Variational analysis of SPM- and IPM-based interactions in cubic non-local nonlinear media

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ABSTRACT

We analytically show that the non-locality of cubic nonlinear media causes an increase of the critical power for self- and induced focusing and influences the condition for signal beam attraction/repulsion in an off-axis geometry.

Keywords: self-phase modulation, induced phase modulation, nonlinear media, non-locality

1. INTRODUCTION

Non-locality is a generic feature arising in physical systems. It appears in optical media with thermal or/and diffusive[1] type of nonlinearity, influences the propagation of electromagnetic waves in plasmas[2] and plays an important role in the Bose-Einstein condensates. In nonlinear optics the non-locality can be described by a spatial dependence of the nonlinear susceptibility tensor. The effect of non-locality is in some sense equivalent to that of saturation, to smooth out the refractive indexes profiles and thereby increase the bright soliton width and decrease the width of dark solitons. Although the non-locality tends to suppress modulation instability in focusing Kerr media, it can not remove it completely[1,3]. Non-local nonlinearity leads to a long-range attraction and formation of stable bound states of otherwise repelling one-dimensional dark solitons[4].

2. THEORETICAL ANALYSES

The non-local, nonlinear term in the generalized nonlinear Schrödinger equation (NLSE) is a convolution of the modulus squared of the electric field amplitude with a non-local response function prescribed by the physical process (interaction kernel). Bright and dark soliton solutions of the NLSE are derived[5].

2.1. Self-phase modulation

Let us consider a single cw or quasi-cw beam propagating through a non-local nonlinear Kerr-type medium. Its evolution is described by the nonlinear Schrödinger equation (NLSE)

$$\mathbf{i}\frac{\partial\psi}{\partial\mathbf{x}}+\alpha\frac{\partial^2\psi}{\partial\mathbf{r}^2}+\mathbf{k}^{\mathbf{SPM}}|\psi|^2\psi+\gamma^{\mathbf{SPM}}\frac{\partial^2|\psi|^2}{\partial\mathbf{r}^2}=\mathbf{0}, \qquad (1)$$

where ψ is the slowly-varying amplitude, $\alpha=\mathbf{1/2k}$, $\mathbf{k}$ is the wave-number, $\mathbf{k}^{\mathbf{SPM}}$ $\left(\mathbf{sign}\left(\mathbf{k}^{\mathbf{SPM}}\right)=\mathbf{sign}\left(\chi^{(3)}_{\mathbf{SPM}}\right)\right)$ is the nonlinear coefficient for self-phase modulation and $\gamma^{\mathbf{SPM}}$ describes the non-locality ($\gamma^{\mathbf{SPM}}=\mathbf{0}$ for a local nonlinear medium). In this work we use the variational method for analyzing the NLSE[6].

Using the Ritz's optimization for the trial function is chosen to be Gaussian

$$\psi(\mathbf{x},\mathbf{r})=\frac{\mathbf{A}(\mathbf{x})}{\omega(\mathbf{x})}\mathbf{exp}\left(\frac{\mathbf{r}^2}{\mathbf{a}^2\omega^2(\mathbf{x})}-\mathbf{i}\frac{\mathbf{k}\rho(\mathbf{x})\mathbf{r}^2}{\mathbf{2}}\right), \qquad (2)$$

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where $\mathbf{A(x)}$ is the complex slowly-varying amplitude of the beam, $\omega(\mathbf{x})$ and $\mathbf{a}$ are normalized and physical beam radii, respectively, at the entrance of the non-local medium, and $\rho(\mathbf{x})$ is the inverse radius of curvature of the beam wavefront. As a result we obtained the following system of ordinary differential equations for variational parameters:

$$\frac{d\omega}{dx} = -\omega\rho \tag{3a}$$

$$\frac{d\rho}{dx} = \rho^2 - \frac{4}{k^2 a^4 \omega^4} + \frac{\sqrt{2} k^{SPM} |A|^2}{k a^2 \omega^4} - \frac{6\sqrt{2}\gamma^{SPM} |A|^2}{k a^4 \omega^6}. \tag{3b}$$

As seen, in the last equation there are two nonlinear terms with opposite signs. After substitution of the initial condition $\omega(\mathbf{x}=0)=1$, the critical value of coefficient of non-locality γ_{crit}^{SPM}, which causes change of the sign of the effective nonlinearity, is

$$\gamma_{crit}^{SPM} = \frac{k^{SPM} a^2}{6}. \tag{4}$$

Utilizing the condition $\rho(\mathbf{x})=0$ we obtained the expression for the critical power for self-focusing in the non-local nonlinear Kerr-type medium

$$P^{SF}\left(\gamma_{NORM}^{SPM} > 0\right) = \frac{\sqrt{2} c \lambda^2}{8\pi^2 n_2^{SPM} n_0} \frac{1}{\left(1-\gamma_{NORM}^{SPM}\right)}, \tag{5}$$

where $\gamma_{NORM}^{SPM} = \gamma^{SPM} / \gamma_{crit}^{SPM}$.

Figure 1 shows the critical power for self-focusing in a local medium $P^{SF}\left(\gamma_{NORM}^{SPM} > 0\right)$ normalized to $P\left(\gamma_{NORM}^{SPM} = 0\right)$. One can clearly see the increase of $P^{SF}\left(\gamma_{NORM}^{SPM} > 0\right)$ and the existence of a critical value of the parameter of non-locality γ_{NORM}^{SPM} (see Eq. (4)). Due to the sign change of effective nonlinearity at higher values of γ_{NORM}^{SPM} there is no possibility to reach the solitary regime (see the shaded region of the Fig. 1).

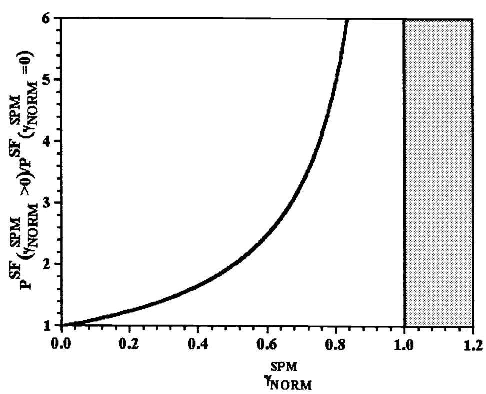

Fig. 1. Relative increase of the critical power for self-focusing in a non-local medium.

2.2. Induced phase modulation

The picture becomes more complicated when two beams (pump and signal) co-propagate in a non-local Kerr-type medium. The NLSE describing the evolution of the signal beam has the form

$$i\frac{\partial \psi_s}{\partial x} + \alpha \frac{\partial^2 \psi_s}{\partial r^2} + \left(k^{IPM}(\Omega_s) |\psi_p|^2 + \gamma^{IPM}(\Omega_s) \frac{\partial^2 |\psi_p|^2}{\partial r^2} \right) \psi_s = 0. \tag{6}$$

Here ψ_s and ψ_p are the complex slowly-varying amplitudes of signal and pump beam, respectively, and γ^{IPM} accounts for the non-locality. The nonlinear coefficient for induced phase modulation is related to the nonlinear susceptibility $\chi_{IPM}^{(3)}$ at a signal frequency Ω_s $\left(\mathbf{sign}\left(\mathbf{k}^{\mathbf{IPM}}(\Omega_s)\right)=\mathbf{sign}\left(\chi^{(3)}_{\mathbf{IPM}}(\Omega_s)\right)\right)$. The lack of the second evolutionary equation in the model is motivated by the supposition that the pump beam propagates as an one-dimensional soliton $(\omega_p(x)=1)$. Following the variational approach, chosing trial functions of Gaussian-type

$$\psi_s(x,r)=\frac{A_s(x)}{\omega_s(x)}\exp\left(-\frac{(r-r_0)^2}{a_s^2\omega_s^2(x)}-i\frac{k_s\rho_s(x)r^2}{2}\right) \tag{7a}$$

$$\psi_p(x,r)=\frac{A_p(x)}{\omega_p(x)}\exp\left(-\frac{r^2}{a_p^2\omega_p^2(x)}\right), \tag{7b}$$

we kept the freedom to analyze off-axis signal beam propagation by introducing an axis displacement $r_0(x=0)$. As a result, we obtained the following system of ordinary differential equations for the variational parameters:

$$\frac{d\rho_s(x)}{dx}=\rho_s^2(x)-\frac{4}{k_s^2a_s^4\omega_s^4}+4\left[\frac{k^{IPM}|A_p|^2a_p}{k_s\omega_p\left(a_s^2\omega_s^2+a_p^2\omega_p^2\right)^{3/2}}\left(1-\frac{4r_0^2}{a_s^2\omega_s^2+a_p^2\omega_p^2}\right)-\right.$$
$$\left.\frac{4\gamma^{IPM}|A_p|^2a_p}{k_s\omega_p\left(a_s^2\omega_s^2+a_p^2\omega_p^2\right)^{5/2}}\left(3-\frac{24r_0^2}{a_s^2\omega_s^2+a_p^2\omega_p^2}+\frac{16r_0^4}{\left(a_s^2\omega_s^2+a_p^2\omega_p^2\right)^2}\right)\right]\exp\left(-\frac{2r_0^2}{a_s^2\omega_s^2+a_p^2\omega_p^2}\right) \tag{8a}$$

$$\frac{d\omega_s}{dx}=-\omega_s\rho_s \tag{8b}$$

$$\frac{dr_o}{dx}=2r_o\rho_s. \tag{8c}$$

Substituting the initial conditions $\omega_s(x=0)=1$; $\omega_p(x)=1$; $\rho_s(x=0)=0$ and introducing normalization of the form $v=r_0(x=0)/a_p$, $u=a_s(x=0)/a_p$, we found a general expression for the critical value of the coefficient of non-locality

$$\gamma_{crit}^{IPM}=\frac{(u^2+1)\left(1-\dfrac{4v^2}{u^2+1}\right)k^{IPM}a_p^2}{4\left(3-\dfrac{24v^2}{u^2+1}+\dfrac{16v^4}{\left(u^2+1\right)^2}\right)}. \tag{9}$$

Assuming that the curvature of the beam wavefront does not change it's sign, we derived from Eq. (8a) a generalized function $\mathbf{G(v,u)}$ describing the initial condition for off-axis signal beam nonlinear attraction/repulsion in non-local medium

$$G(v,u)=\left(1-\gamma_{NORM}^{IPM}\right)\left(u^2+1\right)^{-3/2}\left(1-\frac{4v^2}{u^2+1}\right)\exp\left(-\frac{2v^2}{u^2+1}\right), \tag{10}$$

where $\gamma_{NORM}^{IPM}=\gamma^{IPM}/\gamma_{crit}^{IPM}$.

(a) (b)

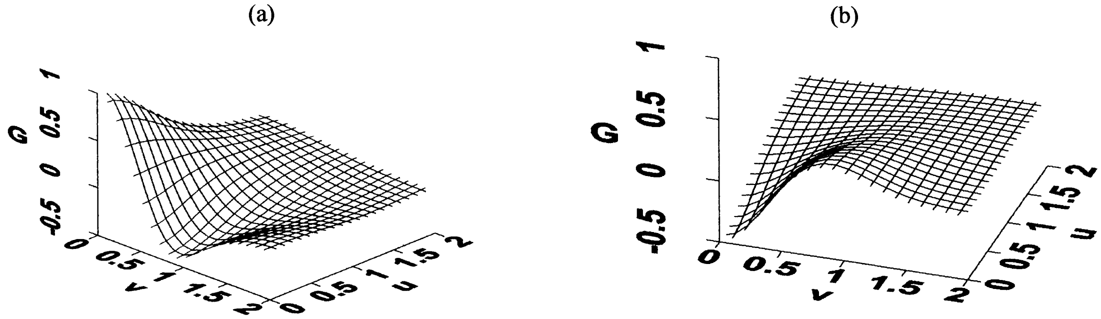

Fig.2. Plot of the function G(v,u) for $\gamma_{NORM}^{IPM}=0$ (a) and $\gamma_{NORM}^{IPM}=1{,}5$ (b).

In Fig.2 we show the function $\mathbf{G}(\mathbf{v},\mathbf{u})$ with $\mathbf{v}=\mathbf{r}_0(\mathbf{x}=0)/\mathbf{a}_\mathbf{p}$ and $\mathbf{u}=\mathbf{a}_\mathbf{s}(\mathbf{x}=0)/\mathbf{a}_\mathbf{p}$. Two characteristic regions in the local case can be distinguished. The first, at $\mathbf{r}_0=0$, corresponds to induced focusing or defocusing in an on-axis geometry, depending on the sign of $\chi^{(3)}_{\mathbf{IPM}}$. The second region is located at $\mathbf{r}_0/\mathbf{a}_\mathbf{p}\approx 0.8$ and corresponds to induced attraction/deflection of the signal beam. Increasing the normalized coefficient of non-locality $\left(\gamma^{\mathbf{IPM}}_{\mathbf{NORM}}=0.5\right)$ the maximum of $\mathbf{G}(\mathbf{v},\mathbf{u})$ decreases. At strong non-locality $\gamma^{\mathbf{IPM}}_{\mathbf{NORM}}>1$ the processes changes from induced focusing/defocusing in on-axis geometry to defocusing/focusing. In an off-axis geometry transformation from induced attraction/repulsion to repulsion/attraction takes place, depending on the sign of $\chi^{(3)}_{\mathbf{IPM}}$. The conditions for induced attraction/deflection are $\mathbf{sign}(\mathbf{G})\mathbf{sign}\left(\mathbf{k}^{\mathbf{IPM}}\right)=\pm 1$, respectively.

Eq. (10a,b) lead to the second-order ordinary differential equation

$$\frac{d^2\omega_s}{dx^2}=\frac{4}{k_s^2a_s^4\omega_s^3}-\left[\frac{4k^{IPM}\left|A_p\right|^2a_p\omega_s}{k_s\omega_p\left(a_s^2\omega_s^2+a_p^2\omega_p^2\right)^{3/2}}\left(1-\frac{4r_0^2}{a_s^2\omega_s^2+a_p^2\omega_p^2}\right)-\frac{16\gamma^{IPM}\left|A_p\right|^2a_p\omega_s}{k_s\omega_p\left(a_s^2\omega_s^2+a_p^2\omega_p^2\right)^{5/2}}*\right.$$
$$\left.\left(3-\frac{24r_0^2}{a_s^2\omega_s^2+a_p^2\omega_p^2}+\frac{16r_0^4}{\left(a_s^2\omega_s^2+a_p^2\omega_p^2\right)^2}\right)\right]\exp\left(-\frac{2r_0^2}{a_s^2\omega_s^2+a_p^2\omega_p^2}\right) \tag{11}$$

From this result, in on-axis geometry $(\mathbf{r}_0=0)$, under plane-wave steady-state condition we obtain an expression for the critical power for induced focusing

$$P^{IF}\left(\gamma^{IPM}_{NORM}>0;p=\frac{a_p}{a_s}>0\right)=\frac{c\lambda_s^2}{16\pi^2n_2^{IPM}n_0}\frac{\left(p^2+1\right)^{3/2}}{p}\frac{1}{\left(1-\gamma^{IPM}_{NORM}\right)}, \tag{12}$$

where $\mathbf{p}=\mathbf{a}_\mathbf{p}/\mathbf{a}_\mathbf{s}$.

In Fig. 3 we present the ratio between the critical powers for induced focusing in non-local and local media for different pump-to-signal beam ratio $\mathbf{p}$:

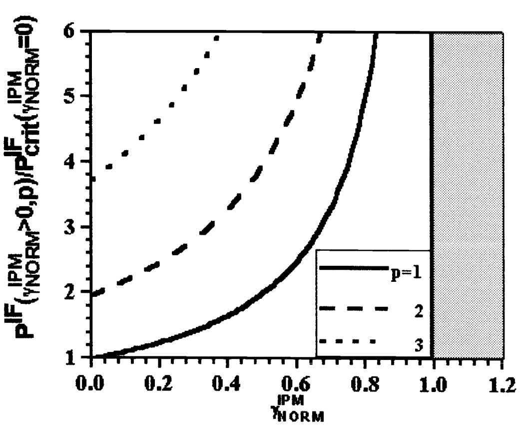

Fig.3. Relative increase of the critical power for induced focusing in a non-local medium for different values $\mathbf{p}$.

The shaded region in Fig.3 corresponds to impossibility of reaching the soliton-like regime. Introducing the normalization $\mathbf{x}'=\mathbf{x}/\mathbf{L}_\mathbf{s}$ ($\mathbf{L}_\mathbf{s}=\mathbf{k}_\mathbf{s}\mathbf{a}_\mathbf{s}^2/2$ - diffraction length) and $\mathbf{p}=\mathbf{a}_\mathbf{p}/\mathbf{a}_\mathbf{s}$, in on-axis case $(\mathbf{r}_0=0)$ Eq. (11) reduces to

$$\frac{d^2\omega_s}{dx'^2}=\frac{1}{\omega_s^3}-\frac{2\sqrt{2}Pp\omega_s}{P^{IF}_{crit}\left(p=1;\gamma^{IPM}_{NORM}=0\right)\left(\omega_s^2+p^2\right)^{3/2}}\left(1-\frac{\gamma^{IPM}_{NORM}\left(p^2+1\right)}{\left(\omega_s^2+p^2\right)}\right). \tag{13}$$

Numerical results are shown in Fig. 4.

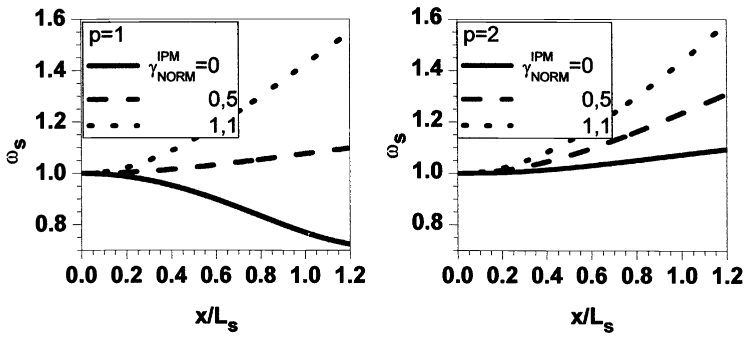

Fig.4. Numerical results by solving Eq. (13) for $\mathbf{P/P^{IF}_{crit} = 1,6}$.

As seen, at $\mathbf{P/P^{IF}_{crit} = 1,6}$ and $\mathbf{p > 1}$ (radius of the pump beam greater than that of the signal) widening of the radius of signal beam is observed as result of the reduced spatial overlapping between two beams. In a non-local medium the non-locality contributes to the beam divergence which leads to an additional increase of the critical power for induced focusing.

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