Initiating self-focusing of beams carrying spatial phase singularities

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In this work we show both experimentally and by numerical simulations that the presence and evolution of ring dark beam and/or on-axis optical vortex nested on a bright background beam noticeably perturb the host background. In a photorefractive nonlinear medium (crystal SBN) these perturbations can initiate self-focusing of the background. By changing the dark ring radius and the presence of an optical vortex and keeping all other experimental parameters unchanged, we can control the dynamics at the initial stage of longitudinal self-focusing and the type of self-focusing structure (single peak or bright ring of variable radius). The presented results may appear especially important in experiments that involve cascaded nonlinear frequency mixing of singular beams, in which accelerated dark beam spreading is accompanied by self-focusing of certain portions of the perturbed host beam.

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1. Introduction

Optical vortices (OVs) are associated with isolated point singularities with a 2π helical phase profile around them, while ring dark waves (RDWs) contain π step-like phase discontinuities curved in a ring and flat phase profiles in- and outside this ring. In the case of continuous-wave laser emission and even in short pulses with small spectral bandwidths single phase masks [1] or computer generated holograms (CGHs, [2, 3]) can create such phase dislocations without the need for additional dispersion compensating elements. Possible methods of creating OVs and RDWs in sub-50-femtosecond laser fields employ CGHs or spatial light modulators aligned within optical schemes with spatial dispersion compensation dispersionless 4f- [4, 5] or 2f-2f setup [6, 7] or a doublepass grating [8] or prism compressor [9]. In the particular case of OVs successful techniques make use of achromatic vortex lens [10], uniaxial crystal [11, 12] or axially symmetric polarizer [13].

In contrast to the evolution of singular beams in selfdefocusing nonlinear media (NLM), their behavior in self-focusing NLM is relatively less explored [14–17]. As self-focusing materials are more common in nature, this is somewhat surprising and could be explained by the intrinsic azimuthal and modulational instabilities of the beam [18–21]. However, many intriguing perturbative nonlinear processes (e.g. cascaded four-wave mixing assisting the white light generation) and non-perturbative processes like high harmonic generation take place in transparent nonlinear media with positive nonlinearities. For example, the generation of dispersion-free highintensity OV has allowed the investigation of nonlinear vortex beam filamentation in air [22] and water [21]. The use of high intensity beams increases the possibility to observe cascaded nonlinear processes [23, 24]. Experimental evidence of cascading has only been shown, however, in terms of modulated intensity profiles [25], and nonlinear interaction has only been observed up to first cascading order [26]. It was recently shown [27] that the OVs, impressed on the fundamental pump beam, survive the highly nonlinear process of high-harmonic generation [28, 29] reaching the extreme ultra-violet spectral range.

Particular interest in singular dark beams (OVs, onedimensional dark beams, and RDWs) is motivated by their ability to propagate as dark spatial solitons or dark solitary waves and to induce gradient optical waveguides in bulk self-defocusing NLM [30–32]. Necessary but not sufficient condition for this is to propagate them in a NLM of negative nonlinearity, in which the dark beam diffraction is compensated for by the nonlinearity of the medium. In contrast, the positive (e.g. Kerr or photorefractive) nonlinearity leads to accelerated dark beam broadening and energy density redistribution on the host background beam. As a result, controllable initiation of self-focusing of the bright structures on the host background could be expected [15, 33, 34].

In this work we show both experimentally and by nu-

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merical simulations that the presence and evolution of ring dark beam and/or on-axis optical vortex nested on a bright background beam are noticeably perturbing the host background beam and, in a photorefractive nonlinear medium, can initiate its self-focusing. Geometrycontrolled conditions here are the dark ring radius and the presence of an optical vortex. Thus, at the initial stages of the processes, we can relatively easily control the self-focusing longitudinal dynamics and the type of self-focusing structure (single peak or bright ring of variable radius).

2. Experimental setup

To demonstrate this concept, in the reported experiment we used a continuous-wave frequency-doubled Nd: YVO_4 laser at a wavelength of 532nm. The experimental setup is shown in Fig. 1. The desired phase singularities are generated by a set of binary CGHs, fabricated photolithographically with a grating period of $30\mu m$. The first-order diffracted beam carrying the phase dislocation(s) was focused by a lens FL at the front face of a 6mm long SBN photorefractive crystal of a crosssection $6mm \times 10mm$. When focusing the incoming pure Gaussian background beam down to $20\mu m$ at the front facet we measured it to be $310\mu m$ wide at the exit facet. This implies that the $6mm \log NLM$ ensures nonlinear beam propagation length of about six Rayleigh diffraction lengths L_d . The polarization of the laser beam was parallel to the crystalline *c*-axis, thus the beam experienced strong photorefractive nonlinearity due to the high electro-optic coefficient r_{33} in the SBN. The crystal was biased by an externally applied electric field E_0 ranging from 300V/cm to 400V/cm. The front or the back facet of the crystal was imaged with a lens IL onto a charge-coupled device (CCD) camera, both moving on a common translation stage (see Fig. 1). Special attention is paid to the alignment of the CGHs in order to place the dark singular beams well centered with respect to the illuminating beam. The power of the background beam was adjusted in order to initiate only weak selffocusing of the pure Gaussian beam (see e.g. Fig. 4, left graph and lower left pair of frames; 6 intensity, close to the 10When nesting a ring dark wave the beams peak intensity remains the same at large ring radii and decreases at relatively small ring radii. The key point in this work is that under these self-focusing conditions the nonlinearly-accelerated dark beam broadening is able to initiate well pronounced bright background beam selffocusing. We refrained from increasing further the background beam power and the crystal illumination time to more than 240s in order to avoid filamentation in the SBN in a necklace-like bright beam structure and to keep the slowly-varying envelope approximation of the model equation (1) valid.

3. Numerical model

To numerically simulate the beam propagation inside the biased photorefractive SBN crystal (dc field applied along the x direction parallel to the crystalline c axis),



Fig. 1. Experimental setup: B - Gaussian background beam illuminating the computer-generated hologram (CGH). D - diaphragm selecting the first-order diffracted beam. FL - focusing lens (f = 3.5cm). SBN - biased photorefractive crystal with marked orientation of the c-axis. Horizontal arrow - beam's polarization. IL and CCD - imaging lens and charge-coupled device camera moving on a common translation stage to image the input or output facet of the crystal.

one has to solve the following system of equations [35, 36]

$$i\frac{\partial A_j}{\partial z} + \frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)A_j - \gamma(E_{sc} + E_0)A_j = 0 , (1)$$

for the transverse components A_j of the slowly-varying optical field amplitude. $\gamma = (1/2)(2\pi/\lambda)^2 x_0^2 n_0^4 r_{eff}$ is a nonlinear material parameter accounting for the corresponding term of the electrooptic tensor ($r_{eff} = r_{33}$ for SBN). When positive, γ accounts for the self-focusing nonlinear response of the crystal. E_{sc} is the space-charge field related to the electrostatic potential Φ ($E_{sc} = -\partial \Phi/\partial x$). E_0 is the external field applied along the c axis perpendicular to the beam propagation direction. The electrostatic potential Φ is modeled by the equation [34, 35]

$$\nabla^2 \Phi + \nabla \Phi \cdot \nabla \ln(1+I) = E_0 \frac{\partial}{\partial x} \ln(1+I) , \quad (2)$$

where $I = |A_1|^2 + |A_2|^2$ is the total light intensity normalized to the dark irradiance of the SBN crystal. The last term in Eq. 2 accounts for the drift of the charge carriers. In the above notations, the refractive index of the medium is modulated via the Pockels effect according to the relation $n^2 = n_0^2 + n_0^4 r_{33} \partial \Phi / \partial x$, assuming that the incident beams are polarized along x direction (crystalline c axis). All material parameters used in the numerical simulations correspond to the typical values of SBN crystals [37, 38] ($r_{33} = 180pm/V$, $n_0 = 2.3$ and $\gamma = 4.3.10^{-4}m/V$) and E_0 is set to 400V/cm. In all measurements presented here the beam's polarization is kept parallel to the crystaline c axis. That is why we set $|A_2(z=0)| << |A_1(z=0)|$ and, effectively, $I \approx |A_1|^2$.

The input electric-field amplitude of the beam carrying the optical vortex is assumed to be of the form

$$A_1(x, y, z=0) = B(x, y) \tanh[r(x, y)/r_0] \exp[i\phi(x, y)] ,$$
(3)

where r and ϕ are the radial and azimuthal polar coordinates corresponding to the Cartesian coordinates xand y. The width w of the super-Gaussian background beam

$$B(x,y) = B_0 \exp\{-\left[\sqrt{(x^2 + y^2)/w^2}\right]^{14}\}, \quad (4)$$

is chosen to be more than 10 times larger than the OV width r_0 . We generated the optical vortices by binary amplitude computer-generated holograms in which the point phase dislocation is encoded in one interference line with a fork-like splitting. In that case the vortex beam is described with confluent hypergeometric function even in many articles this beam is approximated as a Laguerre-Gaussian (LG) mode [39]. r-vortices described by LG functions are found, for example, in beams emerging from cylindrical waveguides. In this case the OV core half-width at half-intensity maximum r_0 is related to the width of the background Gaussian beam w by $r_0 \approx 0.59 w$ [40]. Function tanh is used, for example, to describe optical vortex solitons [31] and solitons in Bose-Einstein condensates [41] for which the width of the bright background beam should be much larger than this of the dark beam. In our experiment we measured that the width of the OV is equal to this of the background beam. Moreover, in our simulations we had to provide "additional space" on the background for a RDW. That is why we used tanh-function for describing the input dark beams. The 7-th power of the super-Gaussian beam was chosen arbitrary and was found to have a negligible influence on the numerical data.

When the beam is carrying a single ring dark wave of radius Δ and ring arc width r_0 , the input electric-field amplitude of the beam is assumed to be of the form

$$A_1(x, y, z=0) = B(r) \tanh\{|r-\Delta|/r_0\} \exp[i\psi(r)] .$$
 (5)

The initial phase $\psi(r) = -\pi/2$ for $r < \Delta$ and $\psi(r) = +\pi/2$ for $r > \Delta$ of the RDW clearly shows the presence of a pair of diametrical phase jumps across the dark ring.

We solved the model equations by means of a modification of the split-step Fourier method with a computational window spanning over 1024x1024 grid points. As a standard test (for calibration purposes only and inverting the sign of the nonlinear parameter γ to be negative) we modeled the formation of a fundamental one-dimensional *tanh*-shaped dark spatial soliton. All further numerical simulations were carried out at this background beam intensity. Since the presence of an optical vortex and/or ring dark wave decrease the initial peak intensity of the background in dependence of the particular dark structure, we will further express all propagation distances in units of diffraction lengths $L_d = (2\pi/\lambda)n_0r_0^2$.

We carefully inspected our data during the calculations and restricted the propagation distances to lengths, at which the peak intensities remained less than 6 times higher than the initial one. This was done in order to keep the slowly-varying envelope approximation valid (see Eq. 1). That is why we will present numerical results for propagation distance $z/L_d = 2$. It is worth mentioning, that at this distance the pure Gaussian background beam, under perfect conditions and at the already specified intensity, increases its intensity by 6% only, close to the 10% value estimated from the experimental data (see Fig. 2, last column).



Fig. 2. Upper block of frames: Input beams (at z = 0) and their numerically simulated reshaping at the exit of the photorefractive NLM (at $z = 2L_d$, after weak self-focusing) for dark rings of initial normalized radii Δ/r_0 . Number below each frame - background beam peak intensity at the respective propagation distance. Lower row of frames: Experimental beam profiles recorded at the exit of the 6mm long SBN crystal for the same initial values of Δ/r_0 . Numbers below each frame - relative increase of the beam's peak intensity at the exit facet of the crystal. Last column - nonlinear evolution of a pure Gaussian background beam.

4. Results and discussion

Quite an intriguing insight into the bright beam reshaping and self-focusing initiation can be gained when following the background beam evolution along the NLM with initial dark rings of different normalized radii Δ/r_0 nested on it (see Fig. 2). In the upper block of frames in Fig. 2 we show the input beams (at z = 0) and their numerically simulated reshaping at the exit of the photo refractive NLM (at $z = 2L_d$, after weak self-focusing) for different Δ . The numbers below each frame indicate the background beam peak intensity at the respective propagation distance. In the lower row of frames in Fig. 2 we show experimental beam profiles recorded at the exit of the SBN crystal for the same initial values of Δ/r_0 . Numbers below each of these frames indicates the relative increase of the beam's peak intensity at the exit facet of the crystal. For comparison, the last column presents the nonlinear evolution of a pure Gaussian background beam. The typical crystal exposure time was 90s, except for $\Delta/r_0 = 1$ (240s). The bias voltage in this measurement was set to 300V and the crystal illumination was terminated (as done in all measurements) prior to saturate the CCD camera. Note, that in this set of simulations, in agreement with the experiment, the fastest longitudinal dynamics is observed for $\Delta/r_0 = 2$. For $\Delta/r_0 = 1$ the most intense parts of the beam are initially located outside the dark ring and, as seen in both simulations and experiment, a ring of relatively large radius starts to self-focus. For $\Delta/r_0 = 2$ and 3, however, the intense areas of the bright beam are near its axis and self-focusing in a single filament initiates, but with quite different intensity growth rates. It is worth mentioning that, in contrast to the previous



Fig. 3. Horizontal cross-sections of some of the experimental frames shown in Fig. 2. Left graph - $\Delta/r_0 = 1$ and 2. Right graph - $\Delta/r_0 = 5$ and 6. Bias voltage - 300V. Crystal illumination time - 240s for $\Delta/r_0 = 1$ and 90s in all other cases.

cases, for $\Delta/r_0 = 3$ the discrepancy between the calculated and measured transverse beam profiles and peak fluxes is relatively large. The reason for this remains still an open question. For $\Delta/r_0 = 4$ the central part of the background starts to self-focus in a ring with a small radius. Since for $\Delta/r_0 = 5$ and 6 the most intense parts of the background are within the dark ring and become compressed in space by its broadening, the patterns become richer containing on-axis peaks surrounded by shrinking bright rings of different radii. Note that in all cases the peak power densities exceed this of a pure Gaussian beam at the same nonlinear propagation distance. The close inspection of the pairs of numerical and experimental data for the same initial Δ/r_0 shows that the ring ellipticity in our simulations is slightly higher than in the experiment due to the SBN anisotropy, but the agreement is qualitatively good.

In Fig. 3 we show horizontal cross-sections of some of the recorded experimental frames presented in Fig. 2. The curves in the left graph refer to $\Delta/r_0 = 1$ and 2, whereas in the right graph we show the data for $\Delta/r_0 = 5$ and 6. As seen (Fig. 3, left, solid curve), the most confined dark ring with $\Delta/r_0 = 1$ gives birth to a central peak and a surrounding ring of nearly equal peak power densities. In contrast, the initially wider ring with $\Delta/r_0 = 2$ forces the central part of the background to self-focus in a dominating on-axis peak, whereas the self-focusing of the outer-lying ring is negligible (Fig. 3, left, dashed curve). For the two broadest rings with $\Delta/r_0 = 5$ and 6 (Fig. 3, right graph) the on-axis peak is still dominating the surrounding pairs of rings, but the peak-to-peak ratio is smaller. The positions of the peaks (i.e. the ring radii (see Fig. 2)) also depend on Δ . All curves shown in Fig. 3 can be directly compared, since all conditions and CCD camera settings are kept unchanged during these measurements.

Next we studied the background-beam reshaping and initial self-focusing in the NLM when two coaxial dark rings were nested on it. Special attention was paid to the evolution of the Gaussian background beam. Its cw



Fig. 4. Left graph: Initial (t = 0) horizontal cross-section of the Gaussian background beam at the output of the SBN crystal (solid curve) and the same after illumination time t = 180s (dashed curve). Right graph: The same for Gaussian background beam carrying two coaxial dark rings. Bias voltage 300V, beam power $110\mu W$. Respective pairs of images: Experimental frames from which the horizontal cross-sections are extracted.

power was chosen to be $110\mu W$ - just enough to initiate weak background beam self-focusing in the NLM (Fig. 4, left graph, dashed curve). Under the same conditions the input host beam carrying the coaxial RDWs (Fig. 4, right graph, solid curve) shows clear self-focusing of its central peak and narrowing of the broad coaxial bright ring (same graph, dashed curve). The increase in the peak power density is approximately four times. The exposure time (180s) and the bias voltage (300V) were chosen in a way to ensure data recording without any CCD camera saturation. We show the respective pairs of images from which the horizontal cross-sections are extracted below each graph. Note that the left frame below the right graph in Fig. 4 is intentionally saturated to make the input outer bright ring (and the subsequent initiation of self-focusing) visible. The curves, however, are extracted directly from the raw experimental data. This figure strongly confirms that the accelerated broadening of the coaxial RDWs and the energy density redistribution on the host background beam are able to initiate controllable self-focusing of the bright structures on the background.

In the last series of measurements presented here (Fig. 5) we compare the nonlinear evolution of a host beam carrying single OV, single RDW, and an OV with a coaxial RDW. In order to ensure the same experimental conditions using the whole dynamic range of the CCD camera we set the beam power to $80\mu W$ and the bias voltage to 400V. The only difference (in order to avoid detector saturation) was the SBN crystal illumination time - 120s for the single ring dark wave, and 240s in the two other cases. In the respective graphs we show the initial (solid curves) and final horizontal

cross-sections (dashed curves) from the recorded frames. To the right of each graph we show numerically calculated (for $z = 2L_d$) and experimentally recorded (at the exit facet of the SBN crystal) power densities of the singular beams carried by the bright background beams. The righmost images are interferograms confirming the presence of phase jumps or abrupt phase changes on the OV axes and across the RDWs after the initiation of the beam's self-focusing. Clear indication for the OV point phase singularity is the splitting of one of the interference fringes in two (see the left arrows in the first and in the last interferogram). In the second and in the third interferogram the right arrows are intended to guide the eye to regions in which interference fringes terminate due to abrupt radial phase changes across the RDWs. Note also the difference in the fringe periods on the left hand side of the RDW vs. the same period on the right hand side of the same RDW due to the "terminated" fringes (better visible in the middle interferogram). It is clearly seen that, depending on the particular singular dark beam and due to their accelerated broadening along the medium with positive nonlinearity, one can controllably force the background beam to start self-focusing in a single ring (Fig. 5, upper row), in a single dominating peak (same figure, middle row), or in a pair of coaxial bright rings with nearly equal local power densities (Fig. 5, lower row).

The numerical simulations shown in Fig. 6 provide some additional insight into the possibility to rule the host beam self-focusing by changing the RDW radius when an on-axis OV is present. For RDWs with small initial radii ($\Delta/r_0 = 1$ and 2) the wings of the RDW and the OV overlap too much and the region of high local intensity is lying outside the RDW. Thus, initial background beam self-focusing in relatively broad rings should be expected. In contrast, for reasonably broad initial RDWs ($\Delta/r_0 = 4$ and 5 in Fig. 6) the points where the intensities are the highest form a circle between the OV and the RDW. Hence, the background beam should start to self-focus in rings of smaller radii (in isotropic NLM) or in smaller ellipses when anisotropic NLM is used. In the intermediate case $\Delta/r_0 = 3$ we observed (see Fig. 6) inner ring decaying into two peaks (which rotate along the NLM due to the presence of the OV) and outer self-focusing ellipse. In the limiting case $\Delta/r_0 = 6$ the initial stage of the self-focusing within the broad ring is accompanied by modulational instability, which is most pronounced along the c axis of the photorefractive material.

5. Conclusion

In this work we show both experimentally and by numerical simulations that the presence and evolution of ring dark beam and/or on-axis optical vortex nested on a bright background beam are noticeably perturbing the host background and, in a photorefractive nonlinear medium, can initiate self-focusing of the background. Geometry-controlled conditions here are the dark ring radius and the presence of an optical vortex. In pho-



Fig. 5. Initial (t = 0; solid curves) and final (t = 240s/120s;dashed curves) power density distributions of background beams carrying an optical vortex (upper row), a ring dark wave (middle row) and coaxial optical vortex and ring dark wave (lower row) at the exit of the SBN crystal. The dashed curves in the 2-D graphs are extracted from the respective experimental frames shown to the right. For comparison, the corresponding numerical data are shown too. Bias voltage -400V, beam power - $80\mu W$, illumination time t = 120s for the single ring dark wave, and t = 240s in the two other cases, in which an on-axis vortex is present. Right experimental frames - interferograms at the NLM exit of the weakly self-focused singular beams with a reference Gaussian beam with a spherical wavefront. Arrows are provided to guide the eve to local features of the interference patterns confirming the presence of spatial phase dislocations.

torefractive media the presented results indicate that parallel all-optical guiding of optical signals (see Fig. 6 in [42]) may appear feasible at wavelengths, for which the NLM is not photosensitive. The results may appear especially important in experiments which involve cascaded nonlinear frequency mixing of singular beams, in which accelerated dark beam spreading is accompanied by self-focusing of certain portions of the perturbed host beam.



Fig. 6. Input beams (at z = 0) and their numerically simulated reshaping at the exit of the photorefractive NLM (at $z = 2L_d$, after weak self-focusing) in the presence of both singly-charged on-axis vortex and outer ring dark wave of varying radius Δ/r_0 . Number below each frame - background beam peak intensity at the respective propagation distance.

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