Dispersion control in a folded 4-f system for shaping femtosecond laser pulses

N. Dimitrov, P. Lazarova, L. Stoyanov, I. Stefanov, A. Dreischuh

Department of Quantum Electronics, Faculty of Physics, Sofia University "St. Kliment Ohridski", 5, J. Bourchier Blvd., Sofia-1164, Bulgaria

ABSTRACT

In this work we study the influence of the additional second-order dispersion introduced in sub-45 femtosecond laser pulses by intentional misaligning a folded 4-f otherwise dispersionless system. The theoretically calculated pulse durations are found to be in a good agreement with the respective experimental data from frequency-resolved optical gating measurements.

Keywords: femtosecond pulses, dispersionless optical system, 4-f system, second order dispersion, transform-limited pulse, pulse shaping.

1. INTRODUCTION

Dispersionless 4-f setups[1] are frequently used for various pulse- and beam shaping applications including creation of spatial phase singularities in broadband femtosecond beams/pulses[2,3]. Eventual misalignments in such systems are expected to result in an uncompensated group-delay dispersion (GDD), in spatial and angular chirp, time-vs.-angle coupling, and pulse front tilt. This later pulse distortion is understood as an arrival time difference which depends on the transverse beam coordinate[4]. In this work we study the influence of the additional second-order dispersion introduced in sub-45 femtosecond laser pulses by intentional misalignment of a folded 4-*f* otherwise dispersionless system (4FDS; see Fig. 1).

In the first part of this work we derived analytical expressions for the above mentioned parameters vs. misalignments of the folded 4FDS. Our approach is based on the 4x4 ray-pulse matrices introduced by Kostenbauder[4] and generalized by Duarte[5] and Trebino and co-workers[6]. In essence, the 4x4 matrices connect the input and output ray and pulse coordinates to each other. The spatial and temporal characteristics of the pulse are represented in a ray-pulse vector $(x, \theta, t, \nu)^T$. The spatial coordinates position (x) and slope (θ) are the same as in the ordinary 2x2 matrices. The coordinate system is defined by the path of a diffraction limited reference beam of a central Hertzian frequency ν. When the pulses are transform-limited, this path corresponds to a well-defined arrival time at each transverse plane[4]. In this 4x4 matrix approach the action of an optical element can be described as:

$$\begin{pmatrix} x \\ \theta \\ t \\ \nu \end{pmatrix}_{out} = \begin{pmatrix} A & B & 0 & E \\ C & D & 0 & F \\ G & H & 1 & I \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ \theta \\ t \\ \nu \end{pmatrix}_{in} = \begin{pmatrix} \frac{\partial x_{out}}{\partial x_{in}} & \frac{\partial x_{out}}{\partial \theta_{in}} & 0 & \frac{\partial x_{out}}{\partial \nu_{in}} \\ \frac{\partial \theta_{out}}{\partial x_{in}} & \frac{\partial \theta_{out}}{\partial \theta_{in}} & 0 & \frac{\partial \theta_{out}}{\partial \nu_{in}} \\ \frac{\partial t_{out}}{\partial x_{in}} & \frac{\partial t_{out}}{\partial \theta_{in}} & 1 & \frac{\partial t_{out}}{\partial \nu_{in}} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ \theta \\ t \\ \nu \end{pmatrix}_{in}, \tag{1}$$

where the elements *I* and *F* stand for the group delay dispersion (GDD) and for the angular dispersion, whereas the matrix elements *E* and *G* have the meaning of spatial chirp and pulse front tilt parameters. In this notation the pulse front tilt angle γ is given by $\tan\gamma = cG\frac{\partial x_{in}}{\partial x_{out}}$, where *c* is the speed of light.

Next we calculated the output duration τ_{pulse} of a known initial Gaussian pulse of input width t_0 and field amplitude $\boldsymbol{E}$

$$\boldsymbol{E(t, z=0) = E_0 e^{-(1+i a)(t/t_0)^2}} \tag{2}$$

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after passing through the intentionally misaligned 4FDS and accumulating the calculated amount of GDD. For this purpose we used the simple relation given by Diels and Rudolph [3]

$$\boldsymbol{\tau_{pulse}} = \sqrt{\frac{4}{\xi} * (\xi^2 + \eta^2)} \begin{cases} \xi = \frac{{t_0}^2}{4(1+\mathrm{a}^2)} \\ \eta = \frac{\mathrm{a}{t_0}^2}{4(1+\mathrm{a}^2)} - \frac{\boldsymbol{GDD}}{2} \end{cases} \quad . \tag{3}$$

In the above relation the longitudinal coordinate z=0 denotes the entrance of the 4FDS system, t_0, as stated, is the initial temporal pulse duration, a is the initial chirp parameter, and GDD is the introduced group delay dispersion by the misaligned 4-f system.

2. THEORETICAL ANALYSIS OF THE MISALIGNMENTS IN A FOLDED 4-F SYSTEM

By placing a mirror in the Fourier plane of a classical 4-f system one can fold it. The mirror (see Fig.1) is sending back the beam/pulse to pass twice the elements, and, effectively, we have the same behavior as in a single-pass classical system, but with the half of its elements. In Fig.1 we show a sketch of the analyzed folded 4f-system consisting of a single diffraction grating, one lens with focal length f, and one mirror. We introduced two different deviations from the precise alignment of the elements - a shift on δf of the perfect lens position, and a synchronous shift on Δf of the lens and mirror as a pair.

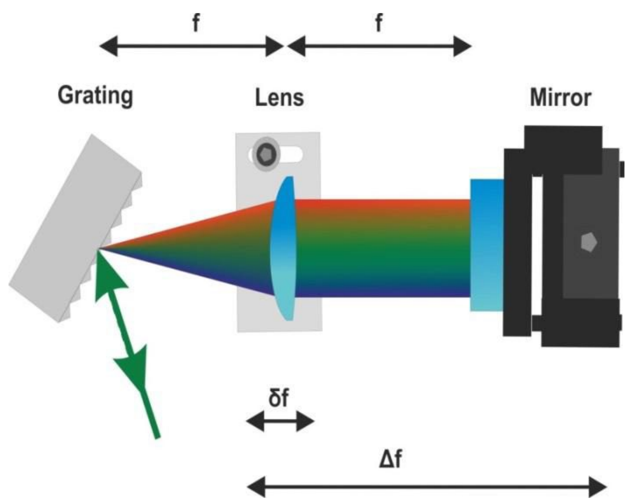

Figure 1. Sketch of the analyzed folded 4f-system. δf– lens offset from the perfect alignment. Δf– synchronous lens and mirror offset from the perfect alignment.

The explicit forms of the matrices for the different elements we have are:

- for the grating on first and on the second pass, respectively:

$$\mathcal{G}_{first\ pass} = \begin{pmatrix} -\frac{Sin[\beta]}{Sin[\alpha]} & 0 & 0 & 0 \\ 0 & -\frac{Sin[\alpha]}{Sin[\beta]} & 0 & \frac{Cos[\beta]-Cos[\alpha]}{v_0\ Sin[\beta]} \\ \frac{Cos[\alpha]-Cos[\beta]}{c\ Sin[\alpha]} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad , \tag{4}$$

$$\mathcal{G}_{second\ pass} = \begin{pmatrix} -\frac{Sin[\alpha]}{Sin[\beta]} & 0 & 0 & 0 \\ 0 & -\frac{Sin[\beta]}{Sin[\alpha]} & 0 & -\frac{Cos[\beta]-Cos[\alpha]}{v_0\ Sin[\beta]} \\ -\frac{Cos[\alpha]-Cos[\beta]}{c\ Sin[\alpha]} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad . \tag{5}$$

Here α is the incident beam-to-grating surface angle and $ß$ is the diffracted beam-to-grating surface angle.

- for the lens:

$$\mathcal{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad , \tag{6}$$

and

- for the free space between the grating and the lens (respectively for the lens-to-mirror distance):

$$\mathcal{T}_1 = \begin{pmatrix} 1 & (f + \delta f + \Delta f) & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathcal{T}_2 = \begin{pmatrix} 1 & (\mathrm{f} - \delta\mathrm{f}) & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{7}$$

According to the procedure of the matrix analysis one has to multiply the respective ray-pulse matrices in reverse order (from the exit to the entrance), i.e.:

$$\mathcal{M} = \mathcal{G}_{second\ pass}.\mathcal{T}_1.\mathcal{L}.\mathcal{T}_2.\mathcal{T}_2.\mathcal{L}.\mathcal{T}_1.\mathcal{G}_{first\ pass} \tag{8}$$

In this way we obtained the following the matrix for the whole system

$$M = \begin{pmatrix} -\frac{f^2+2\delta f(\delta f+\Delta f)}{f^2} & -\frac{2(\delta f+\Delta f)(f^2+\delta f(\delta f+\Delta f))Csc[\beta]^2 Sin[\alpha]^2}{f^2} & 0 & \frac{2(\delta f+\Delta f)(f^2+\delta f(\delta f+\Delta f))(-Cos[\alpha]+Cos[\beta])Csc[\beta]^2 Sin[\alpha]}{\nu_0 f^2} \\ -\frac{2\delta f\, Csc[\alpha]^2 Sin[\beta]^2}{f^2} & -\frac{f^2+2\delta f(\delta f+\Delta f)}{f^2} & 0 & -\frac{2\delta f(\delta f+\Delta f)(Cos[\alpha]-Cos[\beta])Csc[\alpha]}{\nu_0 f^2} \\ -\frac{2\delta f(\delta f+\Delta f)(Cos[\alpha]-Cos[\beta])Csc[\alpha]}{c f^2} & \frac{2(\delta f+\Delta f)(f^2+\delta f(\delta f+\Delta f))(-Cos[\alpha]+Cos[\beta])Csc[\beta]^2 Sin[\alpha]}{c f^2} & 1 & -\frac{2(\delta f+\Delta f)(f^2+\delta f(\delta f+\Delta f))(Cot[\beta]-Cos[\alpha]Csc[\beta])^2}{c \nu_0 f^2} \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{9}$$

This result allows as to extract analytical expressions for dependences between the following physical quantities and the introduced misalignments Δf and δf:

- GDD:

$$\boldsymbol{GDD} = \frac{\boldsymbol{I}}{2\pi} = -\frac{2(\delta\mathrm{f}+\Delta\mathrm{f})(\mathrm{f}^2+\delta\mathrm{f}(\delta\mathrm{f}+\Delta\mathrm{f}))(\mathrm{Cot}[\beta]-\mathrm{Cos}[\alpha]\mathrm{Csc}[\beta])^2}{\mathrm{c}2\pi\nu_0\mathrm{f}^2} \quad . \tag{10}$$

- Pulse front tilt angle γ:

$$\boldsymbol{tan\,\gamma} = c\boldsymbol{G}\frac{\partial x_{in}}{\partial x_{out}} = -\frac{2\delta\mathrm{f}(\delta\mathrm{f}+\Delta\mathrm{f})(\mathrm{Cos}[\alpha]-\mathrm{Cos}[\beta])\mathrm{Csc}[\alpha]}{\mathrm{f}^2}\frac{\partial x_{in}}{\partial x_{out}} \quad . \tag{11}$$

- Angular dispersion

$$\boldsymbol{F} = -\frac{2\delta f(\delta f+\Delta f)(Cos[\alpha]-Cos[\beta])Csc[\alpha]}{\nu_0 f^2} \quad . \tag{12}$$

- Spatial chirp

$$\boldsymbol{SC} = \frac{2(\delta f+\Delta f)(f^2+\delta f(\delta f+\Delta f))(-Cos[\alpha]+Cos[\beta])Csc[\beta]^2 Sin[\alpha]}{\nu_0 f^2} \quad . \tag{13}$$

In this way, knowing the net GDD introduced by the intentionally misaligned 4FDS system (Eq. 10), it is possible to predict the output pulse duration τ_{pulse} using Eq. 3.

3. EXPERIMENTAL DATA AND ANALYSIS OF THE RESULTS

In our experimental setup the laser source we used was a commercial Ti:Sapphire mode-locked femtosecond oscillator Ti-Light (Quantronix). After additional extra-cavity pulse compression with a cascade of 8 chirp mirrors it generates almost non-chirped pulses with temporal duration of about 45 fs ($1/e^2$), emitted at a 80 Mhz repetition rate, with average output power of about 160 mW. The pulse durations are measured by a frequency-resolved optical gating device GRENOUILLE. Folded 4FDS is based on the scheme represented by Fig. 1, where the diffraction grating is with 1200 l/mm, the lens is cylindrical with 12 cm focal length, and we have a flat silver mirror as a back reflector. The lens-mirror pair is mounted on micrometric translation stage, and the lens itself is independently movable being positioned on another translation table. That allowed us to make two sets of pulse duration measurements. In the first one we varied the position of the lens only (by δf), while in the second one we simultaneously changed the lens and folding mirror positions (by Δf) keeping the lens-to-mirror distance unchanged. For every chosen value of possible misalignment δf or Δf we applied our theoretical model and calculate the expected pulse duration at the output of the 4-f system. Figure 2 shows the comparison between the obtained theoretical and experimental results in the case of an intentional misalignment δf of the lens only. In Fig. 3 we present a comparison between the obtained theoretical and experimental results in the case of an intentional synchronous misalignment Δf of the lens and mirror pair. As seen, the agreement is fairly good. This approach of fine tuning the 4FDS allowed us to scale continuously the output pulse duration from 45 fs to 70 fs.

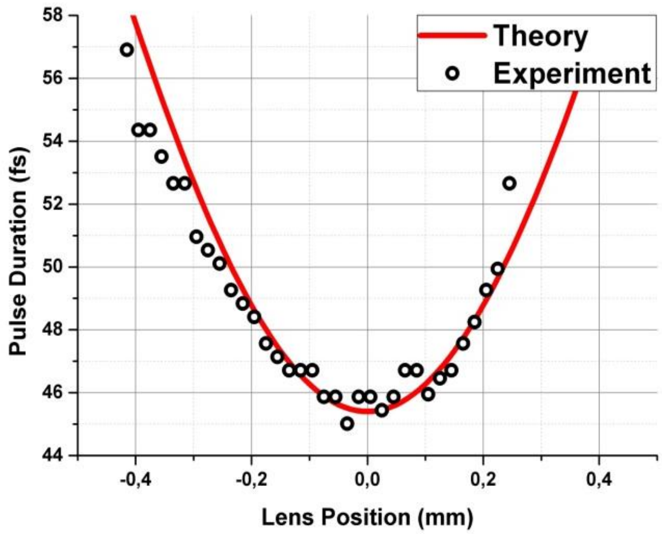

Figure 2. Dependence of the output pulse duration on the intentional misalignment δf of the lens only. Black circles - experimental data; red curve– numerical results.

4. CONCLUSION

We successfully tuned the input sub-45 fs pulses from a mode-locked Ti:Sapphire laser to 70 fs by intentionally misaligning otherwise dispersionless folded 4-f optical system. The analytical results obtained by a matrix approach are in a fair agreement with the experimental data. The knowledge about the alignment tolerances of such folded 4-f system is important in view of our future plans to replace the folding mirror by a spatial light modulator and to implement a genetic algorithm for femtosecond beam/pulse shaping.

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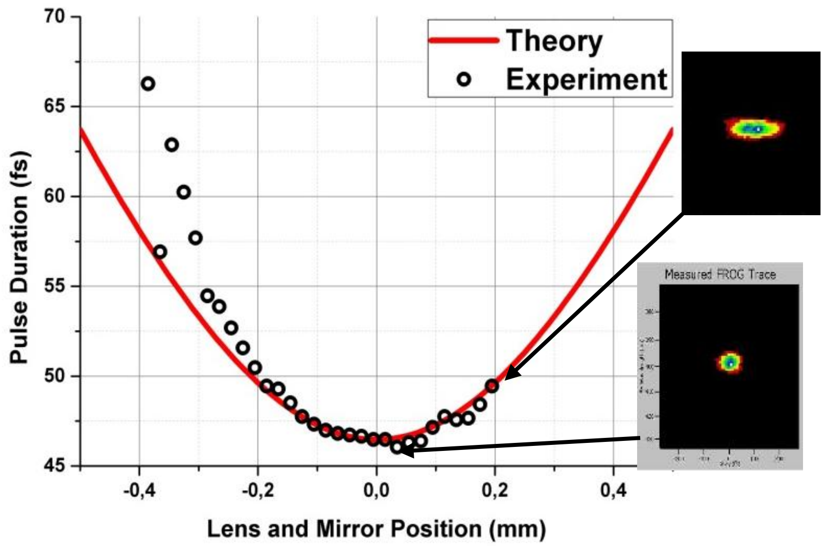

Figure 3. Dependence of the output pulse duration on the intentional synchronous misalignment Δ*f* of the lens and mirror pair. Black circles - experimental data; red curve– numerical results. Insets - measured FROG traces at specific positions.