Stability of one-dimensional dark spatial solitons of finite second transverse extent

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ABSTRACT

In this work we analyze the nonlinear evolution of mixed edge-screw phase dislocations and provide arguments on the existence of one-dimensional dark spatial solitary waves of finite length in bulk Kerr nonlinear media. The characteristic phase gradients force the dark beam to steer in space. An all-optical switching scheme is proposed and critically evaluated with respect to stability and deflection control. Experimental results are reported on the decay of quasi-2D dark spatial solitons into finite-length 1D dark spatial solitons. The role of the saturation of the nonlinearity is discussed.

Keywords: nonlinear optics, dark spatial soliton, vortex soliton, mixed phase dislocation

1. INTRODUCTION

Optical waves can contain dislocations along which the phase is indeterminate and the electric field amplitude is zero. This concept introduced in the wave-theory by Nye and Bery[1] has allowed for the clarification of the structure and properties of edge, screw, and mixed edge-screw dislocations. As a result of the counteraction and cancellation of diffraction and nonlinearity, in a self-defocusing nonlinear medium (NLM) such dark beams preserve their characteristics and form 1D dark spatial solitons[2] (1D DSSs) and optical vortex solitons[3] (OVSs). Since the DSSs do induce in bulk NLM gradient optical waveguides capable to transmit information pulses, the possibility to control the direction and magnitude of the DSS steering appears to be of a practical interest.

The mixed edge-screw phase dislocations considered in this work consist of a pair of opposite semi-helices on π. Their spatial offset determines the length of the edge dislocation and the dark beam. An inherent feature of the 1D DSSs of finite length is the spatial steering. It can be controlled by both the topological charges (TCs) of the semi-helices and the dislocation length.

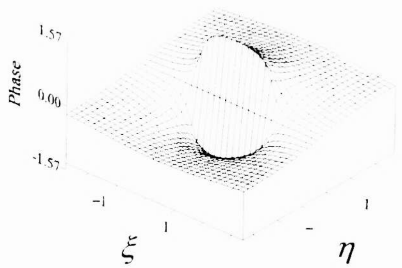
2. MATHEMATICAL DESCRIPTION

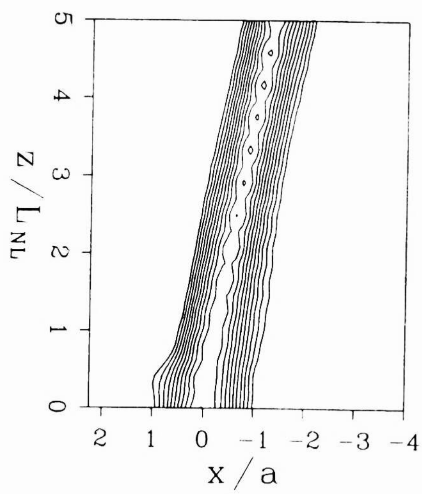
In a bulk homogeneous and isotropic NLM the (2+1)-dimensional evolution of the background beam with the phase dislocation nested in is described by the nonlinear Schrödinger equation. The slowly-varying electric-field amplitude is chosen to be of a *tanh*-shape

$$E(x,y)=\sqrt{I_0}\,B(r_{1,0}(x,y))\tanh\left[r_{\alpha,\beta}(x,y)/a\right]\exp\left\{i\Phi_{\alpha,\beta}(x,y)\right\} \tag{1}$$

where $r_{\alpha,\beta}(x,y)=\sqrt{x^2+\alpha(y+\beta b)^2}$ is the effective cartesian/radial coordinate and $\alpha=0$ for $|y|\le b$ and $\alpha=1$ for both ($\beta=-1,\ \ y>b$) and ($\beta=1,\ \ y<-b$). The phase distribution containing two offset semi-helices of TCs $\pm$ 1/2 separated by a phase step on π is modeled by $\Phi_{\alpha,\beta}(x,y)=-\beta\arctan\left[\alpha x/(y+b\beta)\right]+(1-\alpha)\,\mathrm{sgn}(x)(\pi/2)$.

In Fig. 1 we present a 2D plot of the initial phase distribution of the dark beam considered.





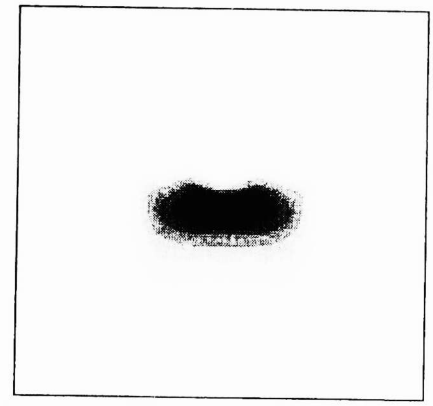


Figure 1 Mixed screw-edge phase dislocation consisting of a pair of opposite semi-helices with TCs $\pm 1/2$ separated by an 1D phase jump on π.

Figure 2 Steering perpendicular to the edge dislocation at b/a=2.75. The most outer contours correspond to the 1/e intensity-level of the background beam.

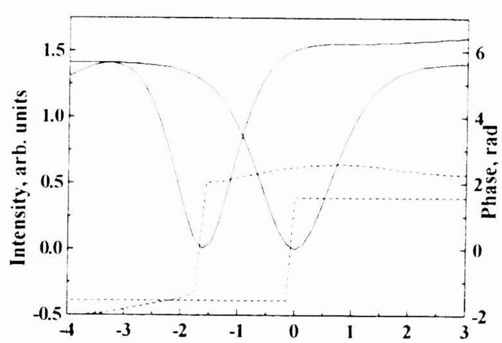
Figure 3 Typical dark beam intensity distribution (grayscale coded) during the initial evolution stage $\zeta<1.5L_{NL}$ (I_0a^2=1.4, b/a=2.75).

3. NONLINEAR DYNAMICS

The asymmetry in the phase distribution (see Fig. 1) is indicative for the differences in the transverse evolution of the dark stripe. An infinite one-dimensional π-phase jump implies a zero transverse velocity for truly 1D DSS. The mixed phase dislocation causes dark beam steering toward the region with an initially lower phase (Fig. 2). The quantity I_0a^2 is chosen to be equal to $\sqrt{2}$, which corresponds (at b=a and a single, on-axis 2π screw dislocation) to the OVS constant[3,4]. In our case (b/a=2.75) the 'lack of energy' exceeds $\sqrt{2}$ times the soliton constant in 1D and a gray dispersive wave is emitted parallel to the edge dislocation on the higher-phase region (Fig. 3). The later results in a weak effective broadening (see Fig. 2) of the central part of the stripe at $\zeta<1L_{NL}$, where L_{NL} stands for the nonlinear length.

3.1. Soliton tests

The soliton nature of the formation is proven by analyzing the reproducibility of the 1D amplitude and phase distributions across the background beam (Fig. 4), by the existence of a soliton constant identical to that in the pure 1D case, and by the conservation of the number of the dark beams (Fig. 5) at distances up to ζ=$6L_{NL}$. Since the ends of the dislocation are constituted by a pair of opposite phase helices, their slight attraction along with the beam steering leads to a monotonic increase in the stripe length at the expense of its shortening. As a consequence, the 1D DSSs of finite extent have a 'gray' final evolution state.



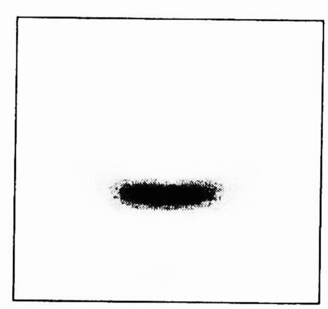


Figure 4 One-dimensional intensity (solid curves) and phase profiles (dashed) of the 1D DSS of finite second extent at y=0 (right pair) and ζ=$5L_{NL}$ (left pair).

Figure 5 Intensity distribution of the 1D DSS at ζ=$5L_{NL}$ (see Fig. 4) demonstrating conservation of the total number of the dark structures.

3.2. Stability

For an initially considerably longer than wider dark beam (b/a=11, see Fig. 6a) we observed decay into a chain of optical vortices with alternating TCs. This instability scenario known for the plane dark solitons[5] is confirmed experimentally in both isotropic[6] and anisotropic[7] nonlinear media. The ratio b/a=4 is still larger as compared to the critical one and the stripe bends and decays in a pair of OVSs of opposite TCs (Fig. 6b). Assigning formally TCs $\pm$ 1/2 to the semi-helices, the total TC remains conserved. In the limiting case b=0 and Φ=const under the same model conditions we observed the formation of a ring dark solitary wave[8] with its typical phase portrait, nonzero transverse velocity and reducing contrast along the propagation axis (Fig. 6c).

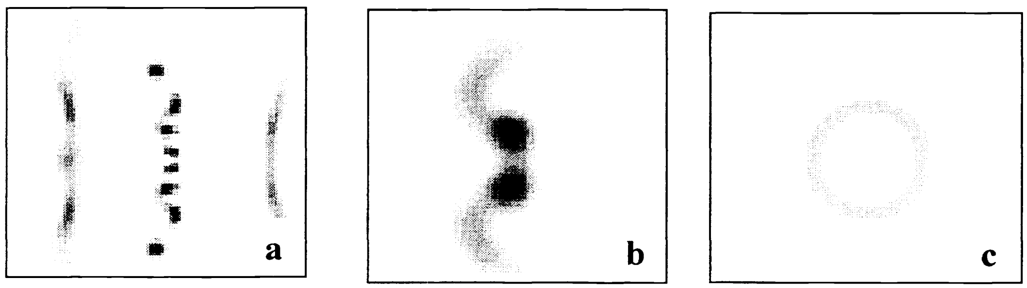


Figure 6 Final stage (ζ=5L_{NL}) of the development of snake-instability (a) at I_0a^2=1.4 and b/a=11, dark beam decay into a pair of interacting OVSs of opposite TCs (b) at b/a=4, and generation of a ring dark solitary wave at b=0.

The characteristic initial slight beam expanding, bending and emission of dispersive waves is still present at b/a=2.75, but these perturbations are not critical in spatial frequency and 1D DSS of finite length forms (Fig. 5).

3.3. Parallel guiding and switching

Due to their finite length the solitons described can be aligned on a 'dashed' line (Fig. 7a). If both edge dislocations have the same orientation, the neighboring semi-helices appear with opposite TCs. The latter can give raise to an attraction and the neighboring vortices can annihilate (for example at ζ=7L_{NL} and an initial separation Δy=2b). By increasing the offset (e.g. Δy=3b, *Fig. 7a*) the interaction is much weaker and non-critical with respect to annihilation. Varying the mutual orientation of the edge dislocations only, it is possible to steer the two dark beams in the same (Fig. 7b) or in opposite directions (Fig. 7c). When the neighboring edge dislocations carry the same 'charges', the steering angle is considerably larger (Fig. 7c). In view of the repulsive interaction between OVSs of equal TCs leading to their rotation, this could be intuitively understood by the enhanced bending of the phase lines in-between. At the initial stage of the nonlinear evolution the neighboring semi-helices of equal charges of 1/2 do rotate slightly giving rise to the dark soliton steering in opposite directions. The slightly larger steering of the neighboring ends of the solitons as compared to this of the outer ones supports such an interaction scheme. If an OVS is formed instead of one of the 1D DSSs it should preserve (at a large enough spatial offset) its spatial position on the background. So, the switching between different initial phase profiles will enable to control the steering of the 1D DSSs of finite length. Eventually, this allows to deflect guided streams of optical information in space into several distinct channels (Fig. 7).

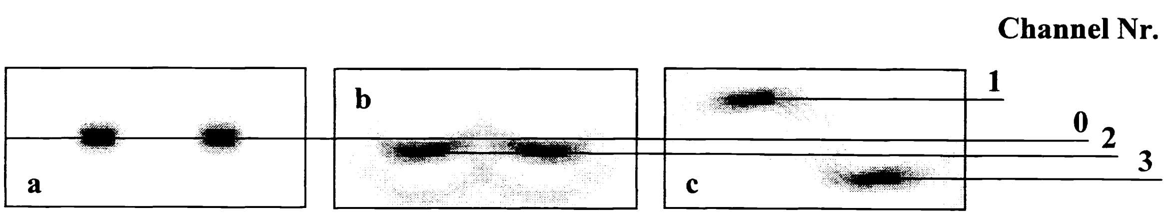


Figure 7 Controllable steering and interaction of a pair of 1D DSSs of finite width at Δy=3b and ζ=0 (a) and 7L_{NL} (b,c). The steering direction is controlled by changing the edge dislocations from identical phase jumps (b) to opposite (c) ones. Solid lines – set of four well separated information channels.

The main results from a numerical simulation on the guiding and deflection of a probe signal beam (Fig. 8) can be summarized as follows. During the initial evolution stage (up to 1.6L_{NL}), in which the soliton forms and starts to steer itself and the probe wave, the signal losses are about 9%. Nearly 91% of the signal energy remains located in the all-optical waveguide at $10L_{NL}$. The saturation of the nonlinearity is found to improve the soliton modulational stability without affecting the transverse velocity. The steering angle can be controlled by the magnitude of the phase jump of the edge dislocation as shown in Fig. 9. Another way to do that is to change the ratio of the 1D DSS length to width (b/a). In this case special attention should be paid to the modulational stability of the all-optical waveguide induced (see Fig. 6).

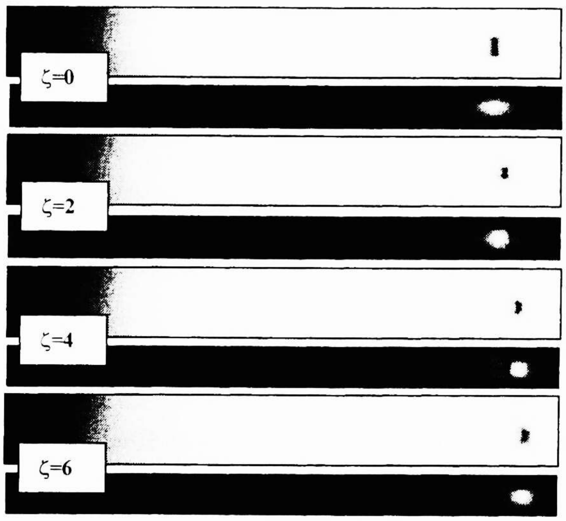


Figure 8 Guiding and switching of a bright probe beam.

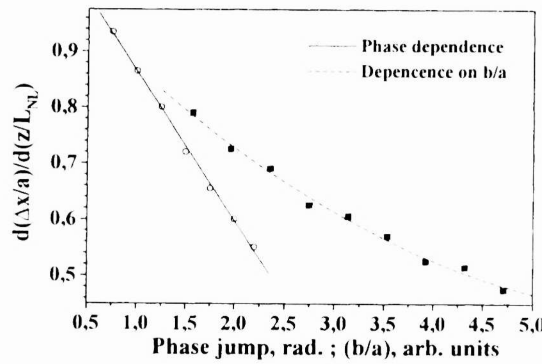


Figure 9 Possible ways to manipulate the steering angle.

4. EXPERIMENTAL OBSERVATION OF 1D DSSs OF FINITE LENGTH

The beam of a single-mode Ar^+-laser (P_{max}=8W at λ=488nm) is used to reconstruct Computer-Generated Holograms[9] (CGHs) produced photolitographically with grating periods of 20μm. The irreducible quantization noise in the reconstructed phase distribution is evaluated to be down to $\pi/24$. The Nonlinear Medium (NLM) is ethylene glycol dyed with DODCI (Lambdachrome) to reach an absorption coefficient of $\alpha=0.107$ cm^{-1} at λ=488nm. The first-diffraction-order background beam with the dark formation nested in is focused on the entrance of the NLM. After passing the desired nonlinear propagation path-length the dark beam is partially reflected by a prism immersed in the liquid and is projected directly on a CCD-array of a 13 μm resolution.

In order to estimate the saturation intensity we carried out a self-bending experiment[2,10]. Assuming that the self-deflection Δy^{SD} is proportional to $I/(1+I/I_{sat})^{\beta}$, the saturation power is estimated to be P=27 mW at β=3.

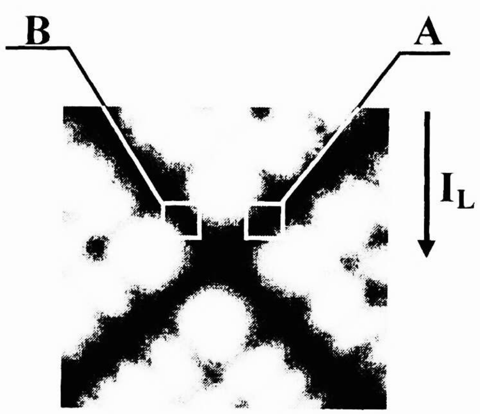


Figure 10. Center - stable quasi-2D dark beam (on-axial alignment); frames A.B - location of the mixed edge-screw phase dislocations; arrow - increase of the local intensity.

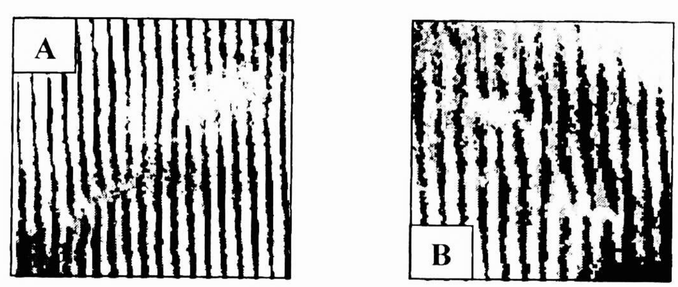


Figure 11. Experimental results depicting the interference pattern resulting from the creation of pairs of mixed edge-screw phase dislocations in sectors A and B of Fig. 10.

The greyscale image in the central part of Fig. 10 refers to a quasi-2D DSS positioned on-axially with respect to the background. The picture is recorded at z=8.5 cm at P=30 mW and is intended to serve for a better intuitive understanding of the instability of the shifted quasi-2D dark beam in the different spatial regions. The snake instability of the 1D dark solitons results in the creation of pairs of oppositely charged OVSs in several areas. The OVSs are easily recognized by the converging of two neighboring equiphase lines in one. In the areas marked with A and B on Fig. 10 we observed an interference pattern of a different type (see Fig. 11A,B). Between interference lines offset at finite distances (approximately 780 μm for the image from Fig. 11A and 550 μm for that of Fig. 11B) the lines become slightly curved, terminate and appear again but shifted by a half pattern period and slightly oppositely curved. It is known, that interference lines shifted by a half period indicate the presence of the 1D π–phase dislocation required for the generation of an 1D DSS. The adjacent dislocations observed are well separated (at approximately 250 μm and 130 μm for the cases presented on Figs. 11A and 11B, respectively) and the smooth interference lines in-between are slightly curved only. This is indicative for the transition of an 1D phase dislocation in a smooth (plane) phase profile by pairs of opposite phase semi-helices on π.

In order to prove qualitatively this explanation we simulated an 1D edge π-phase dislocation of a finite length and the resulting interference pattern. We are convinced to have observed and clearly identified the creation of mixed edge-screw phase dislocations as a consequence of a modulational instability of crossed 1D dark soliton stripes under moderate saturation. Since creation of 1D DSSs of finite length from modulational instability is not well suited for practical purposes, our future experiments will be based on reproduction of CGHs.

5. CONCLUSION

We have shown that the modulational instability in saturated nonlinear medium can lead to the generation of one-dimensional dark beams of finite length which contain mixed edge-screw phase dislocations. In view of the numerical results presented these beams should be classified as steering one-dimensional spatial solitons of finite second transverse extent. Their ability to guide and deflect/switch controllably parallel streams of optical signals is highly promising for future practical application.

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7. REFERENCES

1. J. F. Nye and M. V. Berry, "Dislocations in wave trains," *Proc. R. Soc. London*, **A336**, pp. 165-170 (1974); see also M. V. Berry, "Wave dislocation reactions in non-paraxial Gaussian beams," *J. Mod. Optics*, **45**, pp. 1845-1858 (1998).

2. G. Swartzlander, Jr., D. Andersen, J. Regan, H. Yin, and A. Kaplan, "Spatial dark soliton stripes and grids in self-defocusing materials," *Phys. Rev. Lett.*, **66**, pp. 1583-1586 (1991); G. Allan, S. Skiner, D. Andersen, and A. Smirl, "Observation of fundamental dark spatial solitons in semiconductors using picosecond pulses," *Opt. Lett.*, **16**, pp. 156-158 (1991).

3. G. Swartzlander, Jr., and C. Law, "Optical vortex solitons observed in Kerr nonlinear media," *Phys. Rev. Lett.*, **69**, pp. 2503-2506 (1992).

4. I. Velchev, A. Dreischuh, D. Neshev, and S. Dinev, "Multiple charged optical vortex solitons in bulk Kerr media," *Opt. Commun.* **140**, pp. 77-82 (1997).

5. D. Pelinovsky, Yu. Stepanyants, and Yu. Kivshar, "Self-focusing of plane dark solitons in nonlinear defocusing media," *Phys. Rev.* **E51**, pp. 5016-5026 (1995).

6. V. Tikhonenko, J. Christou, B. Luther-Davies, and Yu. S. Kivshar, "Observation of vortex solitons created by the instability of dark soliton stripes," *Opt. Lett.* **21**, pp. 1129-1131 (1996).

7. A.V. Mamaev, M. Saffman, D.Z. Anderson, and A.A. Zozulya, "Propagation of dark beams in anisotropic nonlinear media: From symmetry-breaking to spatial turbulence," *Phys. Rev.* **A54**, pp. 870-879 (1996); A. V. Mamaev, M. Saffman, and A. A. Zozulya, "Propagation of dark stripe beam in nonlinear medium: Snake instability and creation of optical vortices," *Phys. Rev. Lett.* **76**, pp. 2262-2265 (1996).

8. Yu. S. Kivshar, "Ring dark solitons," *Phys. Rev.* **E50**, pp. R40-R44 (1994); A. Dreischuh, W. Fließer, I. Velchev, S. Dinev, and L. Windholz, "Phase measurements of ring dark solitons," *Appl. Phys.* **B62**, pp. 139-142 (1996).

9. N. R. Heckenberg, R. McDuff, C. P. Smith, and A. G. White, "Generation of optical phase singularities by computer-generated holograms," *Opt. Lett.* **17**, 221 (1992).

10. G. A. Swartzlander, Jr., H. Yin, and A. E. Kaplan, "Self-bending of a cw laser beam in sodium vapour," *Opt. Lett.* **13**, pp. 1011-1018 (1988).