Parametric vector solitons in tetragonal crystals

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We introduce novel types of spatial vector soliton that can be generated in anisotropic optical media, such as tetragonal crystals with third-order nonlinear susceptibility. We demonstrate that these vector solitons provide a nontrivial generalization to both conventional vector solitons of birefringent cubic media and parametric solitons supported by third-order cascaded nonlinearities.

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The study of complex nonlinear dynamics induced by the vectorial nature of the electromagnetic field recently attracted renewed interest because of its experimental realization in soliton physics. The concept of an optical vector soliton is usually associated with birefringent Kerr media, in which interaction of two polarizations of light produces a solitonlike bound state of two orthogonally polarized field components.[1] Vector solitons can also exist as a result of incoherent mutual trapping induced by the cross-phase interaction between the circularly polarized components of light in Kerr media.[2]

In the waveguide geometry, spatial vector solitons can appear as a superposition of the modes of the waveguide induced in a nonlinear medium through self- and cross-phase modulations. In particular, Aitchison *et al.*[3] generated spatial mixed-polarization solitons in AlGaAs planar waveguides and demonstrated the complicated polarization dynamics of solitonlike beams that is related to and can be explained by the existence of linearly and elliptically polarized vector solitons. A special case of such solitons corresponds to the so-called Manakov solitons, for which the ratio of self- to cross-phase modulation is unity and the energy exchange vanishes.[4]

Many studies of the third-order optical effects in centrosymmetric crystals have been performed for nonlinear crystals with cubic symmetry. In particular, spatial vector solitons have been analyzed theoretically and observed experimentally in the cubic crystals.[3–5] However, many interesting effects can be expected in centrosymmetric nonlinear crystals that belong to crystallographic groups with lower symmetry.

To demonstrate the importance of symmetries in nonlinear interactions, we consider the conventional four-wave mixing process for the light propagating along the z axis. For this geometry and cubic symmetry, the relevant components of the third-order susceptibility tensor $\chi^{(3)}_{ijkl}$ can be reduced, because of the space and permutation symmetries, to only three independent components (see, e.g., Ref. 6) as follows: $\chi^{(3)}_{xxxx} = \chi^{(3)}_{yyyy}$, $\chi^{(3)}_{xxyy} = \chi^{(3)}_{yyxx}$, and $\chi^{(3)}_{xyxy} = \chi^{(3)}_{yxyx} = \chi^{(3)}_{xyyx} = \chi^{(3)}_{yxxy}$, where the corresponding changes of the frequencies should be taken into account as well. However, for tetragonal crystals (or, more precisely, crystals of symmetry classes 4, $\bar{4}$, or $4/m$), we find that the space and permutation symmetries allow additional independent components, $\chi^{(3)}_{xyyy} = \chi^{(3)}_{yxyy} = \chi^{(3)}_{yyxy} = \chi^{(3)}_{yyyx}$ and $\chi^{(3)}_{xxxy} = \chi^{(3)}_{xxyx} = \chi^{(3)}_{xyxx} = \chi^{(3)}_{yxxx}$, which are connected by the relation $\chi^{(3)}_{xxxy} = -\chi^{(3)}_{xyyy}$. The additional tensor components $\chi^{(3)}_{xxxy}$ and $\chi^{(3)}_{xyyy}$ may change the properties of nonlinear interactions and vector solitons.[7]

Studies of nonlinear optical interactions in centrosymmetric tetragonal crystals are rather limited. Among a few publications, we would like to mention a study in which measurements of nonlinear components were reported[8] and a theoretical analysis of the nonlinear optical activity[9] in $PbMoO_4$. However, the role of the additional tensor components is extremely important, since the asymmetric components generate parametric coupling between the polarizations that dramatically changes the properties of the vector solitons because of the effective nonlinear optical activity. In this Letter we study both linear and elliptically polarized vector solitons in tetragonal crystals and discuss their stability.

We consider the propagation of optical beams along the z axis and write out the nonlinear polarization terms in the following form:

$$
\begin{aligned}
\mathcal{P}_x = (3/8)[&2\chi^{(3)}_{xxxy}|E_x|^2E_y + 2\chi^{(3)}_{xyxy}|E_y|^2E_x \\
&+ \chi^{(3)}_{xxyy}E_x^*E_y^2 + \chi^{(3)}_{xxxy}E_y^*E_x^2 - \chi^{(3)}_{xxxy}|E_y|^2E_y \\
&+ \chi^{(3)}_{xxxx}|E_x|^2E_x], \\
\mathcal{P}_y = (3/8)[&-2\chi^{(3)}_{xxxy}|E_y|^2E_x + 2\chi^{(3)}_{xyxy}|E_x|^2E_y \\
&+ \chi^{(3)}_{xxyy}E_y^*E_x^2 + \chi^{(3)}_{xxxy}|E_x|^2E_x - \chi^{(3)}_{xxxy}E_x^*E_y^2 \\
&+ \chi^{(3)}_{xxxx}|E_y|^2E_y],
\end{aligned}
\tag{1}
$$

We consider soliton formation in a slab waveguide, and present the field components as $E_x = A_x(x,z)B_x(y)$ and $E_y = A_y(x,z)B_y(y)$, where $B_{x,y}(y)$ are the profiles of the guided modes, and in the following we assume normalization $\int_{-\infty}^{+\infty}|B_{x,y}(y)|^2\mathrm{d}y = 1$. For simplicity, we neglect the difference in the modal profiles,

$B_{x,y}(y) \simeq B(y)$, so that the propagation constants of the guided modes K_x and K_y become close to each other, $|K_x - K_y| \ll K_{x,y} \simeq K$.

To derive the governing model, we average the nonlinear equations over the waveguide cross section (y) and obtain the reduced equations for $A_{x,y}(x,z)$, taking into account nonlinear coupling according to Eq. (1) and the beam diffraction along the x axis in the paraxial approximation:

$$i\,\frac{\partial A_x}{\partial z} + \frac{1}{2K}\,\frac{\partial^2 A_x}{\partial x^2} + K_x A_x + \frac{2\pi\omega^2 S}{Kc^2}\,\mathcal{P}_z(A_x, A_y) = 0\,,$$
$$i\,\frac{\partial A_y}{\partial z} + \frac{1}{2K}\,\frac{\partial^2 A_y}{\partial x^2} + K_y A_y + \frac{2\pi\omega^2 S}{Kc^2}\,\mathcal{P}_y(A_x, A_y) = 0\,, \tag{2}$$

where c is the speed of light, ω is the circular frequency, and $S = \int_{-\infty}^{+\infty} B^4(y)\mathrm{d}y / \int_{-\infty}^{+\infty} B^2(y)\mathrm{d}y$. It is convenient to study Eqs. (2) in normalized form, with the dimensionless variables $Z = z/z_0$ and $X = x\sqrt{2K/z_0}$ and the envelope functions $\{u, v\}(X,Z) = \{E_x, E_y\}(x,z)\exp(-iK_x z)(\omega/c)[3\pi S z_0 \chi^{(3)}_{xxxx}/(4K)]^{1/2}$,

$$i\,\frac{\partial u}{\partial Z} + \frac{\partial^2 u}{\partial X^2} + (|u|^2 + 2\sigma_1|v|^2)u + \sigma_2 v^2 u^* + \gamma u^2 v^* + (2\gamma|u|^2 - \gamma|v|^2)v = 0\,,$$
$$i\,\frac{\partial v}{\partial Z} + \frac{\partial^2 v}{\partial X^2} - \delta v + (|v|^2 + 2\sigma_1|u|^2)v + \sigma_2 u^2 v^* - \gamma v^2 u^* + (\gamma|u|^2 - 2\gamma|v|^2)u = 0\,, \tag{3}$$

where $\delta = z_0(K_x - K_y)$ is the normalized phase mismatch and $\sigma_1 = \chi^{(3)}_{xyxy}/\chi^{(3)}_{xxxx}$, $\sigma_2 = \chi^{(3)}_{xxyy}/\chi^{(3)}_{xxxx}$, and $\gamma = \chi^{(3)}_{xxxy}/\chi^{(3)}_{xxxx}$ are the nonlinear coefficients. With no loss of generality, we consider the case $\delta \geq 0$, as otherwise it is always possible to swap the definition of the envelope functions u and v. Moreover, unless the modes are exactly matched, we can choose the coherence distance as a length scale, $z_0 = |K_x - K_y|^{-1}$, and accordingly normalize the mismatch parameter to unity, $\delta = 1$. Since we neglect absorption, Eqs. (3) conserve the normalized beam power, $P = \int_{-\infty}^{+\infty}(|u|^2 + |v|^2)\mathrm{d}x$.

We now look for soliton solutions of Eqs. (3) in the form $u(x,z) = u_0(x)\exp(i\beta z)$ and $v(x,z) = v_0(x)\exp(i\beta z)$, where β is the nonlinearity-induced shift of the propagation constant. In the limit $\beta \gg 1$, the soliton solutions can be found analytically. First, we find four different solutions for the linearly polarized solitons, where the orthogonal components of the electric field are in phase:

$$\frac{u_0(x)}{C} = v_0(x) = \left(\frac{2C\beta}{C^3 + \sigma C + 3\gamma C^2 - \gamma}\right)^{1/2} \mathrm{sech}(\sqrt{\beta}\,x)\,,$$

where $\sigma = 2\sigma_1 + \sigma_2$, $C = (\kappa/2) \pm \sqrt{(\kappa/2)^2 + 1}$, and $\kappa = \{1 - \sigma \pm [(\sigma - 1)^2 + 16\gamma^2]^{1/2}\}/(2\gamma)$. The corresponding soliton powers can be found as $P_0 = 8\sqrt{\beta}/(1 + \sigma + \gamma\kappa)$. Second, we find solutions for elliptically polarized solitons, where the field components are $\pi/2$ out of phase:

$$u_0(x) = \pm i v_0(x) = [2\beta/(1 + 2\sigma_1 - \sigma_2)]^{1/2}\mathrm{sech}(\sqrt{\beta}\,x)\,, \tag{4}$$

and $P_0 = 8\sqrt{\beta}/(1 + 2\sigma_1 - \sigma_2)$. For large β, two pairs of the linearly polarized solitons have the same power, whereas the elliptically polarized solutions do not depend on γ. This degeneracy is removed for smaller β.

For intermediate values of β, the vector solitons are found numerically. The powers of the vector solitons in tetragonal crystals (when $\gamma \neq 0$) are shown in Fig. 1(b). For comparison, in Fig. 1(a) we show the results for the vector solitons in cubic crystals, which can be found as solutions of Eqs. (3) with $\gamma = 0$. Examples of the soliton profiles are displayed in Fig. 2.

When $\gamma = 0$, Eqs. (3) have simple scalar solutions with either $u = 0$ or $v = 0$, and the linearly and elliptically polarized vector solitons appear at the bifurcation points O_1 and O_2, respectively. However,

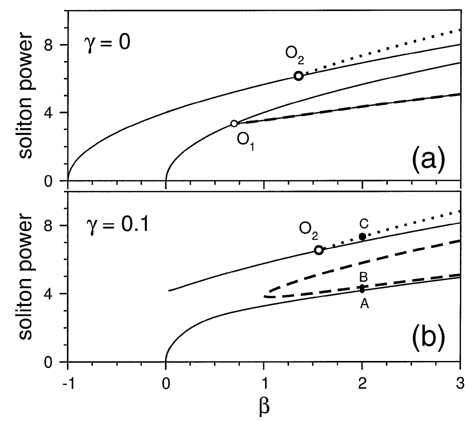

Fig. 1. Bifurcation diagram for vector solitons in (a) cubic and (b) tetragonal crystals for $\sigma_1 = \sigma_2 = 2/3$. Dotted curves, elliptic polarization; solid curves, linearly polarized solitons with in-phase components; dashed curves, linearly polarized solitons with out-of-phase components. O_1 and O_2 are the bifurcation points.

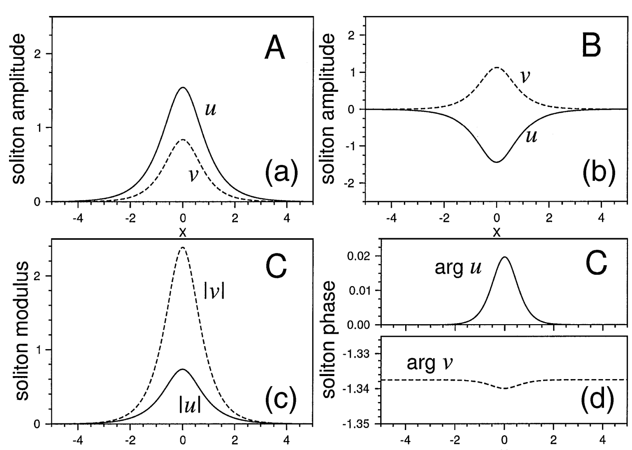

Fig. 2. Characteristic profiles of the vector solitons in tetragonal crystals: (a) in-phase and (b) out-of-phase linearly polarized solitons corresponding to the A and B in Fig. 1(b); (c) amplitude and (d) phase profiles of elliptically polarized solitons [point C in Fig. 1(b)].

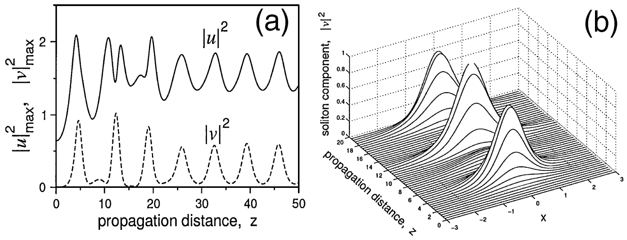

Fig. 3. Generation of type A vector soliton by an x-polarized input Gaussian beam: (a) peak intensities of two polarization components, (b) spatial evolution of the y-polarized beam component.

because of the parametric coupling between the field components in a tetragonal crystal, the solitons always have two nonzero components. This feature resembles the properties of spatial solitons modified by third-harmonic generation.[10]

Conventional vector solitons are invariant with respect to the transformations $u \to -u$ and $v \to -v$, and that is why the power dependencies for out-of-phase and in-phase solitons exactly coincide in Fig. 1(a). However, this degeneracy is lifted when $\gamma \neq 0$, and, in agreement with our analytical results, five branches of vector solitons emerge at large β, as demonstrated in Fig. 1(b). Profiles of the in-phase and out-of-phase vector solitons are shown in Figs. 2(a) and 2(b), respectively. On the other hand, the structure of elliptically polarized solitons becomes less trivial, as the relative phase shift between u and v components is no longer constant across the soliton [see Figs. 2(c) and 2(d)].

Another novel feature is the absence of proper solitons for $\beta < 0$, since in this region the u component is in resonance with linear waves, and spatially localized solutions are possible only when $u \equiv 0$. However, this condition can be satisfied only when $\gamma = 0$, as otherwise the u field is always generated from the v component because of effective nonlinear optical activity. In the region $\beta < 0$, we anticipate the existence of quasi-solitons with oscillating tails, similar to those found earlier for the problem of multistep cascading.[11]

To study stability and dynamics of these novel vector solitons, we perform direct numerical simulations of Eqs. (3) with various input conditions. In Fig. 3, we show generation of a vector soliton by an x-polarized input Gaussian beam. First, we observe the parametric generation of orthogonally polarized component v and then strong interaction of two components, resulting in periodic oscillations. As the radiation is emitted from the interaction region, the bound state converges to a stable vector soliton. Through numerical simulations, we find that only linearly polarized solitons of the types A and B [marked in Fig. 1(b)] are stable, whereas all other soliton branches are unstable. However, for $\gamma > 0$ the y-polarized component is generated in-phase with the other component and, therefore, only the generation of the type A soliton is observed.

In conclusion, we have introduced and analyzed vector solitons in tetragonal crystals. Since the field polarization components are coupled parametrically, these novel solitons provide a nontrivial generalization to both conventional solitons of birefringent cubic media and parametric solitons supported by third-order cascaded nonlinearities.

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