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STATISTICAL PROPERTIES OF SECOND HARMONIC GENERATION IN NONIDEAL 1D NONLINEAR PHOTONIC CRYSTALS

D. Simeonov, S. Saltiel

Department of Quantum Electronics, Faculty of Physics, Sofia University, Sofia 1126, 5 J. Bourchier Blvd.

Abstract. Quasi phase matching structures are used widely in nonlinear optics. Due to different reasons the domain widths have certain distribution. We present here an 1D analytical model that predicts the statistical properties of second harmonic generation (SHG) process in such structures. The strong variance of the efficiency of the SHG process for different samples is described in terms of the distribution of the domain widths. Correlation between the shape and width of the tuning curves and the statistical variance of the domain widths is also found.

1 Introduction

Quasi phase matching (QPM) devices (also called nonlinear photonic crystals) are widely used in the contemporary optical systems. Either induced or caused by technical tolerances, deviations from the ideal periodicity occur in the domain boundaries – the domain widths appear to have certain statistical distribution. Such problems were analytically studied in term of general decrease of the overall second harmonic generation (SHG) efficiency and slight broadening of the tuning curve [1]. Given solutions are acceptable for the near ideal structures. However, experimental results that leave certain unanswered questions, have been reported. In [2] were reported tuning curves for periodically poled Lithium niobate (PPLN) that significantly deviate from these predicted in [1] – multipeak structures have been observed. Recently, it was also reported [3,4] SHG in crystals like multidomain Strontium Barium Niobate (SBN) where the variance of the domain width is much larger. Need of new analytical studies occurs. This paper presents a new computer simulation model for the study of the SHG process in such crystals. The obtained results give clearer view that allows the development of an analytical reasoning for the processes.

2 Analytical model and computer simulation

The development of computer simulated model for SHG in nonideal 1D nonlinear photonic crystals starts from the plane wave equations for SHG based on slowly varying amplitudes approximation in which nondepleted pump and negligible losses for the interacting waves are assumed :

$$-2ik_2\frac{\partial A_2}{\partial z} = \\ = \frac{4\pi(2\omega)^2}{c^2}\left(\vec{e}_2\hat{\chi}^{(2)}:\vec{e}_1\vec{e}_1\right)A_1^2\exp(-i\Delta K z) \quad (1)$$

Like in ideal QPM structures, in natural ones $\hat{\chi}^{(2)}$ changes its sign at the boundary of each domain. However, in the natural structures the domain width is a statistical variable and has its statistical properties: mean, variance and distribution. We consider normal (Gaussian) distribution $\mathrm{N}(\Lambda_\mu,\sigma)$ for QPM structures that deviate from ideal periodicity but do not exclude any other possible distributions for the natural nonlinear photonic crystals like SBN. In order to introduce the domain width and its statistics, the equation (1) is written separately for each domain. Then the SHG for the entire sample is given.

$$A_2(L)=\Gamma\sigma_{12}\int_0^{L_1=0+\Lambda_{\xi1}}\exp(-i\Delta Kz)dz+$$
$$+\Gamma\sigma_{12}\int_{L_1}^{L_2=L_1+\Lambda_{\xi2}}\exp(-i\Delta Kz)dz+$$
$$+\Gamma\sigma_{12}\int_{L_2}^{L_3=L_2+\Lambda_{\xi3}}\exp(-i\Delta Kz)dz+...\quad(2)$$

where $\Gamma=4\pi\frac{i\omega A_1^2}{n_2c}$, $\Delta K=\frac{\pi}{l_c}$ and the coherence length is $l_c=\frac{\lambda}{4(n_2-n_1)}$. Λ_ξ (the domain width) is the statistical parameter that describes the stochastic modulation of $\hat{\chi}^{(2)}$ and has a normal distribution N(Λ_μ,σ). Equation (2) was numerically simulated using a random numbers generator with Gaussian profile as a model for the statistical behavior of the domain width.

3 Results and discussion

We studied the overall SHG efficiency and the tuning curve for the process of SHG as statistical variables in dependence of the variance of the domain width. The results of the calculations show that the statistical variance of the domain width leads to three phenomena: (i) variance of the overall efficiency, (ii) broadening of the tuning curve and (iii) noise structure of the tuning curve. Below we present and discuss these results.

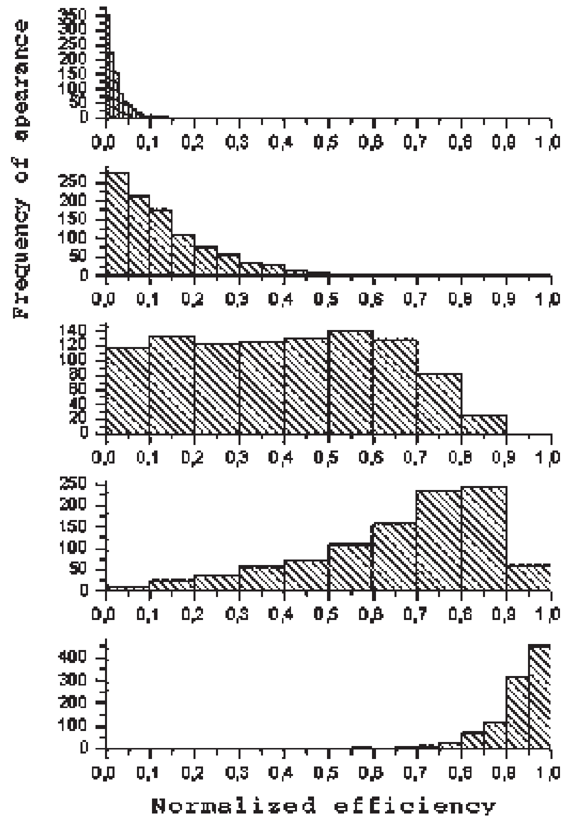

Figure 1. Histograms of the overall efficiencies for domain width variances: $\sigma_1 = 1.02$ μm, $\sigma_2 = 0.62$ μm, $\sigma_3 = 0.22$ μm, $\sigma_4 = 0.12$ μm, and $\sigma_5 = 0.07$ μm from bottom to top respectively.

3.1 Variance of the overall SHG efficiency

The histograms from Figure 1 represent the distribution of the overall efficiency for different samples of a crystal with same general parameters $L = 5000$ μm, $l_c = 8$ μm, $\Lambda_\mu = 8$ μm and different values of the domain width variance. Being a function of an statistical variable the overall efficiency is converged to its mean and, as expected, has its own statistical properties – distribution, variance.

As reported in [1] the dependence of the averaged SHG efficiency on the variance of the domain width is characterized by three zones of decrease. These three zones correspond to the three ones shown in Figure 1. This behavior could be well explained by the central limit theorem of the statistics (CLT) – generally, the variance of the function is proportional to absolute value of the derivate. As expected, the two confinements to the value for ideal QPM (histograms are normalized to it) and the zero efficiency correspond to the two zones of soft decrease of the averaged SHG efficiency when increasing of domain width variance [1]. The dispersed values in the middle zone correspond to the strong decrease of the mean of the overall SHG efficiency [1]. Although these results are important for the production of commercial QPM structures, it is clear that the variance of the overall SHG efficiency with respect to the value that correspond to the ideal structures does not cause many problems.

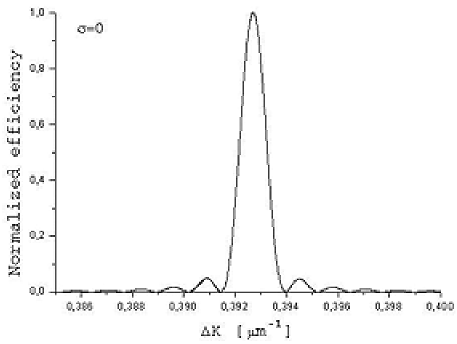

Figure 2. Shape of the tuning curve for domain width variance $\sigma_1 = 0$ μm.

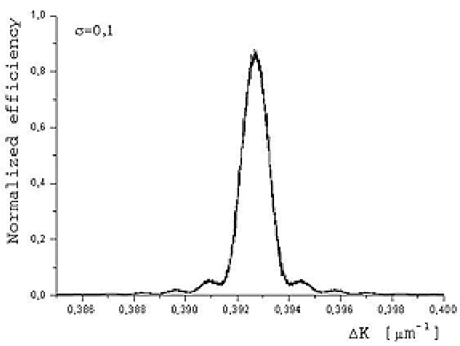

Figure 3. Shape of the tuning curve for domain width variance $\sigma_2 = 0.1$ μm.

3.2 Broadening of the tuning curve

Considering the nonideal QPM structures as possible structures for broadband SHG for short pulses it is important to know the exact behavior of the broadening of the tuning curve with the augmentation of the domain width variance.

In Figures 2, 3, 4 the evaluation of the averaged (over 1000 samples) tuning curve for different values of the domain variance is shown. Together with the augmentation of the width (FWHM) it is observed a suppression and disappearing of the small peaks and lowering the maximal efficiency. Similar figures of evaluated tuning curves for PPLN were experimentally observed and reported [2]. We report here that the broadening of the

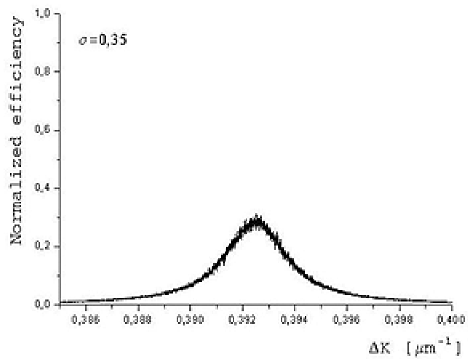

Figure 4. Shape of the tuning curve for domain width variance $\sigma_3 = 0.35$ μm.

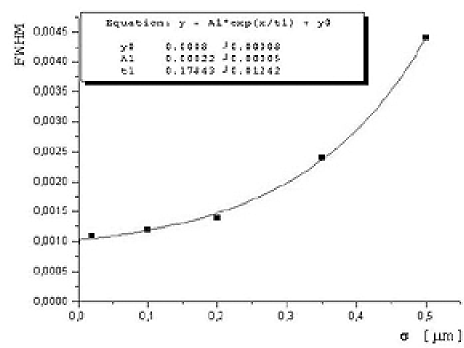

Figure 5. Dependence of the width of the tuning curve on the domain width variance.

tuning curve with the increase of the variance follows exponential law. This is illustrated in 5. The starting value (for $\sigma = 0$ μm.) is the width that corresponds to the ideal QPM structure. The broadening is caused by the enhanced set of reverse lattice vectors in the QPM lattice with induced randomness. The broadening could be explained with the correlation function for the stochastic modulation of $\hat{\chi}^{(2)}$. Although the use of such crystals for broadband SHG is limited because of noise, the dependence of the shape of the tuning curve on the shape and parameters of the domain width distribution is important as method for noninvasive diagnostics of the crystals' properties.

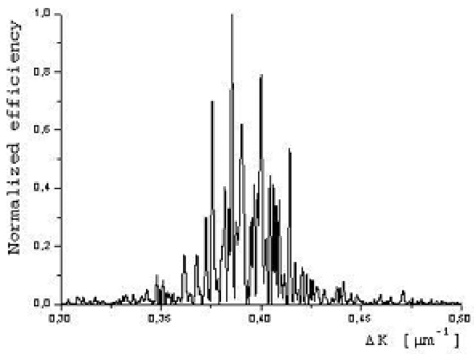

Figure 6. Noise structure of the tuning curve for value of the domain width variance: $\sigma = 0.35\ \mu$m.

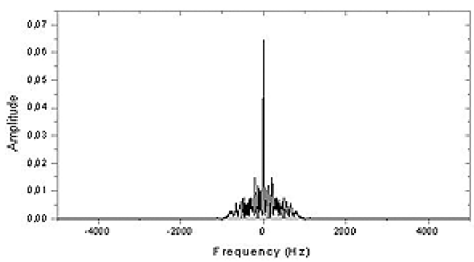

Figure 7. FFT of the noise structure of the tuning curve for value of the domain width variance: $\sigma = 0.35\ \mu$m.

3.3 Noise in the tuning curve

Together with the broadening of the tuning curve with increase of the domain width variance a noisy structure is observed. In Figure 6 is presented the noise in the tuning curve and its spectrum obtained through fast fourier transformation (FFT) is shown on Figure 7. The tuning curve shown in Figure 6 corresponds to these obtained in [2] proving their statical origin. The noise is explained with the random occurrence of the deviations from ideal periodicity in the nonlinear photonic crystal. Generated second harmonic from these deviations interferes with the main SHG process and cause decreases or increases with generally random behavior.

The amount of the noise augment with the increase of the variance of the domain width. However, the noise structure rests generally the same and FFT spectra of the tuning curve could be used as exact and noninvasive diagnostic for the quality of the crystal samples. The converged spectra (the intensity of the noise drops with the increase of its frequency) is also a sufficient reason that allows use of averaged curves for general description and a joker for creation of broadband devices in spite of the noise in the tuning curve.

4 Conclusion

Powerful method is suggested for investigation of nonideal periodical structures. The results obtained for SHG in 1D nonlinear photonic crystals give answers for the statistical behavior of their overall SHG efficiency and tuning curve in case of normally distributed domain widths. Overall efficiency variance together with noisy tuning curve imposes need of special solutions when such structures are going to be used as broadband SHG devices. It has also been revealed the dependence of the tuning curve on the domain width variance. The strong correlations between the parameters allow us to propose the presented in this paper method to be used as a non-invasive method to study the domain distribution in natural nonlinear photonic crystals.

Acknowledgments

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