Collimation and guiding of symbiotic light-beam pairs

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We find conditions for collimation and guiding of two light beams in a nonlinear bulk medium. The spatial parameters of each beam of the symbiotically coupled pair depend critically on the cross-phase modulation from the complementary beam. The diffraction divergence and self-phase modulation can be offset by proper choice of the wavelength, the value, and the sign of the nonlinear susceptibility of the medium.

INTRODUCTION

Self-focusing and self-defocusing have their origin in the intensity dependence of the refractive index of the nonlinear medium.[1] The picture becomes more complicated when two copropagating optical beams affect each other through nonlinear cross-phase modulation (XPM). Since the first experimental observation,[2] several groups have studied XPM with respect to pulse tailoring.[3–7] Recently focusing by induced phase modulation was considered both theoretically[8] and experimentally in multimode fibers.[9] Numerically solving the coupled amplitude equations for the pump and the probe beams, Agrawal[10] showed that induced focusing should be observable in a self-defocusing nonlinear medium in an off-axis geometry.[10] In all these cases, however, the interaction length was inherently limited by the diffraction spreading or self-defocusing of the strong pump beam.

The interplay between nonlinear refraction, anomalous dispersion, and diffraction can lead to modulation instability[11] and the collapse of optical pulses in time[12] and in time and space.[13] It was shown that self-focusing and self-phase modulation (SPM) are unstable in three dimensions; therefore temporal–spatial solitons have been considered mostly with respect to planar waveguides.[13] The scheme of the symbiotic solitary-wave pair was proposed by Lisak *et al.*,[7] and an attractive interaction between two pulses was proposed to result from small initial differences in the relative group velocity or position.

In this paper we report what we believe to be the first model of a symbiotic light-beam pair traveling with constant spatial parameters in a bulk medium. The collimation of the beam pair occurs because XPM-induced focusing compensates for diffraction divergence and self-defocusing. We derive analytical expressions for the critical parameters, incorporating input powers, beam radii, and medium parameters. The conditions for a stable symbiotic solitary pair, propagating without filament formation, are found.

THEORETICAL MODEL

Let us consider two beams, which are cw or quasi-cw, that are copropagating in a bulk medium with cubic nonlinearity. Within this framework we can neglect the group-velocity mismatch and the initial delay between the pulses and use a single laboratory coordinate system. Another reasonable approximation is to assume that the parametric processes are of low efficiency, i.e., that the energy of the beams does not change significantly.

The paraxial, coupled amplitude equations, which describe the evolution of the beams, have the form[8]

$$i\frac{\partial\psi_1}{\partial x} + \alpha_1\frac{\partial^2\psi_1}{\partial r^2} + k^{\mathrm{SPM}}(\lambda_1)|\psi_1|^2\psi_1 + k^{\mathrm{XPM}}(\lambda_1)|\psi_2|^2\psi_1 = 0,$$
$$i\frac{\partial\psi_2}{\partial x} + \alpha_2\frac{\partial^2\psi_2}{\partial r^2} + k^{\mathrm{SPM}}(\lambda_2)|\psi_2|^2\psi_2 + k^{\mathrm{XPM}}(\lambda_2)|\psi_1|^2\psi_2 = 0, \tag{1}$$

where $\alpha_i = -1/(2k_i)$ $(i = 1, 2)$ are the wave vectors and $k^{\mathrm{SPM}}(\lambda_i)$ and $k^{\mathrm{XPM}}(\lambda_i)$ are the nonlinear coefficients for SPM and XPM:

$$k^{\mathrm{SPM}}(\lambda_i) = -\frac{n_2{}^{\mathrm{SPM}}(\lambda_i)k_i}{2n_{0i}} = -\frac{k_i}{2n_{0i}}\frac{3\pi N\chi^{(3)}_{\mathrm{SPM}}(\lambda_i)}{n_{0i}},$$
$$k^{\mathrm{XPM}}(\lambda_i) = -\frac{n_2{}^{\mathrm{XPM}}(\lambda_i)k_i}{2n_{0i}} = -\frac{k_i}{2n_{0i}}\frac{6\pi N\chi^{(3)}_{\mathrm{XPM}}(\lambda_i)}{n_{0i}}.$$

N is the particle density, and $\chi^{(3)}$ denotes the corresponding nonlinear susceptibility. For simplicity we assume that $k^{\mathrm{XPM}}(\lambda_1) = k^{\mathrm{XPM}}(\lambda_2) = k^{\mathrm{XPM}}$.

Equations (1) are solved by the variational method,[5,14] reducing them to a system of coupled ordinary differential equations for the respective variational parameters. The analytical representation of the results is the main advantage of this method. It can be shown that Eqs. (1) are Euler–Lagrange equations for the Lagrangian

$$L = L_1 + L_2 + L_{\mathrm{XPM}}, \tag{2a}$$

where

$$L_1 = (i/2)\left(\psi_1{}^*\frac{\partial\psi_1}{\partial x} - \psi_1\frac{\partial\psi_1{}^*}{\partial x}\right) - \alpha_1\left|\frac{\partial\psi_1}{\partial r}\right|^2 + \frac{k^{\mathrm{SPM}}(\lambda_1)}{2}|\psi_1|^4, \tag{2b}$$

$$L_2 = (i/2)\left(\psi_2{}^*\frac{\partial\psi_2}{\partial x} - \psi_2\frac{\partial\psi_2{}^*}{\partial x}\right) - \alpha_2\left|\frac{\partial\psi_2}{\partial r}\right|^2 + \frac{k^{\mathrm{SPM}}(\lambda_2)}{2}|\psi_2|^4, \tag{2c}$$

$$L_{\mathrm{XPM}} = k^{\mathrm{XPM}}|\psi_1|^2|\psi_2|^2. \tag{2d}$$

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We assume that the beams have a Gaussian form:

$$\psi_1(r,x) = \frac{A_1(x)}{\omega_1(x)}\exp\left[-\frac{r^2}{a_1{}^2\omega_1{}^2(x)} - i\frac{k_1\rho_1(x)r^2}{2}\right], \quad (3a)$$

$$\psi_2(r,x) = \frac{A_2(x)}{\omega_2(x)}\exp\left[-\frac{r^2}{a_2{}^2\omega_2{}^2(x)} - i\frac{k_2\rho_2(x)r^2}{2}\right], \quad (3b)$$

where A_1 and A_2 are the complex, slowly varying amplitudes, ω_1 and ω_2 are the normalized radii of the beams, a_1 and a_2 are the initial physical radii of the beams on the $1/e$ level, and ρ_1 and ρ_2 are functions of the inverse radii of curvature of the respective wave fronts. Finally, the initial conditions for the variational parameters are $\omega_{1,2}(x = 0) = 1$ and $\rho_{1,2}(x = 0) = 0$. Following the variational procedure mentioned above, we derive the following set of differential equations:

$$\frac{d\rho_1}{dx} = -\rho_1{}^2 + \frac{4}{k_1{}^2a_1{}^4\omega_1{}^4} + \frac{\sqrt{2}k^{\mathrm{SPM}}(\lambda_1)|A_1|^2}{a_1{}^2\omega_1{}^4k_1} + \frac{4k^{\mathrm{XPM}}|A_2|^2a_2}{\omega_2k_1(a_1{}^2\omega_1{}^2 + a_2{}^2\omega_2{}^2)^{3/2}}, \quad (4a)$$

$$\frac{d\rho_2}{dx} = -\rho_2{}^2 + \frac{4}{k_2{}^2a_2{}^4\omega_2{}^4} + \frac{\sqrt{2}k^{\mathrm{SPM}}(\lambda_2)|A_2|^2}{a_2{}^2\omega_2{}^4k_2} + \frac{4k^{\mathrm{XPM}}|A_1|^2a_1}{\omega_1k_2(a_1{}^2\omega_1{}^2 + a_2{}^2\omega_2{}^2)^{3/2}}, \quad (4b)$$

$$\frac{d\omega_1}{dx} = \omega_1\rho_1, \quad (4c)$$

$$\frac{d\omega_2}{dx} = \omega_2\rho_2. \quad (4d)$$

When equal initial beam diameters ($a_1 = a_2 = a$) are assumed, the balance conditions for propagation of beams with compensated diffraction divergence [$\omega_{1,2}(x) = 1$, $\rho_{1,2}(x) = 0$] have the forms

$$k^{\mathrm{SPM}}(\lambda_1)|A_1|^2 + k^{\mathrm{XPM}}|A_2|^2 = -\frac{2\sqrt{2}}{k_1a^2}, \quad (5a)$$

$$k^{\mathrm{SPM}}(\lambda_2)|A_2|^2 + k^{\mathrm{XPM}}|A_1|^2 = -\frac{2\sqrt{2}}{k_2a^2}. \quad (5b)$$

Considering single-beam propagation in a nonlinear medium, from Eqs. (5) one can obtain an analytical expression for the critical power for self-focusing.[1] Solutions of Eqs. (5) are valid if $k^{\mathrm{SPM}}(\lambda_1)k^{\mathrm{SPM}}(\lambda_2) \neq (k^{\mathrm{XPM}})^2$ and, in terms of pump powers, they have the form

$$P_1 = -\frac{2\sqrt{2}c}{8.10^7}\frac{\{[k^{\mathrm{SPM}}(\lambda_2)/k_1] - (k^{\mathrm{XPM}}/k_2)\}}{k^{\mathrm{SPM}}(\lambda_1)k^{\mathrm{SPM}}(\lambda_2) - (k^{\mathrm{XPM}})^2}, \quad (6a)$$

$$P_2 = -\frac{2\sqrt{2}c}{8.10^7}\frac{\{[k^{\mathrm{SPM}}(\lambda_1)/k_2] - (k^{\mathrm{XPM}}/k_1)\}}{k^{\mathrm{SPM}}(\lambda_1)k^{\mathrm{SPM}}(\lambda_2) - (k^{\mathrm{XPM}})^2}. \quad (6b)$$

In Eqs. (6) the power is in watts, the wave numbers in inverse centimeters, and the nonlinear coefficients in electrostatic units.

In what follows we discuss three possible regimes for symbiotic beam formation, based on certain nonlinear coefficient sign and value combinations.

Case (a): $n_2{}^{\mathrm{SPM}}(\lambda_{1,2}) > 0$ and $n_2{}^{\mathrm{XPM}} > 0$ [i.e., $k^{\mathrm{SPM}}(\lambda_{1,2}) < 0$ and $k^{\mathrm{XPM}} < 0$]. In this case the pump powers are below the critical powers for self-focusing [see Eqs. (5)]. Spatial modulation instability does not normally occur in this regime.[15] Solutions are possible when $k^{\mathrm{SPM}}(\lambda_1)k^{\mathrm{SPM}}(\lambda_2) \lesseqgtr (k^{\mathrm{XPM}})^2$.

Case (b): $n_2{}^{\mathrm{SPM}}(\lambda_{1,2}) > 0$ and $n_2{}^{\mathrm{XPM}} < 0$ [i.e., $k^{\mathrm{SPM}}(\lambda_{1,2}) < 0$ and $k^{\mathrm{XPM}} > 0$]. The pump powers should be kept above the critical values for self-focusing, and the channeling of the beams is due to the induced defocusing that originates in the XPM process. Solutions are possible only if $k^{\mathrm{SPM}}(\lambda_1)k^{\mathrm{SPM}}(\lambda_2) > (k^{\mathrm{XPM}})^2$.

Case (c): $n_2{}^{\mathrm{SPM}}(\lambda_{1,2}) < 0$ and $n_2{}^{\mathrm{XPM}} > 0$ [i.e., $k^{\mathrm{SPM}}(\lambda_{1,2}) > 0$ and $k^{\mathrm{XPM}} < 0$]. The XPM process counteracts the self-defocusing and the diffraction spreading of the beams. This regime is possible only if $k^{\mathrm{SPM}}(\lambda_1)k^{\mathrm{SPM}}(\lambda_2) < (k^{\mathrm{IPM}})^2$.

SPATIAL STABILITY ANALYSIS

We analyze the stability of the steady state by considering how weak perturbations evolve along the nonlinear medium. If they grow exponentially, the steady-state regime of propagation is unstable. Let the perturbed amplitudes have the form

$$A_i(r,x) = [A_i + \delta A_i(r,x)]\exp(-i\Gamma_i x), \quad (7)$$

where $|\delta A_i| \ll |A_i|$ and $\Gamma_i = \Gamma_i{}^{\mathrm{SPM}} + \Gamma_i{}^{\mathrm{XPM}}$ are the nonlinear parts of the wave vectors, originating from the self-phase-modulation and XPM processes [$\Gamma_i{}^j = (-\omega_i n_i{}^j/2c)|A_i|^2$, j = SPM, XPM]. When Eq. (7) is substituted into Eq. (1) and quadratic and higher-order terms in δA_i are neglected, the perturbations satisfy the following coupled equations:

$$\left\{\frac{\partial}{\partial x} - \alpha_1\frac{\partial^2}{\partial r^2}\right\}\delta A_1 = k^{\mathrm{SPM}}(\lambda_1)|A_1|^2(\delta A_1 + \delta A_1{}^*) + k^{\mathrm{XPM}}(|A_1|^2|A_2|^2)^{1/2} \times (\delta A_2 + \delta A_2{}^*), \quad (8a)$$

$$\left\{\frac{\partial}{\partial x} - \alpha_2\frac{\partial^2}{\partial r^2}\right\}\delta A_2 = k^{\mathrm{SPM}}(\lambda_2)|A_2|^2(\delta A_2 + \delta A_2{}^*) + k^{\mathrm{XPM}}(|A_1|^2|A_2|^2)^{1/2} \times (\delta A_1 + \delta A_1{}^*). \quad (8b)$$

As is usual[16,17] we assume a solution of the form

$$\delta A_i = \delta A_{i0}\exp[i(K_\perp^{(i)}r - h_i x)] + \text{c.c.}, \qquad i = 1,2, \quad (9)$$

where $K_\perp^{(i)}$ is the spatial frequency of the perturbation and h_i is the wave number. Substituting Eq. (9) into Eqs. (8), we get a system of four homogeneous equations for δA_i and $\delta A_i{}^*$. The condition for obtaining a nontrivial solution of this system permits the following spatial-stability conditions to be derived:

$$|K_\perp^{(i)}| \geq K_{\perp\mathrm{crit}}^{(i)}, \qquad i = 1,2. \quad (10a)$$

The critical values of the spatial frequencies are defined by the relations

$$K_{\perp\mathrm{crit}}^{(1)} = \left\{\frac{2}{\alpha_1}[k^{\mathrm{SPM}}(\lambda_1)|A_1|^2 + k^{\mathrm{XPM}}(|A_1|^2|A_2|^2)^{1/2}]\right\}^{1/2}, \quad (10b)$$

$$K_{\perp\mathrm{crit}}^{(2)} = \left\{\frac{2}{\alpha_2}[k^{\mathrm{SPM}}(\lambda_2)|A_2|^2 + k^{\mathrm{XPM}}(|A_1|^2|A_2|^2)^{1/2}]\right\}^{1/2}. \quad (10c)$$

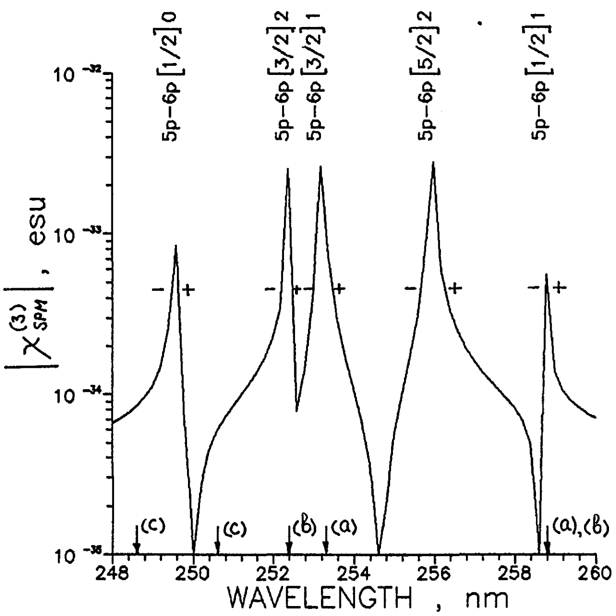

Fig. 1. Dispersion and sign of the $\chi^{(3)}_{\mathrm{SPM}}$ nonlinear coefficient in Xe I, near the $5p^6$–$6p$ resonances. The wavelength pairs (λ_1, λ_2) are denoted for the three cases considered.

For single-beam propagation in a nonlinear medium, this result reproduces a previously obtained one.[15] It is well known that self-focusing becomes stable when diffraction is limited to one spatial dimension, such as in a planar waveguide.[18] This result is of particular importance, since with opposite signs of the nonlinear coefficients $k^{\mathrm{SPM}}(\lambda_{1,2})$ and k^{XPM} the spatial stability conditions can be improved significantly.

RESULTS AND DISCUSSION

As was seen above, the symbiotic-beam-pair formation depends crucially on the values and signs of the nonlinear susceptibilities and the pump powers.

The inert gases seem to be proper nonlinear media. In the vicinity of $\lambda_i + \lambda_j$ two-photon resonances, the $\chi^{(3)}$ nonlinear coefficients are determined mostly by terms of the type

$$\sum_F \frac{{r_{GA}}^2 {r_{AF}}^2}{(\omega_{GF} - \omega_i - \omega_j - \mathrm{i}\gamma_F)}[(\omega_{GA} - \omega_i)^{-2} + (\omega_{GA} - \omega_j)^{-2}], \qquad i,j = 1,2, \quad (11)$$

where r_{ij} denotes the corresponding dipole matrix element and γ_F is the dumping constant of the transition. Far from single-photon resonances the allocation of $\lambda_1 + \lambda_2$, $2\lambda_1$, and $2\lambda_2$ with respect to the two-photon resonances will determine the values and the signs of the nonlinear susceptibilities. Figure 1 shows the dispersion of the $\chi^{(3)}_{\mathrm{SPM}}$ nonlinear coefficient in Xe I ($p = 1$ atm $= 760$ Torr) near the $5p^6$–$6p$ resonances.

The regime of subcritical pump powers, denoted Case (a), can be exemplified through the choice of $\lambda_1 = 253.3$ nm and $\lambda_2 = 258.78$ nm. The corresponding nonlinear susceptibilities are calculated to be

$$\chi^{(3)}_{\mathrm{SPM}}(\lambda_1) = 1.8 \times 10^{-33}\ \mathrm{esu}, \qquad \chi^{(3)}_{\mathrm{SPM}}(\lambda_2) = 1.4 \times 10^{-33}\ \mathrm{esu},$$
$$\chi^{(3)}_{\mathrm{XPM}} = 10^{-32}\ \mathrm{esu}.$$

The powers required for symbiotic-beam-pair formation, $P_1 = 7.2 \times 10^3$ W and $P_2 = 6.8 \times 10^3$ W, are at least two times lower than the critical powers for self-focusing and induced focusing.

As an illustration, for Case (b) one can choose $\lambda_1 = 252.44$ nm and $\lambda_2 = 258.77$ nm, yielding

$$\chi^{(3)}_{\mathrm{SPM}}(\lambda_1) = 2.4 \times 10^{-33}\ \mathrm{esu},$$
$$\chi^{(3)}_{\mathrm{SPM}}(\lambda_2) = 1.7 \times 10^{-32}\ \mathrm{esu},$$
$$\chi^{(3)}_{\mathrm{XPM}} = -1.1 \times 10^{-33}\ \mathrm{esu}.$$

The powers needed for symbiotic nonlinear propagation are $P_1 = 78.3$ kW and $P_2 = 18.5$ kW. The corresponding critical powers for self-focusing are calculated to be 32.5 and 4.8 kW, but this process is prevented from occurring by the XPM-induced mutual defocusing. Let us define the parameter η, which determines the improvement of the spatial modulation stability:

$$\eta_i = \left[\frac{k^{\mathrm{SPM}}(\lambda_i)|A_i|^2 + k^{\mathrm{XPM}}(|A_1|^2|A_2|^2)^{1/2}}{k^{\mathrm{SPM}}(\lambda_i)|A_i|^2}\right]^{1/2}, \qquad i = 1,2. \quad (12)$$

For the case considered, $\eta_1 = 0.75$ and $\eta_2 = 0.86$; i.e., filament formation could be caused by larger transverse spatial perturbation.

The regime denoted Case (c) can be illustrated by choosing $\lambda_1 = 248.6$ nm and $\lambda_2 = 250.6$ nm. The calculated nonlinear susceptibilities are found to be

$$\chi^{(3)}_{\mathrm{SPM}}(\lambda_1) = -8.5 \times 10^{-35}\ \mathrm{esu},$$
$$\chi^{(3)}_{\mathrm{SPM}}(\lambda_2) = -5.9 \times 10^{-35}\ \mathrm{esu},$$
$$\chi^{(3)}_{\mathrm{XPM}} = 5.1 \times 10^{-33}\ \mathrm{esu},$$

which correspond to pump powers of 14.1 and 14 kW for λ_1 and λ_2, respectively. The critical power for induced focusing was calculated to be 7.4 kW. The diffraction divergence and self-defocusing compensation are due to the XPM-induced focusing of the beams. A perturbation of the beam will tend to spread out, and therefore this mode is not critical with respect to optical breaking of the medium.

The values and signs of the nonlinear susceptibilities were calculated by the Feynman-diagrams technique. The accuracy of the calculation is estimated to be within 40%.

The approximation of the beams as complex Gaussians [Eqs. (3)] limits the validity of the results to single-mode beams. The higher-order Gaussian beams should be described by the Laguerre–Gauss functions in the cylindrical geometry.[10] The model presented holds for cw or quasi-cw beams down to the nanosecond range. In the picosecond range the possible initial delay and the group-velocity mismatch can restrict the interaction length and must be carefully considered. The single- and two-photon absorption can also limit the length of the region of the symbiotic laser-beam pair. Assuming that a relative intensity change of less than 1% will not significantly alter the balance condition [Eq. (5)], we can estimate the maximum length for stable mode propagation. The characteristic pulse-to-pulse amplitude stability is within 5% or more (the amplitude varies from pulse to pulse within

5%). Our estimations have shown that, in the wavelength range considered, the single-photon absorption cross section $\sigma^{(1)}$ is of the order of 1.5×10^{-23} cm^2. The two-photon absorption varies between $\sigma^{(2)}_{(\lambda_1)} = 5.7 \times 10^{-30}$ cm^4/W [Case (a)] and $\sigma^{(2)}_{(\lambda_1)} = 3.4 \times 10^{-35}$ cm^4/W [Case (c)]. Therefore the maximum length for stable symbiotic beam propagation extends to 25 cm in the three cases considered.

The mode of propagation analyzed can be quite useful in four-wave mixing experiments in resonant media or photonic switching, for example. The effective interaction length can be extended, compared with that in a tight-focusing configuration, without using a hollow-core capillary.[19] The counteraction of self-phase modulation and XPM [Cases (b) and (c)] can reduce the spectral-broadening enhancement of the pump pulses, resulting in a higher conversion efficiency.

CONCLUSION

We have shown that, when cross-phase modulation and self-phase modulation are taken into account, two copropagating cw or quasi-cw beams of comparable intensity can form a solitonlike symbiotic pair. The beams are symbiotic in the sense that each of them depends critically on the XPM that originates in the second beam. The diffraction divergence and the self-defocusing can be offset by a proper choice of the beam parameters, the sign, and the value of the nonlinear coefficient of the medium.

Although the inert gases are specifically considered, the spatial symbiotic pair could be formed also in multimode fibers, planar structures, and other condensed bulk media by a proper wavelength or polarization of the beams. Conditions are found for stable waveguiding of the beam pair without filament formation, which may have important applications in nonlinear optics, photonic switching, and the transmission of energy and information, for example.

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