFT-dependent interferometric autocorrelation signal in an inverted-field autocorrelator and derive a simplified approximate expression applicable in the case of small PFTs.
2. Matrix analysis

Our calculations are based on the $4 \times 4$ ray-pulse matrices introduced by Kostenbauder [28] and generalized by Duarte [29] and Trebino et al. [30]. In essence, the $4 \times 4$ matrices connect the input and output ray and pulse coordinates to each other. The spatial and temporal characteristics of the pulse are represented in a ray-pulse vector $(x, \vartheta, t, f)$. The spatial coordinates’ position ($x$) and slope ($\vartheta$) are the same as in the ordinary $2 \times 2$ matrices. The coordinate system is defined by the path of a diffraction limited reference beam of a central Hertzian frequency $f$ and, when thought of as being temporally transform-limited, can mark a well-defined arrival time $t$ at each transverse plane [28]. In terms of such coordinates and using a $4 \times 4$ matrix, the action of an optical element can be described as

$$
\begin{pmatrix}
 x_{out} \\
 \vartheta_{out} \\
 t_{out} \\
 f_{out}
\end{pmatrix} =
\begin{pmatrix}
 A & B & 0 & \frac{\partial x_{out}}{\partial f_{in}} \\
 C & D & 0 & \frac{\partial \vartheta_{out}}{\partial f_{in}} \\
 \frac{\partial x_{out}}{\partial x_{in}} & \frac{\partial \vartheta_{out}}{\partial \vartheta_{in}} & 1 & \frac{\partial t_{out}}{\partial f_{in}} \\
 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
 x_{in} \\
 \vartheta_{in} \\
 t_{in} \\
 f_{in}
\end{pmatrix} =
\begin{pmatrix}
 A & B & 0 & E \\
 C & D & 0 & F \\
 G & H & 1 & I \\
 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
 x_{in} \\
 \vartheta_{in} \\
 t_{in} \\
 f_{in}
\end{pmatrix},
$$

where $A$, $B$, $C$, and $D$ are identical to the components of the $2 \times 2$ ray matrix. $E = \partial x_{out}/\partial f_{in}$ and $F = \partial \vartheta_{out}/\partial f_{in}$ describe the spatial and angular chirp, respectively, while $H = \partial t_{out}/\partial \vartheta_{in}$ and $I = \partial t_{out}/\partial f_{in}$ stand for the time-to-angle coupling and for the group delay dispersion. $G = \partial t_{out}/\partial x_{in}$ finally is related to the angle $\alpha$ of the pulse front tilt via [31]

$$
\tan(\alpha) = \frac{\partial (ct_{out})}{\partial x_{out}} = \frac{\partial t_{out}}{\partial x_{in}} \frac{\partial x_{in}}{\partial x_{out}} = \frac{\partial t_{out}}{\partial \vartheta_{in}} \frac{\partial \vartheta_{in}}{\partial x_{out}} = \frac{cG}{c} \frac{\partial x_{in}}{\partial x_{out}},
$$

where $c$ is the speed of light.

2.1. Analysis of a 4$f$-system

In Fig. 1 we show a sketch of 4$f$-system under consideration. We assume that the two identical gratings are perfectly aligned with their grooves being parallel. The displacement of one of the two lenses from its nominal position we denote with $\delta$ and the deviation of one of the gratings away from the focus of the respective lens with $\Delta$. In the following, $f$ and $f_0$ will stand for the focal length of the lens and the center frequency of the pulse spectrum, respectively. According to the familiar procedures of matrix analysis one has
to multiply the respective ray-pulse matrices in reverse order (from the exit to the entrance), i.e.

\[ M_{4f} = G_2 T_{f+\Delta} F T_{f+\delta} T_f F T_f G_1. \]  

(3)

The explicit form of the matrices for the diffraction gratings is

\[
G_1 = \begin{pmatrix}
\frac{-\sin(\phi)}{\sin(\psi)} & 0 & 0 & 0 \\
0 & \frac{-\sin(\psi)}{\sin(\phi)} & 0 & \frac{\cos(\phi) - \cos(\psi)}{f_0 \sin(\phi)} \\
\frac{\cos(\psi) - \cos(\phi)}{e \sin(\psi)} & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
\]

(4)

\[
G_2 = \begin{pmatrix}
\frac{-\sin(\psi)}{\sin(\phi)} & 0 & 0 & 0 \\
0 & \frac{-\sin(\phi)}{\sin(\psi)} & 0 & \frac{\cos(\psi) - \cos(\phi)}{f_0 \sin(\psi)} \\
\frac{\cos(\phi) - \cos(\psi)}{e \sin(\phi)} & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

(5)

Note that the incident angle \( \psi \) of the ray on the first grating is changed to the respective angle \( \phi \) at the second grating. The transition matrices have a much simpler form:

\[
T_f = \begin{pmatrix}
1 & f & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}; \quad T_{f+\Delta} = \begin{pmatrix}
1 & f + \Delta & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}; \quad (6)
\]

and

\[
T_{f+\delta} = \begin{pmatrix}
1 & f + \delta & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}; \quad F = \begin{pmatrix}
1 & 0 & 0 & 0 \\
-1/f & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}. \quad (7)
\]

In this way we arrive for the modeled 4f-system at the matrix \( M_{4f} \)

\[
M_{4f} = \begin{pmatrix}
-1 + \frac{\Delta}{f^2} & -\frac{\Delta \sin^2(\phi)}{\sin^2(\phi)} & 0 & \frac{\Delta (\cos(\phi) - \cos(\psi)) \sin(\psi)}{f_0 \sin^2(\phi)} \\
\delta \sin^2(\phi) & \frac{\Delta (\cos(\phi) - \cos(\psi)) \sin(\psi)}{f_0 \sin^2(\phi)} & 0 & \frac{\Delta (\cos(\phi) - \cos(\psi)) \sin(\psi)}{f_0 \sin^2(\phi)} \\
\frac{\Delta (\cos(\phi) - \cos(\psi)) \sin(\psi)}{f_0 \sin^2(\phi)} & \frac{\Delta (\cos(\phi) - \cos(\psi)) \sin(\psi)}{f_0 \sin^2(\phi)} & 0 & \frac{\Delta (\cos(\phi) - \cos(\psi)) \sin(\psi)}{f_0 \sin^2(\phi)} \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

(8)
From this last matrix it is evident that the misalignment $\Delta$ of the grating determines the signs of the introduced group-delay dispersion (GDD), of the spatial chirp (SC) and of the time-to-angle coupling (T-AC):

$$\text{GDD} = \frac{\partial t_{\text{out}}}{\partial f_{\text{in}}} = -\frac{\Delta}{c f_0} \left[ \cot(\phi) - \cos(\psi)/\sin(\phi) \right]^2; \quad (9)$$

$$\text{SC} = \frac{\partial x_{\text{out}}}{\partial f_{\text{in}}} = \frac{\Delta[\cos(\phi) - \cos(\psi)] \sin(\psi)}{f_0 \sin^2(\phi)}; \quad (10)$$

$$\text{T} - \text{AC} = \frac{\partial t_{\text{out}}}{\partial \vartheta_{\text{in}}} = \frac{\Delta[\cos(\phi) - \cos(\psi) \sin(\psi)]}{c \sin^2(\phi)}. \quad (11)$$

Simultaneously misaligned lenses and gratings introduce pulse front tilt, where the angle of slope $\alpha$ can be determined from the relation

$$\tan(\alpha) = c G \frac{\partial x_{\text{in}}}{\partial x_{\text{out}}} = \frac{\delta \Delta[\cos(\psi) - \cos(\phi)]}{(\delta \Delta - f^2) \sin(\psi)}. \quad (12)$$

When at least one of these two elements is aligned at its nominal position there will be no PFT according to Eq. 12.

### 2.2. Analysis of a $2f$-$2f$ system

A sketch of the $2f$-$2f$ system to be analyzed is shown in Fig. 2. The notations are the same as in Fig. 1, except for additional designations for the different orders of diffracted beams. Multiplying the matrices of the individual optical elements in the following (reverse) order:

$$M_{2f-2f} = FG_2 T_{2f+\Delta} F T_{2f+\delta} G_1 \quad (13)$$

with matrices $G_1$, $G_2$, and $F$ being the same as in the previous case, and

$$T_{2f+\Delta} = \begin{pmatrix} 1 & 2f + \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad T_{2f+\delta} = \begin{pmatrix} 1 & 2f + \delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (14)$$
we derive the matrix $M_{2f-2f}$ of the misaligned $2f-2f$ system

$$M_{2f-2f} = \begin{pmatrix}
-\Delta + f & -\Gamma \sin^2(\psi) & 0 & \Gamma \Omega \sin(\psi) \\
\Delta + f \sin(\psi) & \Gamma \sin^2(\phi) & 0 & -\Omega(\Delta + f) - \Gamma \cos(2\phi) \\
\Delta \cos(\phi) - \cos(\psi) & \Omega \sin(\psi) & 1 & -\Omega \sin(\phi) \\
0 & cf \sin(\psi) & 0 & \frac{2f F_0 \sin(\psi)}{c f} \\
\end{pmatrix},$$

(15)

where $\Gamma = \Delta + f$ and $\Omega = [\cos(\phi) - \cos(\psi)] / \sin^2(\phi)$. In this way we obtain the following relations for the group-delay dispersion (GDD), spatial chirp (SC) and time-to-angle coupling (T-AC):

$$\text{GDD} = \frac{\partial t_{\text{out}}}{\partial f_{\text{in}}} = -\frac{[\Delta + f + \delta(\Delta + f)] [\cos(\phi) - \cos(\psi)]^2}{cf F_0 \sin^2(\phi)}$$

(16)

$$\text{SC} = \frac{\partial x_{\text{out}}}{\partial f_{\text{in}}} = \frac{[\Delta + f + \delta(\Delta + f)] [\cos(\phi) - \cos(\psi)] \sin(\psi)}{f F_0 \sin^2(\phi)}$$

(17)

$$\text{T-AC} = \frac{\partial t_{\text{out}}}{\partial \vartheta_{\text{in}}} = \frac{[\Delta + f + \delta(\Delta + f)] [\cos(\phi) - \cos(\psi)] \sin(\psi)}{c f \sin^2(\phi)}.$$  

(18)

Note that these three spatio-temporal perturbations depend in the same way on the grating and grating-lens-pair offsets. The result for the pulse front tilt angle $\alpha$ is:

$$\tan(\alpha) = \frac{\Delta [\cos(\psi) - \cos(\phi)]}{(\Delta + f) \sin(\psi)}.$$  

(19)

This means that the source of the PFT in a beam after its passage through such a system is the displacement $\Delta$ of the grating-lens pair. The offset $\delta$ of the middle lens alone does not cause PFT.

3. Second-harmonic autocorrelation signal in the presence of pulse front tilt

In this section we consider a Gaussian beam/pulse of width $x_0$ and duration $t_0$. In the absence of a PFT the electric field of this wave is described by

$$E(x, t) = E_0 \exp[-(x/x_0)^2 - (t/t_0)^2] \exp(i \omega t).$$  

(20)

Fig. 3 is intended to visualize the difference between pulses without and with PFT. The right frame of Fig. 3 corresponds to the valid perception that, at
a fixed plane perpendicular to the beam propagation direction, a difference in arrival time that depends on the transverse beam coordinate means an effective pulse lengthening. In a standard Michelson-type autocorrelator the PFT remains hidden [23] (see Fig. 4, left). The picture drastically changes in the inverted-field autocorrelator, where one of the beams is rotated in space by 180° (Fig. 4, right). Corresponding setups for interferometric inverted-field autocorrelation which are suitable for the detection of tilted pulse fronts are described in the literature [22, 23]. Our goal here is to derive an exact analytic expression for the second-harmonic interferometric autocorrelation signal for an arbitrary large pulse front tilt.

Starting with the optical field amplitudes \( E_1 \) and \( E_2 \) of the beams/pulses with opposite pulse front tilts \( -\alpha \) and \( +\alpha \), respectively (inverted-field autocorrelator geometry), the intensity \( I_{2\omega} \) of the second harmonic that is generated is

\[
I_{2\omega} = |E_{2\omega}(x, t, \alpha, \tau_d)|^2 = \left|\left[E_1(x, t, -\alpha) + E_2(x, t, \alpha, \tau_d)\right]\right|^2. \quad (21)
\]

Here \( \tau_d \) is the time delay between the pulses. With a wide aperture photodetector and a detector rise-time much longer than the typical pulse length (conditions easily met with ultrashort pulses) the normalized second-harmonic autocorrelation signal has the form

\[
P_{2\omega}^{AK}(\alpha, \tau_d) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I_{2\omega}(x, t, \alpha, \tau_d)dxdt}{2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left|E_i(x, t, \alpha, \tau_d)\right|^2 dxdt}, \quad (22)
\]

where \( i = 1, 2 \). The natural normalization to the energy of the second-harmonic signal generated from each pulse separately yields a correlation peak-to-background ratio of 8 : 1 at zero tilt of the pulse front. The most time-consuming but straightforward step in this analysis is the correct rotation of the electric field amplitude (20) by an angle \( -\alpha \) and \( \alpha \), respectively. Following the procedure described above, we derived a general expression for the interferometric autocorrelation signal for an arbitrary large pulse front tilt \( \alpha \)

\[
P_{2\omega}^{AK}(\alpha, \tau_d) = 1+2\sqrt{2} \tau_0 \left\{ 8 \exp\left[\frac{-A(\alpha)\tau_d^2}{B(\alpha)}\right] \cos(\omega \tau_d) + \exp\left[-\frac{\tau_d^2}{C(\alpha)}\right] [2 + \cos(2\omega \tau_d)] \right\}, \quad (23)
\]
where
\[A(\alpha) = 12[(x_0^2 + t_0^2) + (x_0^2 - t_0^2) \cos(2\alpha)]\]
\[B(\alpha) = 3x_0^4 + 26x_0^2t_0^2 + 3t_0^4 - 3(x_0^2 - t_0^2)^2 \cos(4\alpha)\]
\[C(\alpha) = t_0^2 \cos^2(\alpha) + x_0^2 \sin^2(\alpha)\]
\[D(\alpha) = x_0^4 + 6x_0^2t_0^2 + t_0^4 - (x_0^2 - t_0^2)^2 \cos(4\alpha)\]

In Fig. 5 we show numerically calculated interferometric autocorrelation signals \(P_{2\omega}^{AK}\) for different values of \(\alpha\). It is evident that the peak-to-background signal ratio of 8 : 1 of the correlation function at \(\alpha = 0\) gradually decreases with increasing the PFT angle \(\alpha\) while the amplitudes of the oscillations in the wings of the correlation functions grow monotonically. This is a clear indication that the autocorrelation curve becomes broader with increasing the pulse front tilt. The effect is more pronounced when we inspect the envelopes of the autocorrelation signals (see Fig. 6). It is even intuitively clear that, at one and the same beam width, the shorter the laser pulse, the stronger the influence of the PFT. Table 1 summarizes results concerning the relative pulse broadening \(\Delta t/\Delta t(\alpha = 0)\) for different pulse duration to beam width ratios vs. tilt angle \(\alpha\). As seen, for \(\alpha = 0.35\) rad a decrease of the pulse duration by a factor of 3 results in an increase of the effective pulse broadening by a factor of more than 2. The shorter pulses appear stronger influenced by the PFT than the longer ones. In the case of a relatively small pulse front tilt \((\tan(\alpha) \approx \sin(\alpha) \approx \alpha)\) the derived general expression (23) simplifies to
\[P_{2\omega}^{AK} = 1 + 4 \exp\left\{-\frac{4\tau_d^2}{3t_0^2}\right\} \cos(\omega \tau_d) + \exp\left\{-\frac{\tau_d^2}{t_0^2 + x_0^2}\right\} [2 + \cos(2\omega \tau_d)]. \quad (25)\]

The validity of this approximation has to be checked carefully. If the tolerable deviation of the approximate result from the exact one is within 5%,
this approximate formula holds for \( \alpha < 0.1 \text{rad} \) (see Fig. 7). Since nearly single-cycle laser pulses require a lot of experimental effort and most of the misalignments of the optical system are probably compensated to reach this regime, Eq. 25 may appear a reasonable and relatively simple first-order approximation in analyzing autocorrelation signals in the presence of small pulse front tilts.

4. Conclusion

In this work we presented a matrix analysis of sensitivity to misalignments of otherwise dispersionless 4\( f \) and 2\( f - 2f \) optical systems with respect to the introduced spatio-temporal distortions. Special attention has been given to the possible creation of a pulse front tilt (PFT). In a 4\( f \)-setup, PFT is introduced when the second lens and the second grating are simultaneously shifted from their exact positions. In a misaligned 2\( f - 2f \) setup, the source of the PFT is the displacement of the grating-lens pair. Since the PFT is expected to lead to an effective broadening of the ultrashort pulses, we analyzed the second-harmonic autocorrelation signal in an inverted-field autocorrelator. The derived exact analytical expression in the case of an arbitrary PFT and the approximation for the case of such a weak distortion show an effective pulse broadening, which is well motivated by physical intuition.

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6. Captions for the figures

Fig.1 – Sketch of the analyzed 4\( f \)-system. \( G_1, G_2 \) - diffraction gratings assumed to be aligned with strictly parallel grooves, \( L \) - lenses with focal length \( f \), \( D \) - diaphragm, \( \delta \) and \( \Delta \) - lens and grating offsets from the perfect alignment.

Fig.2 – Sketch of the analyzed 2\( f - 2f \)-system. \( G_1, G_2 \) - diffraction gratings assumed to be aligned with strictly parallel grooves, \( L \) - lenses with focal
length \( f \), \( D \) - diaphragm, \( \delta \) and \( \Delta \) - grating and grating-lens-pair offset from the precise alignment.

Fig.3 – Electric field amplitude of an ultrashort pulse without (left) and with pulse-front tilt (right).

Fig.4 – Snapshot of the electric field amplitudes of ultrashort optical pulses with pulse-front tilt propagating in a standard (e.g. Michelson-type) autocorrelator (left) and in an inverted-field autocorrelator (right) at a certain pulse delay.

Fig.5 – (Color online) Simulated interferometric autocorrelation signal (Eq. 23) for different values of the PFT angle \( \alpha \).

Fig.6 – (Color online) Simulated envelopes of the interferometric autocorrelation signals (Eq. 23) for different values of the PFT angle \( \alpha \).

Fig.7 – (Color online) Simulated interferometric autocorrelation signals by using the exact (Eq. 23) and the approximate analytical result (Eq. 25) and difference of the calculated signals (bottom curve) for small PFT angle \( \alpha = 0.1 \text{ rad} \).

7. Captions for the tables

Table 1 – Relative pulse broadening \( \Delta t/\Delta t(\alpha = 0) \) for different pulse duration \( t_0 \) to beam width \( x_0 \) ratios vs. pulse-front tilt angle \( \alpha \).

References


