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Self-heating effects in electro-optic light modulators

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The performance of an electro-optic light modulator under high average laser power is studied theoretically and experimentally. Absorption of laser radiation produces a thermal gradient in the electro-optic crystal which degrades the performance of the modulator. Measurements on modulators using KDP and KD*P crystals support the analytical results.

1. Introduction

Propagation of high average power laser radiation through an optical modulator may lead to heating of the electro-optic crystal due to absorption. The spatial distribution of the light beam and surface cooling of the electro-optic crystal produces a steady-state temperature distribution within the crystal. This is the case when either a CW laser or a high repetition rate pulsed laser is employed. A radial thermal gradient will produce a radial strain as well as a dilation of the crystal. This self-induced inhomogeneity in the nonlinear crystal will influence its performance as an optical modulator.

In this paper we consider the influence of self-heating effects on the operation of a longitudinal electro-optic modulator based on a KDP or KD*P crystal. The thermally induced strain and dilation will produce a variation in the refractive index and in the birefringence throughout the crystal. Strain effects in electro-optic light modulators due to rf heating have been studied by Kaminow [1] and a study of the self-induced thermal rotation of light polarization has been reported by Ryzhakova and Tomov [2]. We calculate the thermally induced change in the refractive index and the strain effect in KDP and KD*P modulators and compare the results with experimentally measured values. KDP and KD*P were selected for this study since they are the crystals most often used in practical devices. At the Nd:YAG laser wavelength ($\lambda = 1.064\ \mu$m) used in these experiments, KDP and KD*P have different absorption coefficients [3] and therefore it is easy to note the significance of the thermal effects.

2. Thermal gradient

We considered the cases of both a laser beam with a uniform amplitude A_0 and radius w, and a laser beam with a Gaussian spatial amplitude distribution:

$$A(r) = A_0 \exp(-r^2/w^2) \tag{1}$$

where r is the distance from the axis of the beam and w is the beam radius. w is assumed to be small compared to the electro-optic crystal cross-section. We also neglected the longitudinal thermal gradient effects. For a media with weak absorption ($\delta L \ll 1$, where δ is the absorption coefficient and L is the crystal length) the steady-state temperature distribution can be approximated by [4].

$$T(r) = T_0 + \frac{P_a}{2\pi HCL}\left(1 - \frac{r^2}{w^2}\right) \tag{2}$$

here $P_a = \delta LP$ is the dissipated laser power in the crystal. $P = C\pi w^2 A_0^2/2$ is the input laser beam power. H is the thermal conductivity and T_0 is a constant determined by the boundary conditions and equals the crystal temperature at $r = w$. The constant $C = 1$ for a Gaussian beam and $C = 2$ for a uniform beam.

The temperature gradient in Equation 2 will produce optical inhomogeneity in the refractive index which is now a function of the local temperature, $n(T)$. It is known that the temperature variation of the index of refraction for ordinary and extraordinary rays is different [5] and therefore the retardation in an anisotropic crystal will be a function of the temperature distribution. For a uniaxial crystal the retardation is

$$\Gamma = \frac{2\pi L}{\lambda}\left[n_o - n_e + \left(\frac{\partial n_o}{\partial T} - \frac{\partial n_e}{\partial T}\right)\Delta T\right] \tag{3}$$

where λ is the light wavelength, n_o and n_e are the indices of refraction for the ordinary and extraordinary rays in unperturbed medium, respectively. ΔT is the temperature increase due to laser radiation heating.

In a typical electro-optic amplitude modulator the crystal is placed between two crossed polarizers. If a linearly polarized laser beam with power P_{in} passes through the front polarizer after the second polarizer (crossed with respect to the front one) the output power is [2]:

$$P_{out} = \frac{P_{in}}{2}\left[1 - (1 + P_{in}^2/4P_c^2)^{-1}\right] \tag{4}$$

where

$$P_c^{-1} = P_{co}^{-1} - P_{ce}^{-1}; \qquad P_{co} = H\lambda/\frac{\partial n_o}{\partial T}\delta L; \qquad P_{ce} = H\lambda/\frac{\partial n_e}{\partial T}\delta L.$$

Here $\partial n/\partial T = 0$ for both polarizers as well as equal absorption coefficients, δ, are assumed for ordinary and extraordinary rays.

In a longitudinal electro-optic modulator without an applied electric field, the light beam propagates along the optic axis of the crystal and $\partial n_o/\partial T = \partial n_e/\partial T$. Thus $P_{out} = 0$ and the amplitude modulator is not transparent for the incident radiation even though there is a radial temperature gradient. When a voltage V is applied along the optic axis the induced birefringence will produce an optical retardation. The changes of the refractive indices are [6]:

$$\begin{aligned} n_{x'} &= n_o - n_o^3 r_{63} V/2L \\ n_{y'} &= n_o + n_o^3 r_{63} V/2L \\ n_{z'} &= n_e. \end{aligned} \tag{5}$$

Here the subscripts x', y' and z' are related to the directions of the major axes of the index ellipsoid in the presence of an external field applied along the z' axis. r_{63} is the corresponding electro-optic coefficient for crystals of the class $\bar{4}2m$. The electro-optic coefficient r_{63} is also temperature dependent [7]. In this case the electrically induced retardation will be

$$\Gamma_1 = \frac{2\pi}{\lambda} r_{63} n_o^3 V\left[1 + 3\frac{\partial n_o}{\partial T}\frac{\Delta T}{n_o} + \frac{\partial r_{63}}{\partial T}\frac{\Delta T}{r_{63}}\right]. \tag{6}$$

The ratio of the output intensity ($I_{out} = AA^*$) to the input intensity when absorption is taken into account is [6]

$$\frac{I_{out}(r)}{I_{in}(r)} = e^{-\delta L}\sin^2\left(\frac{\Gamma_1}{2}\right). \tag{7}$$

If the applied voltage equals the halfwave voltage $V_\pi = \lambda/2n_o^3 r_{63}$, then we have

$$\frac{I_{out}(r)}{I_{in}(r)} = e^{-\delta L}\cos^2\left(\frac{\Gamma_0}{2}\right) \tag{8}$$

where

$$\Gamma_0 = \frac{P_a}{2HLC}\left(\frac{3}{n_o}\frac{\partial n_o}{\partial T} + \frac{1}{r_{63}}\frac{\partial r_{63}}{\partial T}\right)\left(1 - \frac{r^2}{w^2}\right). \tag{9}$$

TABLE I

Parameter	KDP	KD*P	Reference
n_o	1.494	1.495	[3]
L (cm)	2.5	2.5	–
λ (cm)	1.064×10^{-4}	1.064×10^{-4}	–
δ (cm^{-1})	0.054	0.006	[3]
H (W cm^{-1} deg^{-1})	0.019	0.019	[3]
$\frac{\partial n_o}{\partial T}$ (deg^{-1})	-3.2×10^{-5}	-2.74×10^{-5}	[5]*
r_{63} (cm V^{-1})	9.7×10^{-10}	2.43×10^{-9}	[3]*
$\frac{\partial r_{63}}{\partial T}$ (cm V^{-1} deg^{-1})	-5.3×10^{-12}	-3.2×10^{-11}	[7]*
S_{11} (cm^2 N^{-1})	1.75×10^{-7}	1.575×10^{-7}	[3]
S_{12} (cm^2 N^{-1})	-4×10^{-8}	2.1×10^{-8}	[3]
S_{13} (cm^2 N^{-1})	-7.5×10^{-8}	-4×10^{-8}	[3]
S_{33} (cm^2 N^{-1})	2×10^{-7}	2.1×10^{-7}	[3]
α_1 (deg^{-1})	2.49×10^{-5}	2.01×10^{-5}	[3]
α_3 (deg^{-1})	4.4×10^{-5}	4.07×10^{-5}	[3]
P_{66}	5.8×10^{-2}	5.8×10^{-2}	[3]

*The measured values are for the visible region and we assume the same quantities for 1.064 μm.

We note that if the crystal was without absorption ($\delta = 0$, $\partial/\partial T = 0$), then $I_{out}/I_{in} = 1$. The additional retardation is a result of the heating of the crystal and temperature dependence of the refractive index and electro-optic coefficient. The temperature variations of the index of refraction and the electro-optic coefficient for KDP and KD*P were measured by Phillips [5] and Zwicker and Scherrer [7]. From the data presented in Table I we find that in KDP and KD*P the term associated with $\partial r_{63}/\partial T$ is responsible for more than 99% of the total value of Γ_0. In other words it can be said that the half-wave voltage is a function of the temperature and it varies across the beam cross-section.

By integrating Equation 8 over the beam cross-section we obtain the output power. For a laser beam with a uniform intensity distribution we have

$$\frac{P_{out}}{P_{in}} = e^{-\delta L}\left(\frac{1}{2} + \frac{\sin q}{2q}\right) \tag{10}$$

and for a Gaussian beam we obtain:

$$\frac{P_{out}}{P_{in}} = \frac{e^{-\delta L}}{4+q^2}\left(4\cos^2\frac{q}{2} + q\sin q + \frac{q^2}{2}\right) \tag{11}$$

here

$$q = \frac{P_a}{2CHL}\left(\frac{3}{n_0}\frac{\partial n_0}{\partial T} + \frac{1}{r_{63}}\frac{\partial r_{63}}{\partial T}\right). \tag{12}$$

Estimating the output power from Equations 10 and 11 using the data from Table I and an input power of 10 W, we found that the transmitted power is decreased by less than 1% compared to $P_{in}e^{-\delta L}$. Therefore, the effect of thermal changes due to $\partial r_{63}/\partial T$ is not a limitation for the operation of a modulator at these power levels.

3. Strain effect

Inhomogeneous heating of the crystal also produces mechanical strain. The strain induced change of the optical index ellipsoid will tend to degrade the performance of the electro-optic light modulator. Kaminow [1] has studied the strain effects in electro-optic modulators due to rf heating. We will use his approach to calculate the strain effect in the case of laser heating of the crystal.

The appropriate index ellipsoid for KDP is given by

$$\frac{1}{n_0^2}(x^2+y^2)+\frac{z^2}{n_e^2}+2\Delta B_6 xy = 1. \tag{13}$$

Here z is the optic axis and x and y are the twofold crystal axes in the class $\bar{4}2m$. For crystals of that class it is found that

$$\Delta B = p_{66}\Sigma_6 \tag{14}$$

where p_{66} is the elasto-optic coefficient and Σ_6 is a shear strain about the z-axis. For a temperature distribution as in Equation 2 the shear strain is

$$\Sigma_6 = \frac{p_0}{8H}\left(\frac{S_{11}-S_{12}}{S_{11}-S_{13}^2/S_{33}}\right)\left(\alpha_1-\frac{S_{13}}{S_{33}}\alpha_3\right) r^2 \sin 2\theta \tag{15}$$

here s_{ij} are elastic moduli, α_1 and α_3 are the thermal extension coefficients for directions normal to and along the z-axis, respectively. $p_0 = (CP\delta)/(\pi w^2)$ is the average absorbed power per unit volume. θ is the polar angle measured from the x-axis.

The phase retardation for a crystal of length L is:

$$\Gamma_2 = \frac{2\pi n_0^3}{\lambda}\int_0^L \Delta B_6(z)\,\mathrm{d}z \tag{16}$$

and from Equations 14, 15 and 16 we obtain

$$\Gamma_2 = g\frac{r^2}{w^2}\sin 2\theta \tag{17}$$

where

$$g = -\frac{n_0^3 P_a}{4C\lambda H}\left(\frac{S_{11}-S_{12}}{S_{11}-S_{13}^2/S_{33}}\right)\left(\alpha_1-\frac{S_{13}}{S_{33}}\alpha_3\right) P_a. \tag{18}$$

The output power after the second polarizer for a uniform beam is

$$\frac{P_{out}}{P_{in}} = \tfrac{1}{2}\,e^{-\delta L}[1 \pm H(g)] \tag{19}$$

where

$$H(g) = \frac{1}{g}\int_0^g J_o(x)\,\mathrm{d}x \tag{20}$$

and $J_o(x)$ is the zero-order Bessel function. The negative sign in Equation 19 is for the case without an electric field and the positive sign is for the case when a halfwave voltage, V_π, is applied to the crystal. The graph of the function $H(g)$ is presented by Kaminow [1].

For a Gaussian beam distribution we have

$$\frac{P_{out}}{P_{in}} = \left(\frac{1}{2}\pm\frac{1}{(4+g^2)^{\frac{1}{2}}}\right)e^{-\delta L} \tag{21}$$

with the same sign designation as above. If the absorption is absent, $g = 0$, then $P_{out} = P_{in}$ when a halfwave voltage is applied. For large g, $H(g) \to 0$, and from Equations 19 and 21 it follows that $P_{out} \simeq 0.5 P_{in} e^{-\delta L}$ in both cases with and without halfwave voltage applied to the crystal. Therefore in this case no modulation can be observed. The calculated dependence of P_{out}/P_{in} on P_{in} for KDP and KD*P are plotted in Fig. 1 for the case when no voltage is applied to the crystal. When a halfwave voltage is applied at an input power of $P_{in} = 10$ W, we obtain

$$\left(\frac{P_{out}}{P_{in}}\right)_{\mathrm{KDP}} = 0.75 \quad \text{and} \quad \left(\frac{P_{out}}{P_{in}}\right)_{\mathrm{KD^*P}} = 0.98.$$

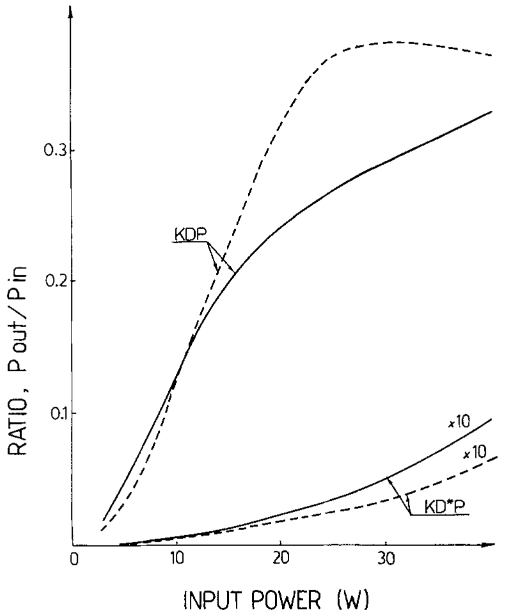

Figure 1 Dependence of the normalized output power (P_{out}/P_{in}) on the input power in an amplitude modulator using KDP and KD*P. No voltage is applied. Gaussian beam: solid curve; uniform beam: dashed curve.

Thus the thermal strain effect produces about a 12% additional loss in KDP and less than a 1% additional loss in KD*P at a power level of 10W.

We note also that the strain induced retardation according to Equation 17 is angular dependent. A uniform input light beam after the second polarizer will exhibit modulation in its cross-section distribution.

4. Experiments

The experimental study of thermally induced effects in electro-optic modulators was carried out using a Nd:YAG laser with repetition rates up to 100 Hz and an average output power up to 15 W. The laser was operated in a multimode regime to give a 0.5 cm diameter output beam with fairly uniform intensity distribution. The electro-optic crystals KDP and KD*P were 2.5 cm long with cross-sections an order of magnitude higher than the beam cross-section.

In the first experiment the electro-optic crystal was properly oriented between two crossed polarizers. The ratio of input to output power was measured as a function of the input power. The results are presented in Fig. 2. The solid curves are calculated assuming a beam with uniform cross-section and crystal data from Table I. The measured values of KDP are in good agreement with those calculated. The approximately 10% difference is most probably due to the actual temperature distribution in the crystal differing from that given by Equation 2. For KD*P the experimental data are about 2.5 times higher than those calculated. In the calculations we used $\delta = 0.006\ \text{cm}^{-1}$ for the absorption coefficient deduced from [3] for a 98.9% deuterated crystal. The losses in KD*P for wavelengths above 0.95 μm strongly depend on the degree of deuteration. No loss measurements were carried out for our crystal and we suspect that the actual loss was greater than $0.006\ \text{cm}^{-1}$.

As noted from Equation 17 it follows that if the strain induced retardation is dominant a cross-sectional modulation of the output beam will be observed. In order to verify this an unfocused He–Ne laser beam was directed collinearly with the Nd:YAG beam. With no thermally induced strain (Nd:YAG power off) no He–Ne laser radiation was recorded after the second polarizer. When the Nd:YAG laser power was increased the light from the He–Ne laser was registered behind the second polarizer. A photograph of the He–Ne laser beam at 5 W Nd:YAG laser power is shown in Fig. 3. The dark cross in the distribution corresponds to the prediction of Equation 17. The dark spot in the centre corresponding

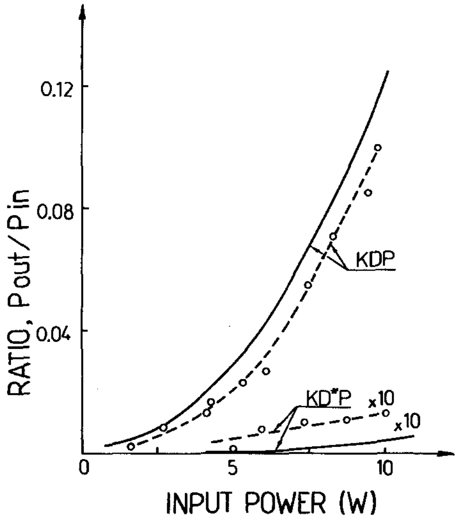

Figure 2 Experimentally measured transmission of amplitude modulators based on KDP and KD*P without an applied voltage. Solid curve: theory; dashed curve: experiment.

to $r = 0$ shows that no birefringence along the beam axis is induced thermally. The same picture was observed with KD*P at a power level of about 15 W.

These results show that the main degradation of electro-optic modulator performance at a high average power level is due to the thermally induced strain in the crystal.

5. Conclusions

In conclusion, we have observed a self-induced thermal strain in longitudinal electro-optic light modulators. The behaviour of the modulator is in good agreement with the existence of a simple radial temperature gradient produced by the absorbed laser radiation. The results show that in high average power lasers the loss in the modulator may limit the maximum operational power. For KDP at $\lambda =$ 1.06 μm the thermal effects are negligible for an average power below 1 W, while KD*P may be used with laser beams almost 10 times more powerful.

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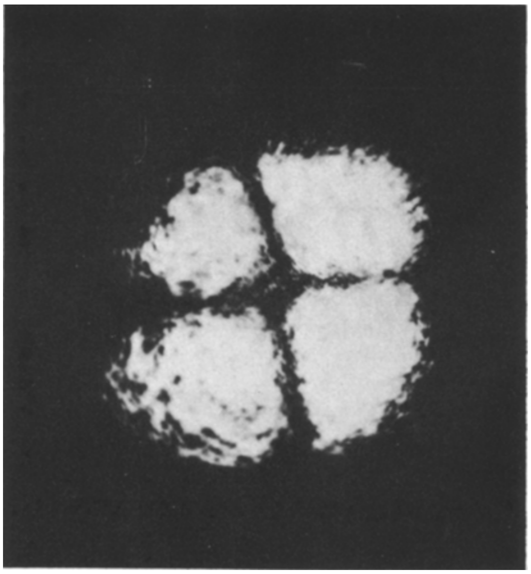

Figure 3 Photograph of the transmitted He–Ne laser beam distribution after the KDP modulator was heated with 5 W average power at 1.064 μm.

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