Focal beam structuring by triple mixing of optical vortex lattices

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Well-known fact is that an ensemble of optical vortices (OVs) exhibits a fluidlike motion [1,2] that depends strongly on the geometrical configuration. For the first time the possibility to stabilize ensembles of equally-charged OVs against rotation by a proper choice of the topological charge of a "control OV" nested in the ensemble center is proved numerically in [2]. Specifically, if an OV with a TC opposite to the TCs of the rest of the OV ensemble is positioned in the center of the structure, the rotation of the entire ensemble is cancelled. An extension of such vortex ensembles towards large stable regular OV lattices is also analyzed in [2] and is studied for the first time experimentally in [3]. An extension to these studies is published in [4-6] where the beam reshaping in the focal plane of a lens due to the initial type of the elementary cells (square or hexagonal) in these optical vortex lattices is studied. Fig. 2 in [4] and Fig. 2 in [5] show the amplitude and phase profiles of a square-shaped and a hexagonal vortex lattice, along with their interference patterns. All these frames clearly show the elementary cell (square-shaped or hexagonal) and the alternating TCs of neighboring OVs. The latter is easily recognized for instance in Fig. 2(b) in [4] by the spiral phase profile around each lattice node (black and white denote phase 0 and phase 2π, respectively) and by the fork-like splitting of the interference lines in Fig. 2(c) in [4] (upward or downward, depending on the sign of the TC, but always with |TC|=1).

In Fig. S1, the Fourier-transformation of the near field intensity distribution of a square-shaped and of a hexagonal optical vortex lattice (OVL) is sketched in row (a). The Fourier transformation of the OVL performed by the thin lens to the artificial far field is denoted by $\mathscr{F}$.

The focal pattern of a square-shaped OVL is always a rhomboidal structure consisting of four peaks (**(a)**; left panel), while the focal pattern of a hexagonal OLV consists of three bright peaks situated in the apices of an equilateral triangle (**(a)**; right panel). When two OVLs are mixed and subsequently focused, their Fourier transformation, according to the Convolution theorem, is equal to the convolution of the focal patterns of the individual Fourier-transformed OVLs. In agreement with the Scaling theorem, the OVL of smaller node-spacing determines the large-scale far-field (focal) structure and vice versa. In Fig. S1, in Case **(b)** the denser OVL is the square-shaped, while in Case **(c)** it is the hexagonal one. As a result, in Case **(b)** the focal structure resembles a rhomb consisting of peaks situated in the apices of triangles, while in Case **(c)** the structure is triangle-like with peaks situated in the apices of rhombs. It is straightforward to show that the Fourier-transformation of triple-mixed OVLs is equal to the convolution of the Fourier-transformations of the individual OVLs **(d)**. In row **(e)** is shown the same as in row **(d)**, but visualized with near-field OVLs (left three panels) and with the resulting focal pattern (right panel).

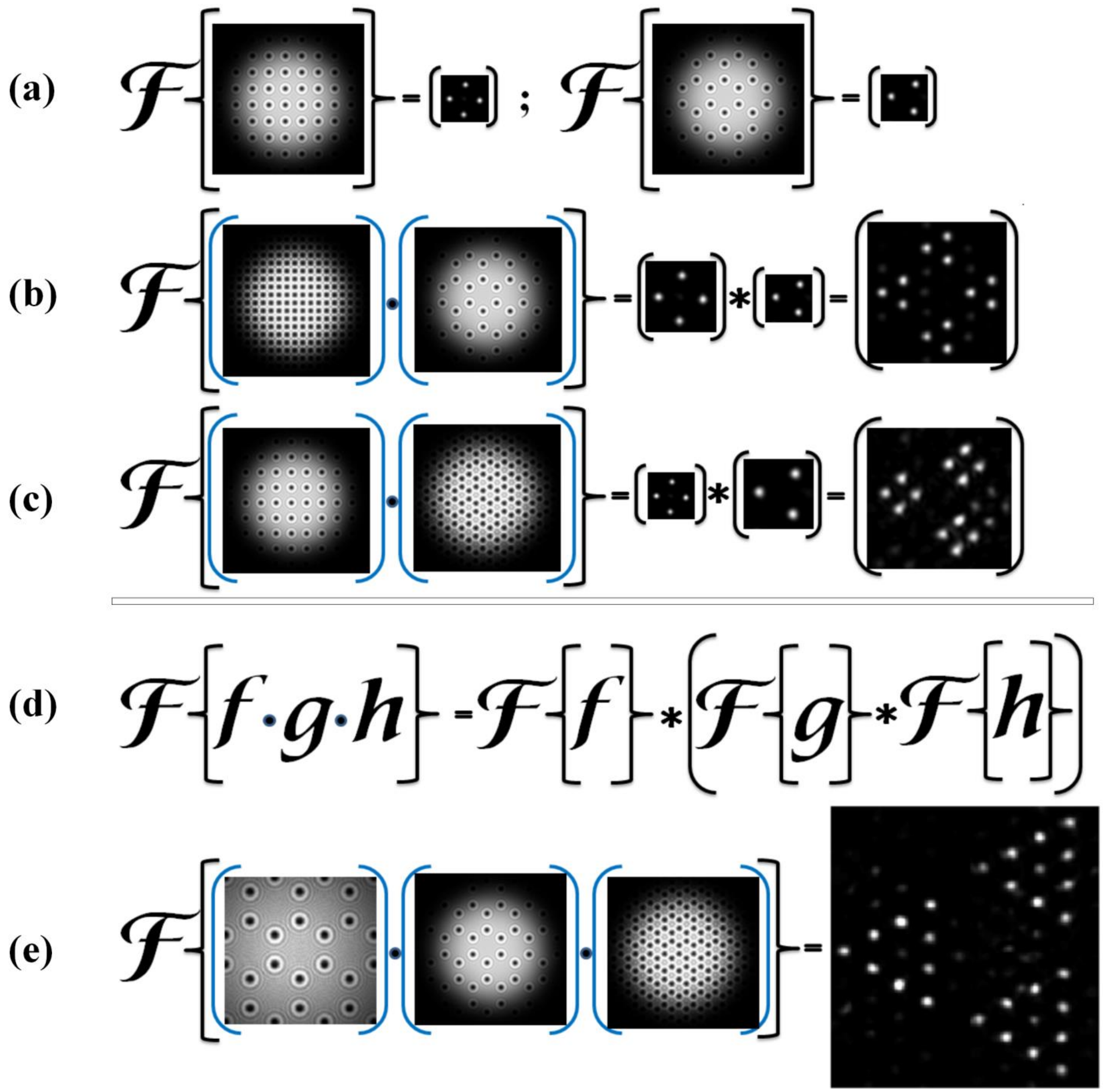

Figure S1. (a-e) Pictorial representation of the underlying results in the case of double and triple mixing of optical vortex lattices (OVLs). (See text for details).

In order to correctly reproduce the square-shaped and hexagonal elementary cells of the used OVLs with OVs of alternative TCs, one may consider adapting analytically the formula describing singly-charged OVs arranged on a circle (see Eq. 3 in [7]). However, there is one main problem with this analysis: Although that it is possible to create square, hexagonal or even a Bravais-type elementary cell, they are all formed on a common background beam and are consisting of the so-called *r*-vortices. Characteristic for them is that their widths are coupled to the width of the host beam. Unfortunately, we were not able to generate analytically large arrays from such beams (cells) on a common background beam and, in our opinion this will not be possible using this approach.

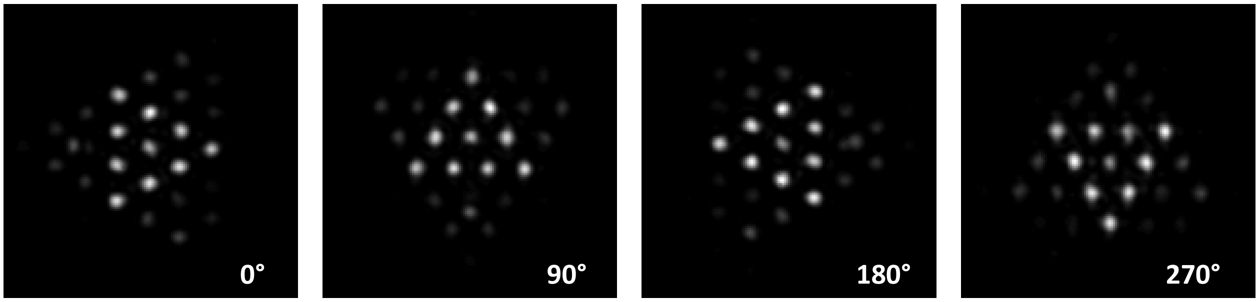

Figure S2. Rotation of the numerical sum (modulo 2π) of the phases of two hexagonal lattices with node spacings δ_{hx1} = 41pix. and δ_{hx2} = 81pix. encoded on SLM2 resulting in the rotation of the focal array of bright beams.

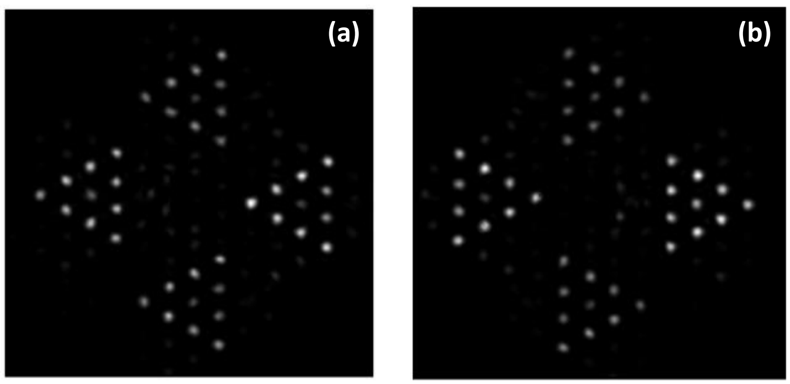

Figure S3. Triple mixing of one square-shaped and two hexagonal OV lattices. Input Gaussian beam is reflected from SLM1, programmed with the phase of a square OVL with Δ_{sq} = 21 pix. In the plane of SLM2 it is subsequently phase modulated with numerically added phases of hexagonal OVLs δ_{hx1} = 41 pix. and δ_{hx2} = 81 pix. **(a)** - Experimentally recorded intensity distribution of the triple mixed OVLs. **(b)** As discussed in the text, by changing the order of the creation of the individual lattices on the SLMs one can rotate the focal structures resulting from the hexagonal phase distributions and, thus, the whole focal array by 180°.

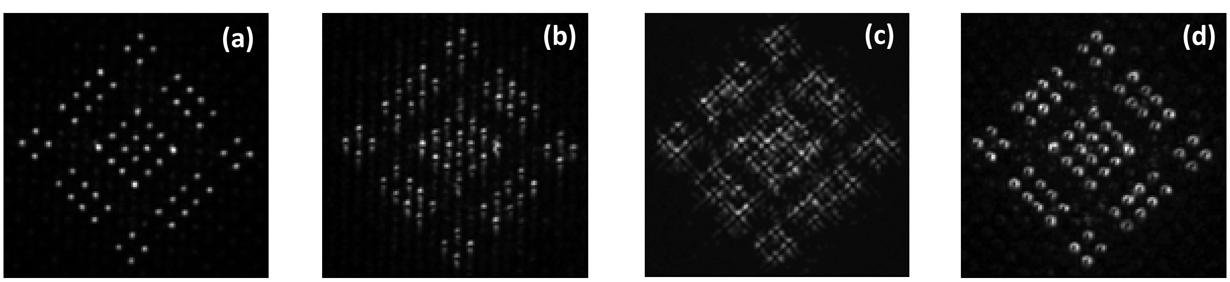

Figure S4. Experimentally recorded focal intensity distributions resulting from triple mixing of square-shaped OV lattices. **(a)** – Triple mixed square-shaped OV lattices with lattice constants Δ_{sq} = 41pix. and the sum δ_{sq1} = 21 pix. + δ_{sq2} = 101 pix. Additional structuring of the far-field intensity profiles of the same triple mixed square-shaped OV lattices by nesting 1-D dark beam **(b)**, a quasi-2-D dark beam **(c),** and by hosting an OV **(d)** in each bright focal beam of the structure.

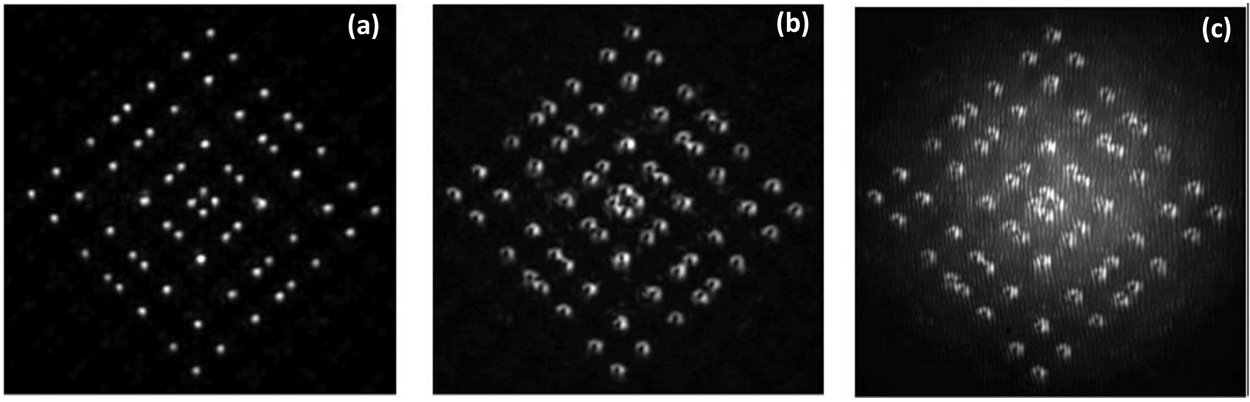

Figure S5. Triple mixed square-shaped OV lattices **(a)** with lattice constants Δ_{sq} = 41 pix. and the sum δ_{sq1} = 21 pix. + δ_{sq2} = 81 pix. Additional structuring **(b)** of the focal intensity profiles by hosting an OV in each bright focal beam and the corresponding interferogram **(c).** Note the upward fork-like splitting of one interference stripe in each bright peak.

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