Topological-charge controlled interaction within ordered structures of optical vortex solitons

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## ABSTRACT

The feasibility to stabilise ensembles of optical vortex solitons with respect to their positions on a common background beam is numerically demonstrated. The topological charge of the vortices is found to rule the propagation characteristics of the ensemble as a whole. The most stable configurations regarding ensemble rotation/repulsion are discussed.

**Keywords**: optical vortex soliton, topological charge, ensemble, interaction, all-optical control

## 1. INTRODUCTION

Optical vortices are characterised as two-dimensional screw dislocations on plane wavefronts. The existence of such dislocations requires that the amplitude of the electric field becomes zero at the dislocation centre. From mathematical point of view the screw phase-profile is described by the multiplier exp(im$\varphi$), where $\varphi$ is the azimuthal coordinate and m is an integer number which sign determines the direction of the dislocation. The counterclockwise direction (m>0) is known to be a positive Topological Charge (TC). In the free-space propagation regime pairs of vortices of the same TCs are found to propagate linearly retaining their relative positions and their positions within the host beam[1]. In contrast, vortices of opposite charges attract each other[1,2], which could lead to their annihilation. Collisions of two OVS pairs and linear arrays of vortices of alternatively TCs investigated[3], demonstrated elastical interaction-character.

In our previous work[4] we have found that the relative topological charge of a pair of Optical Vortex Solitons (OVSs) formed on different background beams rules the interaction characteristics (attraction/repulsion).

## 2. GOAL OF THE ANALYSIS

Because of the negative nonlinear refractive index $n_2$ required for the OVS formation the total refractive index appears lower in the dark beam wings as compared to this in the vicinity of its intensity-dip. As a consequence, OVSs do obey guiding properties[5]. A parallel transmission line based on OVSs in an ensemble should be stable with respect to the disposition of the individual dark beams. The eventual all-optical control of the ensemble evolution will open the way for future parallel switching devices based on OVSs. The analyses presented are aimed to provide feasible solutions to the problems formulated.

## 3. NUMERICAL PROCEDURE

The evolution of the dark beams imposed on a bright background is studied by solving the (2+1)D nonlinear Schrödinger equation:

$$i\frac{\partial A}{\partial z}+\left(2L_{Diff}\right)^{-1}\left\{\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right\}A+L_{NL}^{-1}|A|^2A=0, \tag{1}$$

where A is the complex slowly-varying field amplitude and the dimensionless transverse coordinates $\xi$ and $\eta$ are normalised to the dark beam radii $r_0$ (assumed to be equal). The soliton-mode of propagation of each individual optical vortex implies that the nonlinear and diffraction lengths[6] ($L_{NL}$ and $L_{Diff}$ , respectively) are equal. Eq.1 is solved by means of the 2D generalisation of the split-step Fourier method[6]. The accuracy of the results was confirmed by twice increasing the grid-mesh and by reducing twice the step of the calculation.



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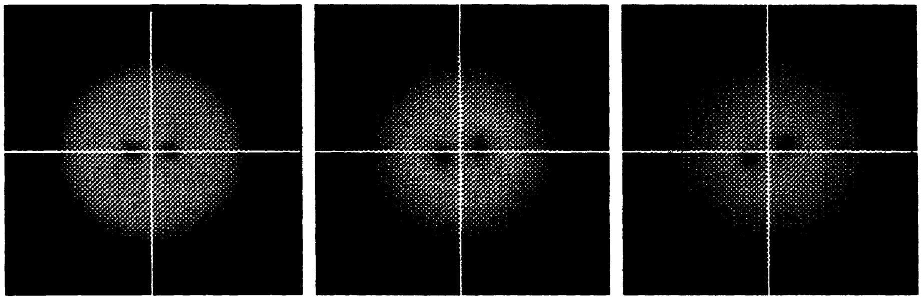
## 4. RESULTS AND DISCUSSION

In order to ensure conditions under which the undesired interaction between axially offset optical vortices and the finite background beam is absent we proved that super-Gaussian beam of a HWHM 15 times the OVS-radius is well suited for our analyses. The explicit form-factor of the background beam used is

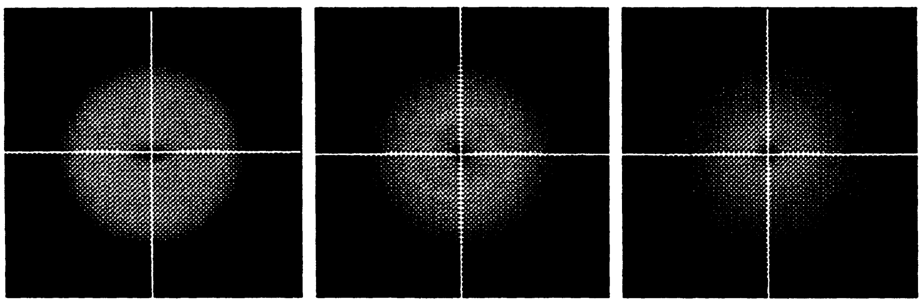
$$B(r) = \exp\left\{-\left(\frac{r}{15}\right)^{16}\right\}, \quad r = \left(\xi^2 + \eta^2\right)^{1/2}. \tag{2}$$

It was found that pairs of OVSs of the same TC rotate round their "centre of gravity" along their nonlinear propagation path. Similar behaviour has also been observed in the linear evolution of pairs of optical vortices, as well as under self-defocusing conditions (by focusing the background beam[7]) .



(a) (b) (c)

Fig. 1. Rotation of a pair of OVSs of m=+1 initially offset at $\Delta$=6.25$r_0$ at the entrance of the nonlinear medium (a), at z=3$L_{NL}$ (b), and z=7$L_{NL}$ (c).



(a) (b) (c)

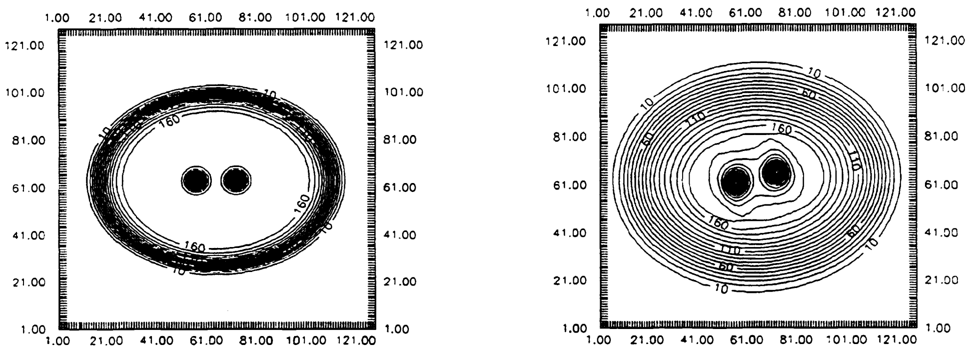
Fig. 2. Axial rotation of an initially elliptical dark beam (a) evolving (z=3$L_{NL}$; b) in a stable OVS of a circular symmetry (c; z=7$L_{NL}$).

Fig. 1 illustrates the rotation of a pair of OVSs initially offset at $\Delta$=6.25$r_0$ (Fig. 1a) up to z=7.0$L_{NL}$ (Fig. 1c). The detailed analysis of the numerical data indicated, that the rotation angle $\theta$ of symmetrical structures consisting of N OVSs is related to the total phase-shift $\Delta\phi$ in the vicinity of the vortices by

$$\theta = \frac{\Delta\phi}{N}, \tag{3}$$

where $\Delta\phi = \Delta\phi_L + \Delta\phi_{NL}$. Most probably this dependence is a simplification of a more general relation, which takes into account the eventual interaction of multiply topological charged vortices.

In the particular case of N=1 rotation of a single OVS nested on-axially on the background beam should be expected. Such behaviour was observed during the simulations by introducing weak ellipticity in the vortex formation (Fig. 2). It was proven, that this rotation takes place without any rotation of the background beam (Fig. 3).




(a) (b)

Fig. 3. Interaction within a pair of OVSs of m=+1. Rotation of the dark beams without any indication of a background-beam rotation is clearly expressed.

The dependence of the angular velocity $(d\theta / dz)$ on the nonlinear propagation path-length was found to decrease monotonically, due to the background beam self-broadening and the intensity-dependence of $\Delta\phi_{NL}$ (Fig. 4). Reversing simultaneously the TCs of the OVSs results in a change of the direction of rotation.

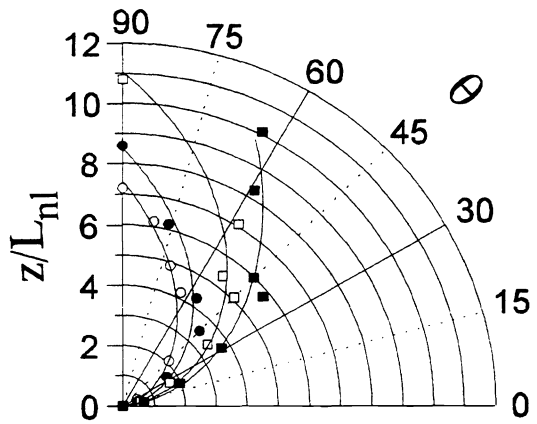



Fig. 4. Polar plot of the OVS rotation angle θ vs. normalised nonlinear propagation path-length (open circles - $\Delta=1.50r_0$; solid circles - $\Delta=2.25r_0$; open squares - $\Delta=3.00r_0$; solid squares - $\Delta=3.75r_0$). Solid lines - best fit.

Our further analyses were directed toward the investigation of the most stable ensemble of OVSs. The most simple structure symmetrical in both the TC distribution and the OVSs disposition is the linear structure consisting of three OVSs. If the TCs are the same the speed of rotation of the ensemble was found to increase. More interesting result was obtained at alternating TC-distribution within the ensemble (Fig. 5,6). The mutual attraction between the OVSs up to $\Delta(z=0)\leq 3.5r_0$ was found to result in a formation of a twin spiral structure and OVSs collapse (Fig. 5). The latter is accompanied by a reversal of the central OVS's topological charge, but the total TC of the ensemble remains conserved. The increase in the initial offset $\Delta$ prevents for the mutual attraction of the vortices, but not for the rotation of the ensemble (Fig. 6). The TCs in this case remain undisturbed. The direction of the rotation, however, is opposite to that, which is expected when the central OVS is absent. On this way other types of optically controlled rotation-switches could be constructed[7].

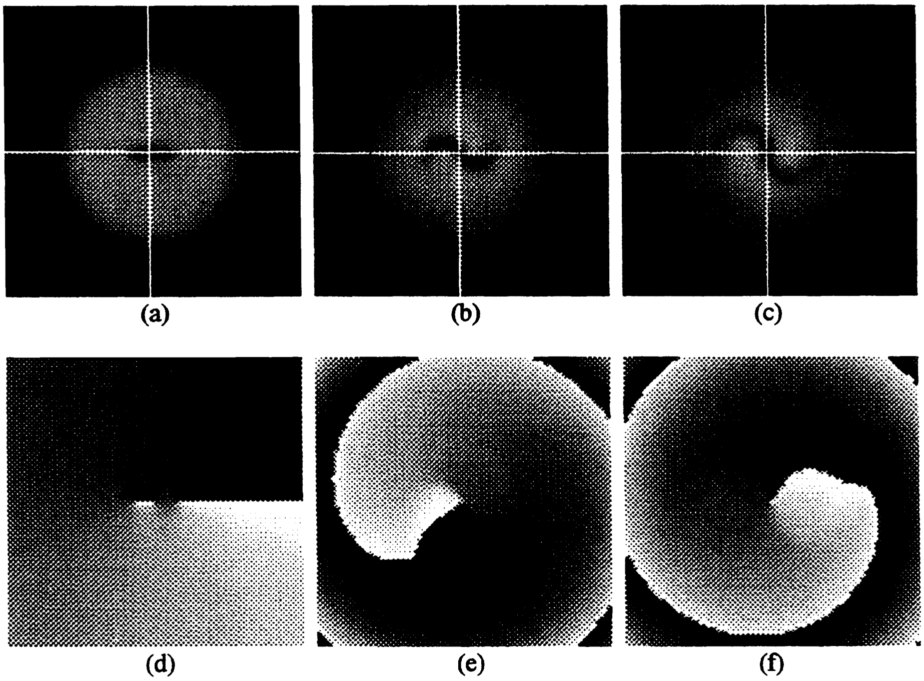



Fig. 5. Interaction within linear array of OVSs initially spaced at $\Delta=2.25r_0$ (a) at m=+1,-1,+1 (d).. Intensity (b) and phase profile (e) at the initial stage of the annihilation process ($z=4L_{NL}$). At the final stage ($z=7.0L_{NL}$) single OVS with m=+1 is formed (c,f). The total topological charge conserves.

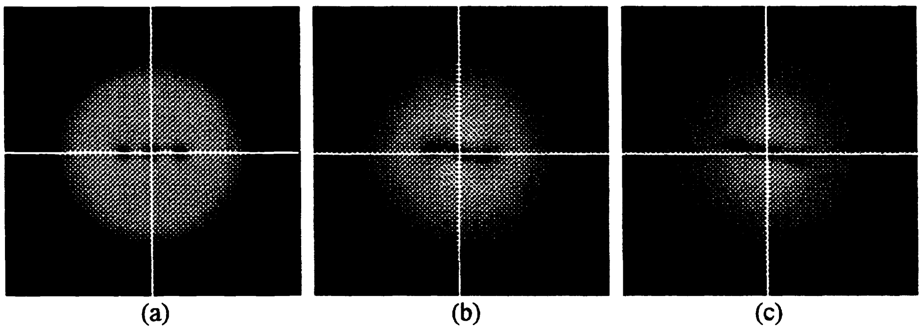



Fig. 6. Interaction within linear array of OVSs initially spaced at $\Delta=3.75r_0$ (a) at m=+1,-1,+1. No indications for OVS attraction and annihilation are seen at $z=3.5L_{NL}$ (b) and $z=7.0L_{NL}$ (c).

Three identical OVSs ordered in a triangle-like ensemble also were found to rotate axially along the nonlinear propagation path in agreement with Eq. 3. The formation of a fourth OVS of the opposite TC at the centre of the ensemble (Fig. 7a) was found to cancel the rotation (Fig. 7b). The symmetry of the structure formed results in a simultaneous suppression of the attraction within each pair of OVSs incorporating the central one. As a result this ensemble was found to propagate as a "static" structure along the nonlinear medium, thus providing the possibility parallel streams of binary-coded optical information to be transmitted.

If each OVS within the ensemble is of the same TC, the rotation speed increases significantly, but the relative disposition of the dark beams remains undisturbed (Fig. 7c). This interaction configuration may appear useful for future switching applications based on OVSs.

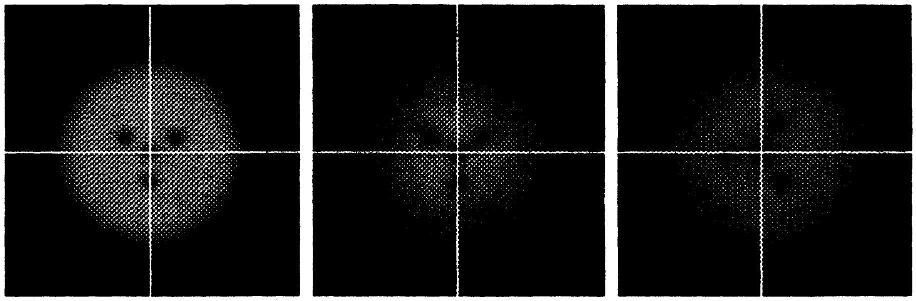



Fig. 7. Interaction within a triangle-like ensemble of OVSs of the same TCs initially spaced at $\Delta=17.0r_0$ (a). By inserting an additional OVS of opposite TC the ensemble stabilises (b). If the fourth TC is the same the angular rotation of the ensemble increases (c). Figs. 7b-c refer to $z=7.0L_{NL}$.

## 5. CONCLUSION

Qualitatively, the possibility to control and stabilise (at least partially) ensembles of OVSs by the topological charge of a single "control" OVS nested in is proved with up to seven input odd dark formations. Extension of the present analyses toward ordered structures of multiply-charged OVSs are under way.

## 6. ACKNOWLEDGEMENTS

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