Nonlocal solitons

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ABSTRACT

We review our recent works on spatial solitons in nonlocal nonlinear media. In particular, we discuss stabilization of two dimensional bright solitons and vortex ring solitons as well as interaction of dark solitons.

1. INTRODUCTION

A soliton is a localised wave that propagates without change through a nonlinear medium. Such a localized wave forms when the dispersion or diffraction associated with the finite size of the wave is balanced by the nonlinear change of the properties of the medium induced by the wave itself. Solitons are universal in nature and have been identified in such physical systems, as fluids, plasmas, solids, matter waves, and classical field theory.

The soliton concept is an integral part of studies of the coherent excitations of Bose-Einstein condensates (BECs).[4] Such BECs, inherently have a spatially nonlocal nonlinear response due to the finite range of the inter-particle interaction potential. Spatial nonlocality, which is already an established concept in plasma physics,[5–7] means that the response of the medium at a particular point is not determined solely by the wave intensity at that point (as in local media), but also depends on the wave intensity in its vicinity. The nonlocal nature often results from a transport process, such as atom diffusion,[8] heat transfer[9, 10] or drift of electric charges.[11] It can also be induced by a long-range molecular interaction as in dipolar Bose Einstein Condensate[12] or nematic liquid crystals.[13, 14] Nonlocality is thus a generic feature of a large number of nonlinear systems. It has also recently become important in optics.[15–17] Although nonlocality can have a considerable impact on many nonlinear phenomena, studies of nonlocal nonlinear effects are still in their infancy.

In this paper we review our recent results on some aspects of soliton formation and interaction in nonlocal nonlinear media. In particular, we will discuss the role of nonlocality on stability of optical beams and spatial solitons as well attraction of dark spatial solitons.

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2. MODEL

We consider an optical beam propagating along the z-axis of a nonlinear medium with the scalar electric field $E(\vec{r}, z) = \psi(\vec{r}, z)\exp(iKz - i\Omega t) + c.c.$ Here $\vec{r}$ spans a D-dimensional transverse coordinate space, K is the wavenumber, Ω is the optical frequency, and $\psi(\vec{r}, z)$ is the slowly varying amplitude. We assume that the refractive index change $N(I)$ induced by the beam with intensity $I(\vec{r}, z) = |\psi(\vec{r}, z)|^2$ can be described by the phenomenological nonlocal model

$$N(I) = s\int R(\vec{\xi} - \vec{r})I(\vec{\xi}, z)d\vec{\xi}, \tag{1}$$

where the integral $\int d\vec{r}$ is over all transverse dimensions and $s = 1$ ($s = -1$) corresponds to a focusing (defocusing) nonlinearity. The response function $R(\vec{r})$, which is assumed to be real, localized and symmetric (i.e. $R(\vec{r}) = R(r)$, where $r = |\vec{r}|$), satisfies the normalization condition $\int R(\vec{r})d\vec{r} = 1$. This model of nonlinearity leads to the following nonlocal nonlinear Schrödinger (NLS) equation governing the evolution of the beam

$$i\partial_z\psi + \frac{1}{2}\nabla_\perp^2\psi + N(I)\psi = 0. \tag{2}$$

The width of the response function $R(r)$ determines the degree of nonlocality. For a singular response, $R(r) = \delta(r)$ the refractive index change becomes a local function of the light intensity, $N(I) = sI(\vec{r}, z)$, i.e., the refractive index change at a given point is solely determined by the light intensity at that very point, and Eq.(2) simplifies to the standard NLS equation

$$i\partial_z\psi + \frac{1}{2}\nabla_\perp^2\psi + s\psi|\psi|^2 = 0. \tag{3}$$

With increasing width of $R(r)$ the light intensity in the vicinity of the point $\vec{r}$ also contributes to the index change at that point. While Eq. (1) is a phenomenological model, it nevertheless adequately describes several physical situations where the nonlocal nonlinear response is due to various transport effects, such as heat conduction or diffusion of molecules or atoms.

3. COLLAPSE SUPPRESSION AND STABILIZATION OF SPATIAL SOLITONS

Collapse is a phenomenon well-known in the theory of wave propagation in nonlinear focusing media. It refers to the situation when strong self-focusing of a beam leads to a catastrophic increase (blow-up) of its intensity in a finite time or after a finite propagation distance.[18–20] Collapse has been observed in plasma waves,[21] electromagnetic waves or laser beams,[22] BEC's,[23] and even capillary-gravity waves on deep water.[24]

In the past there have been a few attempts to determine the role of nonlocality in development of the collapse of finite-size beams. Turitsyn proved analytically the arrest of collapse for a specific choice of the nonlocal nonlinear response.[25] Recently Perez-Garcia *et al.* discussed the collapse suppression in the case of weak nonlocality, in a direct application for a BEC.[26] The analysis of the collapse conditions in case of general BEC nonlocal model is difficult and has so far only been done numerically.[27]

Here we present an analytical approach to beam collapse in nonlocal media, which is based on the technique introduced in,[28] and extended later in.[29] We consider the general case of *symmetric, but otherwise arbitrarily shaped, non-singular response functions* and prove rigorously that a collapse cannot occur.

For localized or periodic solutions Eqs. (1) and (2) conserve the power (in optics) or number of atoms (for BEC) P and the Hamiltonian H,

$$P = \int Id\vec{r}, \quad H = \frac{1}{2}||\nabla\psi||_2^2 - \frac{1}{2}\int NId\vec{r}, \tag{4}$$

where $||\psi||_2^2 \equiv \int |\psi|^2 d\vec{r}$. In the local limit when the response function is a delta-function, the nonlinear response has the form $N(I) = sI$, as in local optical Kerr media described by the conventional NLS equation and in BEC's described by the standard Gross-Pitaevskii equation. In this local limit multidimensional beams with a power

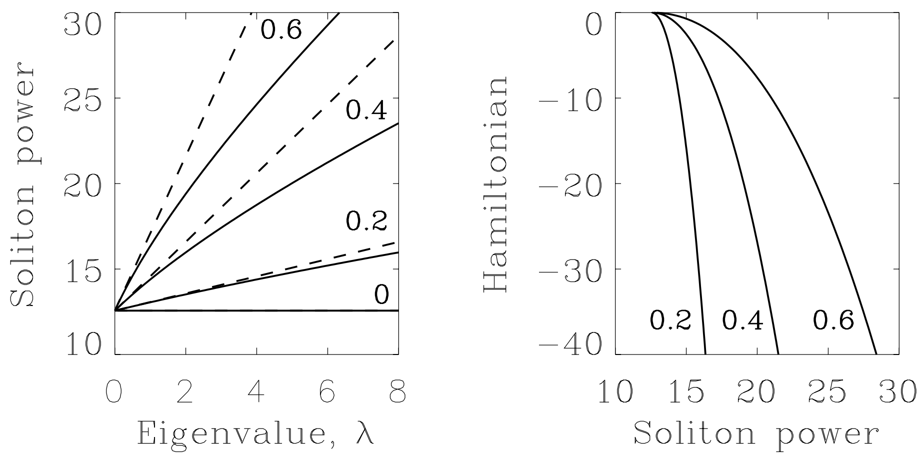

Figure 1. 2D variational results with Gaussian response and trial function. Left: Soliton power (solid) versus eigenvalue λ for different degrees of nonlocality, σ=0, 0.2, 0.4, and 0.8. Dashed lines show the weakly nonlocal approximation. Right: Corresponding Hamiltonian versus power diagrams. (After Ref. [28])

higher than a certain critical value will experience unbounded self-focusing and *collapse* after a finite propagation distance.

It can be easily shown that in the two extreme limits of a weakly and highly nonlocal nonlinear response the collapse is prevented.[7,30] Here we consider the general case of *symmetric, but otherwise arbitrarily shaped, non-singular response functions.* Introducing the D-dimensional Fourier transform and its inverse, it is straightforward to show that for $N(I)$ given by Eq. (1), when s=1 the following relations hold[28]

$$\begin{aligned} |\tilde{I}(\vec{k})| &= |\int I(\vec{r})\mathrm{e}^{i\vec{k}\cdot\vec{r}}d\vec{r}| \leq P, \\ \int NId\vec{r} &= \frac{1}{(2\pi)^D}\int \widetilde{R}(\vec{k})|\tilde{I}(\vec{k})|^2 d\vec{k}. \end{aligned} \tag{5}$$

For any response functions for which the spectrum $\widetilde{R}(k)$ is absolute integrable, we then have

$$|\int NId\vec{r}| \leq P^2 R_0, \quad R_0 \equiv \frac{1}{(2\pi)^D}\int |\widetilde{R}(k)|d\vec{k}, \tag{6}$$

and hence we get $|H| \geq ||\nabla\psi||_2^2/2 - R_0P^2/2$. This inequality shows that the gradient norm $||\nabla\psi||_2^2$ is bounded from above by the conserved quantity $2|H| + R_0P^2$ for all symmetric response functions under the only requirement that their spectrum is absolute integrable. It represents a rigorous proof that a collapse with the wave-amplitude locally going to infinity cannot occur in BEC's or Kerr media with a nonlocal nonlinear response for any physically reasonable response functions, such as Gaussians, exponentials, etc.

We emphasize that although the nonlocality prevents collapse, it does not remove the collapse-like dynamics of high-power beams. To the contrary, as long as the beam width is much larger than the extent of the nonlocality (width of the response function), the beam will naturally contract as in the local case. However the collapse will be arrested when the width becomes comparable with that of the response function. Finally, during the stationary phase of the beam propagation, its width will be comparable with the extent of the nonlocality.

The stabilizing effect of the nonlocality can be further illustrated by the properties of the stationary solutions of Eqs. (2). As the simplest example we consider nonlocal optical bulk Kerr media with a Gaussian response

$$R(r) = \left(\frac{1}{\pi\sigma^2}\right)^{D/2} \exp\left(-\frac{r^2}{\sigma^2}\right) \tag{7}$$

To capture the main (qualitative) physical effects we use the approximate, variational technique[28] which uses the following Gaussian ansatz $\psi(r,z) = \phi(r)\exp(i\lambda z)\alpha\exp(-(r/\beta)^2))$. In Fig. 1 we show the power and Hamiltonian of the stationary solutions in 2D. The dashed lines give the reults of the weakly nonlocal approximation with N given by[29]

$$N(I) = s(I + \gamma\nabla_\perp^2 I),\ \gamma = \frac{1}{2D}\int R(r)d\vec{r} \tag{8}$$

from which $\alpha^2 = 4\lambda$ and $\beta^2 = 2/\lambda + 2\sigma^2$ is found, resulting in the power

$$P_s = 4\pi(1 + \sigma^2\lambda), \tag{9}$$

where 4π is the (λ-independent) power of the Gaussian approximation to the soliton solutions of the standard 2D NLS equation (also the collapse threshold power), which is recovered in the local limit σ=0.

In the 2D NLS equation the collapse is a critical collapse and the stationary solutions are "only" marginally unstable with $dP_s/d\lambda$=0.[19] Typically any perturbation will act against the self-focusing, with several effects, such as non-paraxiality and saturability, completely eliminating the possibility of a collapse.[20] This is also the case with nonlocality, as evidenced from Fig. 1 and the simplified expression (9), which shows that any finite width of the response function (non-zero value of σ) implies that $dP_s/d\lambda$ becomes positive definite. According to the (necessary) Vakhitov-Kolokolov criterion[31] the soliton solutions therefore (possibly) become *linearly stable*.

4. VORTEX RING SOLITONS

The stabilizing effect of the nonlocality on localized nonlinear structures can be further illustrated by considering the propagation of a vortex beam in a self-focusing medium. Such a beam is characterized by a bright ring with a helical phase front with the so-called charge, defined as a closed loop contour integral of the wave phase modulo 2π. A typical example is the Gaussian-Laguerre beam

$$\psi(\vec{r}) = r\exp\left[-(r/r_0)^2\right]\exp(i\phi), \tag{10}$$

where r and ϕ are the radial and angular coordinates, respectively. This beam represents a vortex of charge one. Beams of such structure have been considered as candidates for vortex-type solitons in nonlinear self-focusing media.[32, 33]

The formation and dynamics of bright vortex solitons has been extensively studied in the context of optics[34–37] and BECs with an attractive inter-particle potential.[38] The early studies of vortex beams in saturable self-focusing nonlinear media demonstrated their intrinsic azimuthal instability.[39]

Recently stable propagation of vortex rings demonstrated the in the case of competing third-order self-focusing and fifth-order self-defocusing nonlinearities.[36, 37] Yang and Pelinovsky[33] showed that vortex ring can be stable in focusing medium when it co-propagates with another mutually incoherent beam. Another method of stabilizing vortex solitons utilizing a partially coherent light beam has also been proposed.[40]

Here we show that spatially nonlocal focussing nonlinearity supports *stable propagation* of vortex solitons of arbitrary charge for a sufficiently high degree of nonlocality.[41]

Since for any reasonable response function the physical properties do not depend strongly on its shape we choose here to work with the following Gaussian response

$$R(\mathbf{r}) = \frac{1}{\pi\sigma_0^2}\exp(-|\mathbf{r}|^2/\sigma_0^2). \tag{11}$$

As we are interested in circularly symmetric solutions (vortex rings) we look for the stationary solution in a cylindrical coordinate system

$$\psi(\mathbf{r},z) = \psi(r,\phi,z) = u(r)\exp(im\phi)\exp(i\Lambda z) \tag{12}$$

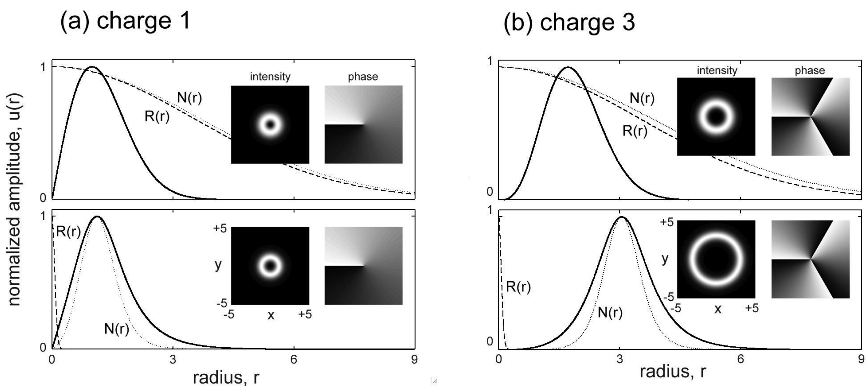

Figure 2. Examples of ring vortex solitons in a nonlocal medium with charge m=1 (a) and m=3 (b). Solid line - normalized amplitude profile, dotted line - refractive index profile, dashed line - profile of the nonlocal response function. Top row - strongly nonlocal regime ($\sigma_0 = 5,\ \sigma = 1, \Lambda = 26.0783$). Bottom row - weak nonlocality ($\sigma_0 = 0.1,\ \sigma = 1, \Lambda = 2.0198$). (After Ref. [41])

where r and ϕ are the radial and angular coordinates, $u(r)$ represents the radial structure of the solution, Λ is the propagation constant, and m denotes the vorticity (charge). In this case, Eq. (2) assumes the following form

$$\partial_r^2 u(r) + \frac{1}{r}\partial_r u(r) - \left(\frac{m^2}{r^2} + \Lambda\right) u(r) + u(r) \int_0^{2\pi}\int_0^{\infty} R(|\mathbf{r} - \mathbf{r}'|)|u(r')|^2 r' \, \mathrm{d}r' \mathrm{d}\phi = 0 \tag{13}$$

To find exact stationary solutions to the nonlocal equation (13), we resort to an iterative numerical procedure (see Ref.[41] for details). Illustrative results of the soliton-finding algorithm are presented in Fig.2 showing amplitude profiles of ring solitons and their corresponding contour plots with charge $m = 1$ (a) and $m = 3$ (b) for strongly (top) and weakly (bottom) nonlocal regimes. These plots also show the profile of the nonlocal response function $R(r)$ (dashed line) and the nonlinearity-induced index profile $N(I)$ (dotted line). Note that in the strongly nonlocal regime the index profile is very broad and resembles the shape of the nonlocal response function rather than the intensity profile of the vortex (as in the local case).

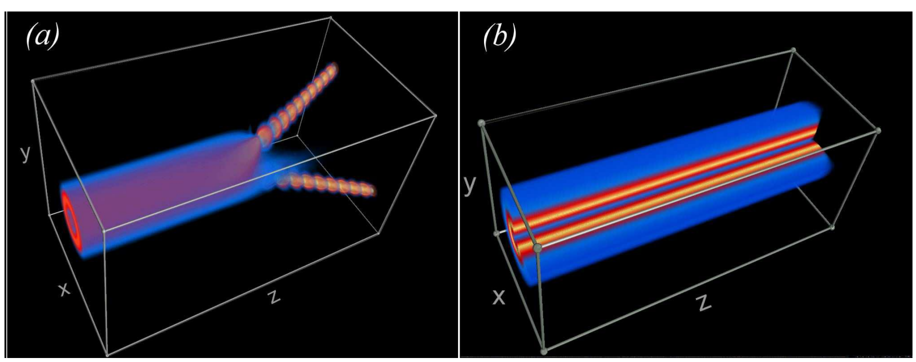

Figure 3. Propagation of nonlocal charge $m = 1$ ring vortex solitons. (a) unstable propagation in the weakly nonlocal case with $\sigma_0/\sigma = 0.1$ and $\sigma = 1,\ \Lambda = 2.0198,\ (x, y) \in [-10, 10] \times [-10, 10]$ and $z \in [0, 5]$. (b) stable propagation in the highly nonlocal case with $\sigma_0/\sigma = 10,\ \Lambda = 101.0199$ and $\sigma = 1,\ (x, y) \in [-30, 30] \times [-30, 30]$ and $z \in [0, 50]$. (After Ref. [41]).

In order to investigate the stability of the vortex soliton solutions we simulate their propagation by directly solving the original nonlocal evolution Eq. (2) using the numerically-obtained exact solutions as initial conditions.

Our simulations reveal that the stability of vortex solitons is determined by the degree of nonlocality of the nonlinearity. In the weakly and moderately nonlocal regime all vortex ring solitons experience azimuthal instability and break into filaments after a certain propagation distance, as seen in Fig. 2(a). The nonlocality decreases the maximum growth-rate of the instability and thereby increases the length over which the solitons remain stable, but is too weak to remove the instability completely.[29] However, when the nonlocality becomes large the resulting vortex ring propagates in a stable fashion exhibiting only small-scale oscillations of its amplitude and radius due to an initial destabilizing perturbation. These oscillations are manifestations of the so-called internal modes of the nonlocal solitons discussed by Krepostnov *et al.*.[42] An illustrative example of stable propagation is displayed in Fig. 2(b). Importantly, stabilization of the vortex ring is not restricted to charge $m = 1$ but was also observed for higher charges (up to $m = 5$).

In nonlocal media the role of confining potential is played by the self-induced nonlocal waveguide structure $N(|u(r)|^2)$. As the nonlocality tends to average and smooth out spatial variations of the beam intensity, soliton perturbations exert a reduced effect on the nonlinearity-induced potential which, being sufficiently broad and deep (see top graphs in Fig. 2), confines the vortex ring and inhibits its decomposition.

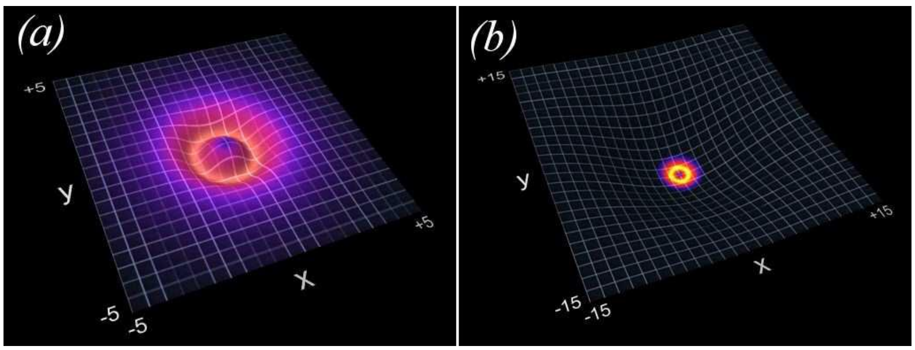

Figure 4. Transverse structure of the nonlinearity-induced potential nesting the ring vortex soliton. (a) weakly nonlocal (and unstable) case from Fig. 2.(a); (b) strongly nonlocal case (stable propagation of the vortex soliton). Simulation parameters are the same as in Fig. 2. (After Ref. [41]).

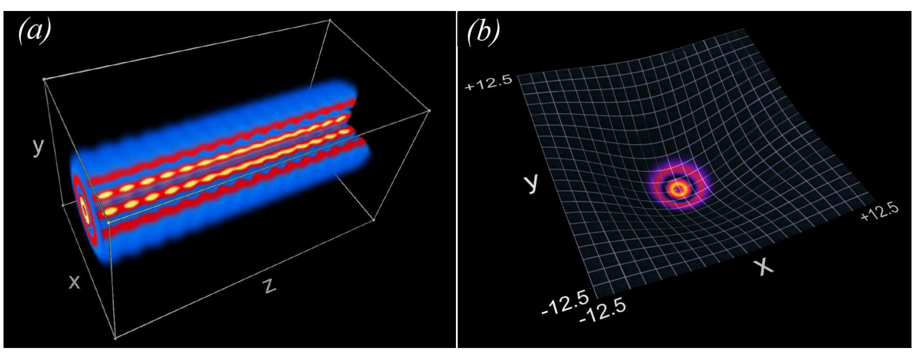

Figure 5. Stable propagation of a double ring nonlocal vortex soliton with charge m=1, $\sigma_0 = 9$, $\sigma = 1$, $(x, y) \in [-25, 25] \times [-25, 25]$ and $z \in [0, 25]$ (a) surface plot of the intensity distribution; (b) transverse structure of the soliton-induced potential. (After Ref. [41])

Finally, in Fig. 5 we illustrate the dynamical behavior of the higher order single charge vortex soliton, which, in the highly nonlocal regime is approximated by the function

$$\psi(r, \phi) = 4\sqrt{2}\sigma_0^2\sigma^{-4}r(1 - r^2/(2\sigma^2)) \exp(-r^2/(2\sigma^2)) \exp(i\phi). \tag{14}$$

This structure has a form of two out-of phase bright rings. As the plot shows, this two-ring vortex soliton propagates in a stable manner exhibiting only internal oscillations caused by the fact that the initial condition was only an approximate solution to the original nonlocal equation. Interestingly, as the plots show, the inner and outer rings oscillate with different frequencies, which may indicate the simultaneous excitation of different internal modes.

So far we have considered only individual vortex ring solitons. However, very interesting, from the point view of potential applications is the aspect of soliton interaction. We investigated numerically interaction of nonlocal ring solitons. An example of their interaction is illustrated in Fig.6. In this particular case both vortex rings have opposite charge (± 1). As the plots show solitons strongly attract exhibiting mutual oscillations. It turns out that after few cycles of collision the vortex structure of solitons is destroyed. This appears to be a generic behavior. Nonlocal vortex solitons always attract independently of their vorticities or phases.

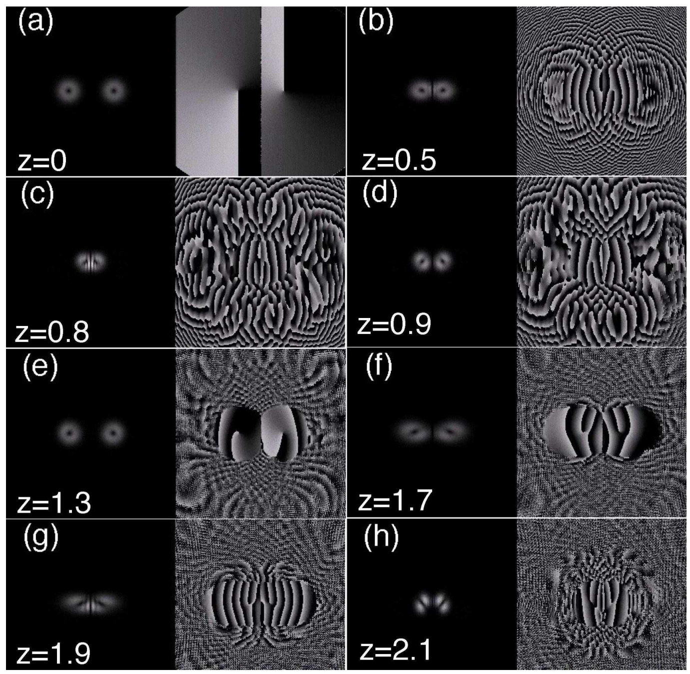

Figure 6. Collision of initially parallel vortex ring solitons with opposite charges (m1=-m2=1) $\sigma_0 = 9$, $\sigma = 1$, $(x, y) \in [-25, 25] \times [-25, 25]$ and $z \in [0, 25]$. The left part of each frame shows intensity distribution while the right part illustrates corresponding phase structure.

Recently independent studies of ring soliton formation in focusing nonlocal media has been published by Zaliznyak *et al.*.[43]

5. ATTRACTION OF DARK NONLOCAL SOLITONS

Because of the specific nature of the nonlocality, which results in spatial advancing of the nonlinearity far beyond the actual spatial location of the beam, it is natural to expect strong influence of the nonlocality on interaction of well separated localised waves and solitons. For instance, in case of two nearby optical beams, each of them will induce a refractive index change extending into the region of the other one, thereby affecting its trajectory. One can show that in a self-focusing medium nonlocality always provides an attractive force between interacting bright solitons. This effect has been recently demonstrated for the interaction of bright solitons formed in a liquid crystal.[44, 45]

It is commonly accepted that in a local medium dark solitons always repel[46] Here we show that the attractive force provided by the nonlocality may change drastically characetr of interaction of dark solitons leading to their attraction and even formation of their bound states.[47]

We will consider interaction of one-dimensional solitons. Without loss of generality, we assume the exponential response function $R(x) = (2\sigma)^{-1} \exp(-|x|/\sigma)$. The nonlocal Schrodinger equation with this response function is

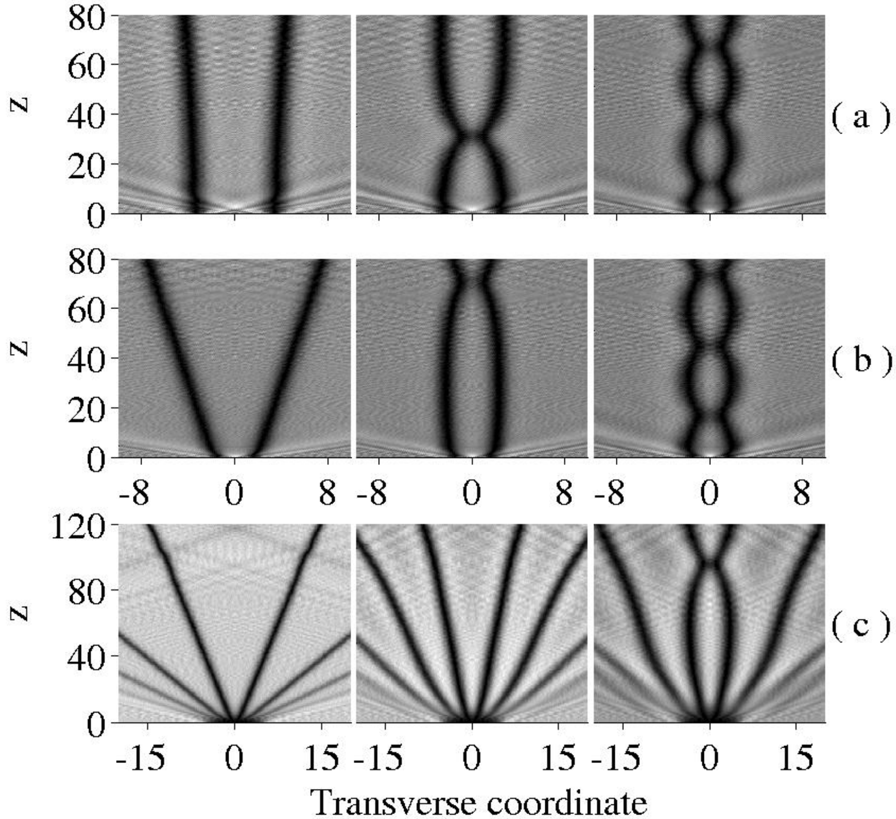

Figure 7. Attraction of dark nonlocal solitons formed either by two closely spaced phase jumps (a,b) or a dark notch (gap) in the initial cw background intensity (c). In (a) the phase jump is π and the degree of nonlocality is σ=2, while the initial soliton separation is x_0=5.5, 4, 2.5 from left to right. In (b) the phase jump is 0.95π and x_0=2.5, while σ=0.1, 1, 2. In (c) the width of the intensity gap is 7.5, while σ=0.1, 3, 6. (After Ref. [47]).

equivalent to that describing stationary states of parametric solitons.[49, 50] It has beens shown that this model predicts the existence of stable multiple dark soliton solutions above a certain critical value of the nonlocality parameter σ.

The ability for dark solitons to form bound states and their subsequent stability is a direct consequence of the *nonlocality-induced long range attraction of solitons*. This effect can be qualitatively explained using the self-guiding concept. In a local defocusing medium the refractive index change corresponding to two distant dark solitons has the form of two waveguides separated by a region of lower refractive index (a potential barrier). In the presence of nonlocality the effect of the convolution term in Eq. (2) is to decrease the index difference between these two separate waveguides (lower the barrier) thereby allowing light to penetrate the area between solitons. This, consequently, manifests as soliton attraction. The attraction of solitons can be clearly demonstrated by simulating numerically the interaction dynamics of two nearby solitons. The results of these simulations are summarized in Fig. 7(a-c). The plots show trajectories of dark solitons generated by two nearby phase jumps (a,b) or a gap in the incident cw background. These results clearly demonstrate that as the nonlocality parameter σ becomes comparable with the separation of the solitons they strongly attract and trap each other and subsequently propagate together as a bound state exhibiting transverse oscillations.

In order to investigate experimentally interaction of dark nonlocal solitons[48] we used liquids with thermal nonlinearity as a nonlinear medium. Absorption of light results in raising the temperature of the liquid, and subsequently decreasing its refractive index leading to the self-defocusing effect. Due to the heat conduction in the liquid the thermal nonlinearity is *inherently nonlocal*. First we studied interaction of dark solitons in thermal medium by solving numerically ensuing model equation. As an initial condition we used a broad Gaussian beam with two closely placed π phase jumps. Such initial conditions (Fig.8(a)) result in the formation of two, 1-dimensional "black" dark solitons of a zero transverse velocity. The nonlocal interaction of solitons leads to oscillatory behavior of their trajectories (Fig. 8(b)). For comparison, Fig. 8(c) illustrates the propagation of two dark solitons in a *local* saturable nonlinear medium. Clearly the attraction is gone and those solitons gradually move apart as they propagate. In Fig. 8(d,e) we show experimental results on the separation between the dark solitons during their propagation, obtained in experiments with paraffin oil doped with iodine as a nonlinear medium. Solitons were excited in the liquid using appropriate phase masks. The light intensity distribution along the propagation direction could be monitored by placing a mirror mounted on a translation stage. However, because of the finite size of this mirror the first 19mm of the propagation distance could not be accessed. Plot in Fig.8(e) depicts the intensity distribution of the solitons as a function of their propagation. The dashed line follows the intensity minima thereby depicting soliton trajectories. For initial separation of 59 μm the repulsive force between solitons is strong and can not be compensated by the nonlocality induced attraction. Therefore, for this separation the two dark solitons repel. For the cases of initial separation of 91, 101 and 117 μm the dark soliton trajectories exhibit spatial oscillations. The solitons come closer than their initial separation. The presence of this oscillation is a direct proof of the interplay of repulsive and *nonlocality-mediated* attractive forces acting between the solitons. In fact, such behavior indicates that the repulsion and attraction are of comparable strength so the solitons are actually close to form their bound state.

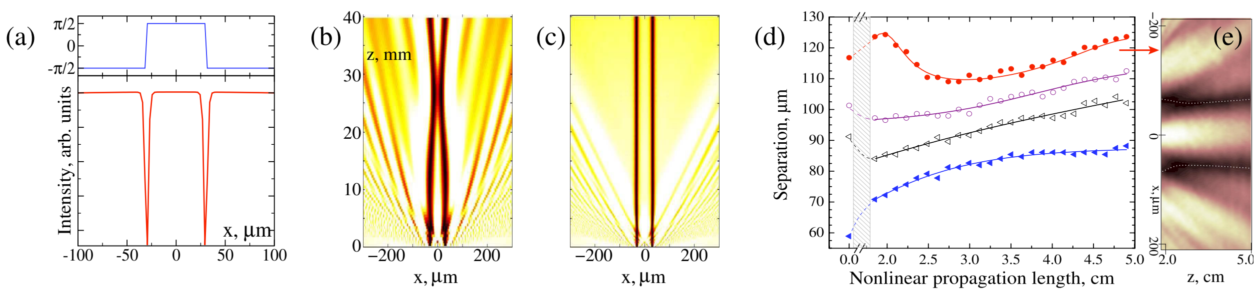

Figure 8. (a) Initial phase (top) and intensity (bottom) profile of two dark solitons. (b) Numerically found trajectories of interacting nonlocal (b) and local (c) solitons. (d) Experimentally determined separation of interacting nonlocal dark solitons as a function of the propagation distance for initial separation of 116.8 μm - closed circles; 101.3 μm - open circles; 91.2 μm - open triangles; 58.9 μm - closed triangles. Grey stripe indicates experimentally unaccessible region of soliton propagation. (e) Oscillating trajectories of nonlocal solitons in case of initial separation of 117μm. (After Ref. [48])

6. CONCLUSIONS

In conclusion, we discussed some properties of optical beams in nonlocal nonlinear media. We showed that unlike local Kerr media where multidimensional beam are unstable, nonlocal Kerr-type nonlinearity arrests a collapse of beams and allows for the formation of stable multi-dimensional solitons. In addition we showed that ring vortex solitons become stable in nonlocal medium when the degree of nonlocality is sufficiently high. Finally, we demonstrated theoretically and experimentally that dark solitons exhibit nonlocality mediated attraction.

7. ACKNOWLEDGEMENT

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