Multistep cascading in periodically poled directional couplers

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We suggest a novel scheme for multistep cascaded parametric processes based on a periodically poled nonlinear directional coupler and derive the conditions for simultaneous phase matching of the second-harmonic generation and sum-frequency mixing processes. We predict an efficient wavelength conversion by the cascaded third-harmonic generation from the third optical communication window (1.53–1.56 μm) to visible in a $LiNbO_3$ poled waveguide coupler. © 2005 Optical Society of America

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Recent years have demonstrated a lot of interest in the physics and applications of multistep parametric optical processes.[1] In particular, the frequency converters for the third-harmonic generation (THG) that use a single quadratic medium have been studied more extensively, and a conversion efficiency of about 20% has been achieved experimentally.[2–4] Many of these concepts are based on different types of periodic and quasi-periodic structures with a varying sign of the second-order susceptibility. However, the main disadvantage of the schemes suggested so far is that the phase matching is achieved only at some fixed values of the fundamental wavelength. Obtaining continuity for a given multistep cascaded nonlinear process with respect to the fundamental wavelength is a serious drawback of all known methods of parametric double-phase-matching processes. In order to overcome this limitation, in this Letter we suggest a novel approach for double-phase matching based on a periodically poled nonlinear directional coupler (NLDC) and demonstrate it for cascaded THG.

Using two identical parallel coupled channel waveguides for second-harmonic generation (SHG) was first suggested by Maier.[5] A directional coupler supports two modes, symmetric and antisymmetric modes, characterized by two different propagation constants, at both fundamental and second-harmonic frequencies. The difference between the propagation constants can be used to compensate for the material index dispersion and achieve phase matching. Later, Bozhevol'nyi *et al.*[6] demonstrated this technique experimentally, with the addition of electro-optic tuning for facilitating the phase-matching condition. More recently, SHG in NLDCs was discussed theoretically from the viewpoint of a spatial periodic modulation of the fundamental light intensity along the propagation direction,[7] and the phase-matching conditions for this process were derived from the coupled-mode equations.[8] In this Letter we suggest the use of a quadratic nonlinear directional coupler for double-phase-matched parametric processes. In particular, for the first time to our knowledge we demonstrate the possibility of efficient THG in such NLDCs. The suggested approach leads to broadband frequency conversion, which is in sharp contrast with other known methods employing one-dimensional nonlinear periodical structures, in both the waveguide and bulk geometries. We also consider the case of double-phase matching in a NLDC with periodic poling and show that this approach is suitable for a design of compact frequency converters based on standard crystals such as $LiNbO_3$ or KTP and operating in any desirable spectral range.

We consider a nonlinear directional coupler created by two weakly coupled identical waveguides a and b, as shown in the inset of Fig. 1. For definiteness, we are interested in the cascaded THG process that couples the fields of the fundamental (A_1), second harmonic (A_2) and third harmonic (A_3) in the first waveguide, a, and the corresponding values B_j ($j=1,2,3$) for the second waveguide, b. Then the process of cascaded THG in such a NLDC can be described, in the plane-wave approximation, by a sys-

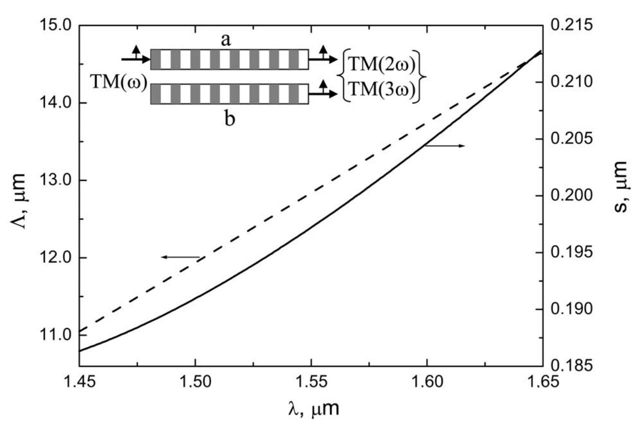

Fig. 1. Design of a periodically poled NLDC in $LiNbO_3$ at temperature $T=25$ °C for double-phase matching (Table 1, type 1b).

tem of six coupled equations:

$$i\frac{\mathrm{d}A_1}{\mathrm{d}z} = \kappa_1 B_1 + p\sigma_1 A_1^* A_2 \exp(-i\Delta k_1 z) + p\sigma_2 A_2^* A_3 \exp(-i\Delta k_2 z),$$

$$i\frac{\mathrm{d}A_2}{\mathrm{d}z} = \kappa_2 B_2 + \sigma_1 A_1^2 \exp(i\Delta k_1 z) + 2p\sigma_2 A_1^* A_3 \exp(-i\Delta k_2 z),$$

$$i\frac{\mathrm{d}A_3}{\mathrm{d}z} = \kappa_3 B_3 + 3\sigma_2 A_1 A_2 \exp(i\Delta k_2 z),$$

and three more equations for B_j obtained from the above equations by the corresponding substitutions $A_j \to B_j$ and $B_j \to A_j$. Here κ_j are the coupling coefficients for the fundamental, second-, and third-harmonic waves, respectively. When $p=0$, the system describes the THG process without depletion, and it can be solved analytically. In this case the efficiency of the THG process depends on the phase-mismatching parameters $\Delta k_1 = \beta_2 - 2\beta_1$ and $\Delta k_2 = \beta_3 - \beta_2 - \beta_1$, where β_j are the corresponding propagation constants for the fundamental, second-, and third-harmonic waves, respectively. Furthermore, we assume that all fields have the same polarization (along the Z crystal axis), and consequently $\sigma_1 = \sigma_2 = (2\pi d_{zzz}/\lambda_1 n_2) f(z)$, where the dispersion of the refractive indices is neglected, and $f(z)$ can describe a periodical poling pattern; thus we set $f(z)=1$ when no poling is used.

First, we consider NLDC without periodical poling. To find the phase-matching conditions for the SHG and THG processes we solve the system of equations for $A_{1,2}$ and $B_{1,2}$ analytically at $p=0$ and $f(z)=1$, when only the fundamental-frequency field is sent to waveguide a, $A_1(z) = a_0 \cos(\kappa_1 z)$, $B_1(z) = a_0 \sin(\kappa_1 z)$, and more complicated but explicit expressions for $A_2(z)$ and $B_2(z)$. Analyzing these results, we find three possible phase-matching conditions for the SHG process: (a) $\Delta k_1 = \kappa_2$, with the solution for the second-harmonic field in each waveguide at the exact phase matching: $A_2(z) = -B_2(z) = -(i/2)\sigma_1 a_0^2 z \exp(i\kappa_2 z)$; (b) $\Delta k_1 = 2\kappa_1 - \kappa_2$ with the solution $A_2(z) = -B_2(z) = -(i/4)\sigma_1 a_0^2 z \times \exp(-i\kappa_2 z)$; and (c) $\Delta k_1 = -(2\kappa_1 + \kappa_2)$ with the same solutions as in case (b).

Phase-matching conditions (a) and (b) were derived earlier.[7,8] Here we find the third phase-matching conditions (c) for SHG. In this case, we can solve the system of coupled equations analytically and obtain the general analytical solutions for the third-harmonic fields in both waveguides and find the respective phase-matching condition for THG. Results for the phase-matching conditions and the corresponding third-harmonic amplitudes (in waveguide a) are summarized in Table 1.

In order to test the practicality of this double-phase matching, we consider a NLDC created by two identical slab waveguides with thickness w and separation s formed by neutron exchange in $LiNbO_3$ or KTP. We assume that the waveguides have a steplike refractive index with $n_c = n_b + 0.1$, where n_c is the refractive index of the core and n_b of the base. The coupling constants are defined in the case of TE modes as[9] $\kappa_{\mathrm{TE},j} = 2h_j^2 p_j \exp(-p_j s)/\{\beta_j (w + 2/p_j)(h_j^2 + p_j^2)\}$, and in the case of TM modes as[10,11] $\kappa_{\mathrm{TM},j} = 2n_{c,j}^2 h_j^2 p_j \times \exp(-p_j s)/\{n_{b,j}^2 \beta_j Q_j [1 + 2(h_j^2 + p_j^2) n_{c,j}^2/(n_{b,j}^2 p_j Q_j)]\}$, where $p_j = \sqrt{\beta_j^2 - k_{b,j}^2}$, $h_j = \sqrt{k_{c,j}^2 - \beta_j^2}$, $Q_j = w(h_j^2 + p_j^2 n_{c,j}^4/n_{b,j}^4)$, $n_{c,j} = n_{b,j} + 0.1$.

In order to achieve the double-phase matching for the fundamental wavelength in the region 1.5–1.6 μm, we vary thickness, distance, and the mode type (TE or TM) in a NLDC fabricated in $LiNbO_3$ or KTP. As a result, we were not able to satisfy any pair of the phase-matching conditions from Table 1 simultaneously, at least for these two materials. The reason is that both Δk_1 and Δk_2 are quite large, and they can not be compensated by any realistic values of κ_1, κ_2, and κ_3.

A well-known way to overcome this difficulty is to introduce a periodical change of the sign of the second-order nonlinearity along the NLDC's longitudinal direction, i.e., to consider the conventional quasi-phase-matching (QPM) processes. The period of this modulation is an additional parameter that could allow double-phase matching for realistic parameters of the directional coupler. In order to find the double-phase-matching conditions for a NLDC with a QPM structure, we need simply to replace $\Delta k_{1,2}$ in Table 1 with $\Delta k_{1,2} - (2\pi/\Lambda) m_{1,2}$, where Λ is the QPM period and m_1 and m_2 are the QPM orders used for the cascading. Using this approach and considering the multistep cascading THG interaction of the type $\mathrm{TM}(\omega) + \mathrm{TM}(\omega) \to \mathrm{TM}(2\omega)$ and

Table 1. Double-Phase-Matching THG Conditions in a Directional Coupler and Amplitude $A_3(z)$ of the Third-Harmonic Field in Waveguide a, $\alpha \equiv (\kappa_1 + \kappa_3)^{-1}$

Type	Δk_1	Δk_2	$A_3(z) = -(3/8)\sigma_1\sigma_2 a_0^3 z T_k$
1a	κ_2	$\kappa_1 + \kappa_3 - \kappa_2$	$T_{1a} = \exp(i\kappa_3 z)[z^2 - iz\alpha \exp(2i\kappa_1 z)]$
1b	κ_2	$-\kappa_1 - \kappa_3 - \kappa_2$	$T_{1b} = \exp(-i\kappa_3 z)[z^2 + iz\alpha \exp(-2i\kappa_1 z)]$
2a	$2\kappa_1 - \kappa_2$	$\kappa_2 - \kappa_1 - \kappa_3$	$T_{2a} = T_{1b}/2$
2b	$2\kappa_1 - \kappa_2$	$\kappa_2 + \kappa_1 + \kappa_3$	$T_{2b} = T_{1a}/2$
3a	$-2\kappa_1 - \kappa_2$	$\kappa_2 - \kappa_1 - \kappa_3$	$T_{3a} = T_{1b}/2$
3b	$-2\kappa_1 - \kappa_2$	$\kappa_2 + \kappa_1 + \kappa_3$	$T_{3b} = T_{1a}/2$

$TM(\omega)+TM(2\omega)\rightarrow TM(3\omega)$, we are able to find appropriate device parameters w, s, Λ that could ensure double-phase-matched THG for the fundamental wavelengths in the spectral interval $1.45\ \mu m<\lambda_{fund}<1.65\ \mu m$ for the devices fabricated from $LiNbO_3$ or KTP, for all types of phase-matching processes listed in Table 1.

As an example, in Fig. 1 we show the optimal values of s and Λ versus the fundamental wavelength λ_{fund} for a NLDC made from $LiNbO_3$ with the thickness $w=0.5\ \mu m$. This device employs the double-phase-matching conditions of the type 1b (see Table 1) and the QPM orders $m_1=1$ and $m_2=3$. At $\lambda_{fund}=1.55\ \mu m$ ($s=0.197\ \mu m, \Lambda=12.84\ \mu m$), the calculated coupling coefficients are $\kappa_1=0.0907\ \mu m^{-1}$, $\kappa_2=0.0878\ \mu m^{-1}$, and $\kappa_3=0.0414\ \mu m^{-1}$, so that the phase mismatches $\Delta k_1=0.5772\ \mu m^{-1}$ and $\Delta k_2=1.2483\ \mu m^{-1}$ can be compensated by the balance equations $\Delta k_1=\kappa_2+(2\pi/\Lambda)m_1$ and $\Delta k_2=(2\pi/\Lambda)m_2-\kappa_1-\kappa_2-\kappa_3$.

To demonstrate the possibility of achieving a reasonable efficiency of conversion to the third harmonic by using this type of frequency converter, we solve the full system of coupled equations numerically at $p=1$ (i.e., including the depletion effects) and $f(z)=(-1)^{\mathrm{int}(2z/\Lambda)}$ for the structure made in a $LiNbO_3$ substrate with the parameters considered above for $\lambda_{fund}=1.55\ \mu m$. The results for the efficiency of the conversion into the third harmonic are shown in Fig. 2.

In Fig. 3 we demonstrate that, by a simple change of temperature, the constructed NLDC structure allows frequency tripling over a wide spectral range. For each value of the fundamental wavelength, the conversion into the third harmonic is optimized by changing the temperature of the sample (the optimal value is shown on the left). We believe that these results provide the first proposal for a periodical photonic structure that can allow double-phase matching over a broad spectral range. We note that in Figs. 2 and 3 the conversion in only one of the waveguides is shown. The conversion in the second waveguide is in fact an overlapping curve, so the resulting efficiency is two times larger.

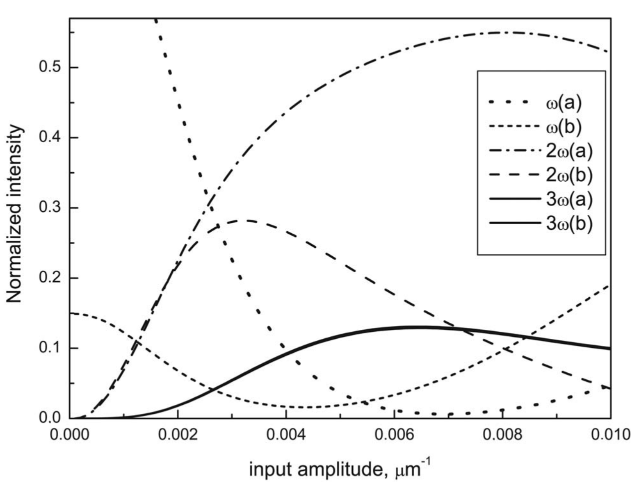

Fig. 2. Normalized output intensities of the fundamental (at $\lambda_{fund}=1.55\ \mu m$), and second and third harmonics for the type 1b phase-matching conditions in a 1 mm long periodically poled NLDC versus the input amplitude $\sigma_1 a_0$.

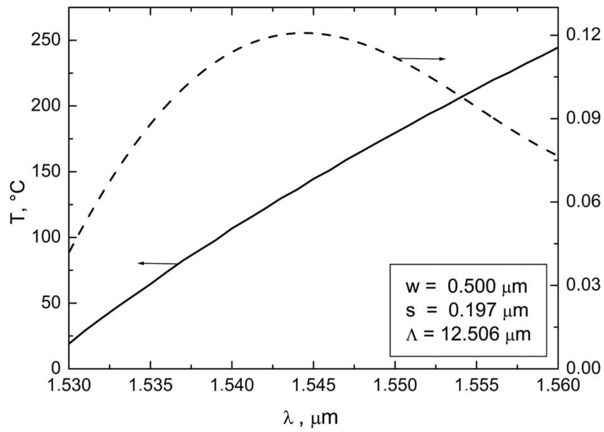

Fig. 3. Tuning curve for THG efficiency (right axis) and the sample temperature (left axis) for a 1 mm long NLDC. The input amplitude is $\sigma_1 a_0=0.006\ \mu m^{-1}$.

In conclusion, we have suggested a novel approach to double-phase matching based on mode coupling in two periodically poled waveguides with second-order nonlinearity. Our approach allows achieving simultaneous phase matching over a very broad spectral region, in contrast to what can be realized by employing other methods based on the use of one-dimensional periodically poled structures. The important aspect of our scheme is additional design parameters such as waveguide dimensions and the distance between waveguides. We have applied our ideas to analyze the cascaded generation of the third harmonics, but we believe the proposed approach is rather general and can be employed for other types of parametric processes that require that several phase-matching conditions be satisfied simultaneously.

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