# Group-velocity-matched multistep cascading in nonlinear photonic crystals

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We demonstrate that simultaneous phase- and group-velocity-matched cascaded parametric processes can be achieved in two-dimensional quadratic nonlinear photonic crystals by proper matching of noncollinear processes with front tilting of the fundamental pulse. We present examples of cascaded third- and fourth-harmonic generation in a poled $LiNbO_3$ planar structure. © 2006 Optical Society of America

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Frequency conversion of ultrashort optical pulses is important for many applications of optical parametric devices in communications, signal processing, and spectroscopy. It is well known that the conversion efficiency and phase-matching bandwidth of femtosecond nonlinear optical devices is dramatically limited by the group-velocity mismatch (GVM) between the fundamental- and second-harmonic pulses. Therefore, efficient frequency conversion for ultrashort pulses can be achieved by matching both phase and group velocities.

However, only a few birefringence crystals possess narrow spectral regions where simultaneous phase matching and GVM can be achieved for single nonlinear parametric process. A more practical method, that can be employed for any spectral region is based on tilting of the front of the pump pulses. In a spectral domain, the tilting of the pulse front is equivalent to an angular frequency dispersion.[1] The use of front-tilted pulses for compensating GVM has been proposed and applied for efficient conversion with a different parametric process including second-harmonic generation[2,3] (SHG) and collinear three-wave mixing.[4,5] Since then, front-tilted pulses have been employed for both collinear and noncollinear parametric interactions in birefringence crystals.[6–9] Ultrafast cascaded quadratic nonlinear processes based on the front-tilted pulses have been further developed for generation of temporal and spatiotemporal optical solitons.[10,11]

Recently, group-velocity-matched SHG has been demonstrated by using noncollinear ("tilted") quasi-phase matching (QPM) in periodically poled lithium niobate.[12,13] This noncollinear QPM technique, in combination with spectral angular dispersion, allows substantial broadening of the bandwidth of SHG.[14] In a recent experiment,[15] cascaded third-harmonic generation (THG) by tilted femtosecond pulses in a structure formed by two crossed one-dimensional QPM gratings was reported.

In this Letter, we suggest that simultaneous phase-matching and GVM compensation of several parametric processes can be achieved in two-dimensional structures by proper matching of noncollinear processes with front tilting of the fundamental pulse. Our approach is based on the use of two-dimensional nonlinear photonic crystals[16] (2DNPCs) as an extension of the fundamental concept of the QPM optical processes in one-dimensional structures. In past years, it was predicted[17,18] and demonstrated experimentally[19–23] that 2DNPCs are promising for realizing double-phase-matched cascaded second-order parametric processes. In particular, THG and fourth-harmonic generation (FHG) in single 2DNPC have been reported in Refs. 21 and 23, respectively. However, most of these experiments employed nanosecond pulses, and accounting of GVM was not necessary. Nevertheless, using picoseconds and femtosecond pulses for direct and cascaded parametric processes strongly depends on GVM of the interacting waves.

Here we show theoretically that 2DNPC (see the inset in Fig. 1) are very suitable for realizing phase-matched multistep cascaded second-order parametric processes with complete compensation for GVM by proper matching noncollinearity of the parametric

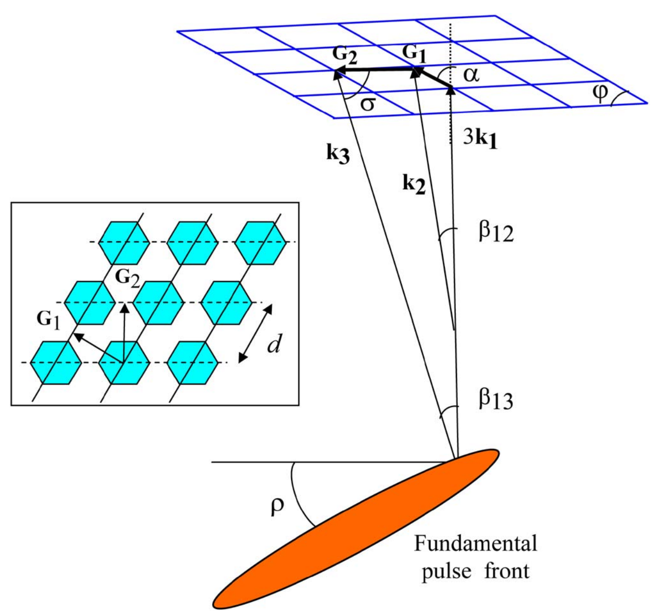


Fig. 1. (Color online) Geometry for THG with simultaneous double-phase matching and GVM compensation in 2DNPC. The grid shows a reciprocal lattice. Inset, hexagonally poled asymmetric two-dimensional nonlinear photonic crystal.

 

process and by front tilting of the fundamental pulse. We demonstrate our approach for two examples, cascaded phase- and group-velocity-matched THG and FHG processes in a single crystal, and show how to design 2DNPC structures for achieving simultaneously double-phase matching and GVM compensation.

We start our analysis by considering nonlinear parametric interactions of tilted-front pulses in 2DNPC with a simple case of GVM-compensated phase-matched SHG processes. We consider a two-dimensional hexagonally polled lithium niobate structure[19] with a distance between the inverted area $d$ (see the inset in Fig. 1). The reverse lattice is formed by two principal lattice vectors at angle 60°, each of them with the magnitude $G_1 = G_2 = 4\pi/d\sqrt{3}$. In general, the wave vectors of the fundamental and second-harmonic (SH) waves are noncollinear with the angle of noncollinearity $\beta_{12}$ defined by the relation $\cos\beta_{12} = (4k_1^2 + k_2^2 - G_1^2)/4k_1k_2$, where $G_1$ is the one of the principal vectors that is employed to phase match the SHG process. Compensation for GVM is achieved by tilting of the front of the fundamental pulse. The required tilting angle $\rho$ can be found from a simple geometric relation by noting that the time of flight for the SH pulse should coincide with that for the fundamental pulse. As a result, we find that the tilting angle $\rho$ that will compensate for GVM is defined by the angle of noncollinearity and the group velocities $v_1$ and $v_2$,

$$\tan\rho = \frac{v_1 - v_2\cos\beta_{12}}{v_2\sin\beta_{12}}, \qquad (1)$$

and this result coincides with the condition of the GVM-compensated SHG in one-dimensional QPM structures.[13]

We now move to the main goal of this Letter and study simultaneous phase matching of several parametric processes with compensation for GVM between all waves. We first consider the THG process achieved by cascading SHG and sum-frequency mixing processes of the fundamental and SH fields in a single crystal.[24] To obtain the GVM compensation for all pairs of waves, we use a 2DNPC structure with an asymmetric (orthorhombic) lattice,[17] where the principal reciprocal vectors $G_1$ and $G_2$ and angle $\varphi$ should be found from the condition of the GVM compensation. The phase-matching geometry of this cascaded process is sketched in Fig. 1. For a given value of the tilting angle $\rho$ the requirement of group-velocity matching for all waves implies fixed angles of noncollinearity that depends on the group-velocity dispersion, which are defined from the relations

$$\tan\frac{\beta_{12}}{2} = \frac{v_2\tan\rho - \sqrt{v_2^2\cos^{-2}\rho - v_1^2}}{v_1 + v_2}, \qquad (2)$$

and the similar expression for $\tan(\beta_{13}/2)$ where we should make the following change: $v_2 \to v_3$.

With known angles $\beta_{12}$ and $\beta_{13}$, entrance angle $\alpha$, principal reciprocal vectors of the 2DNPC $G_1$ and $G_2$, and the angle between them, $\varphi = \beta_{13} + \sigma - \alpha$, are calculated by using trigonometrical theorems. Figure 2 show the periods $d_1$ and $d_2$ calculated from the relations: $d_{1,2} = 2\pi/G_{1,2}\sin\varphi$, and the angle $\varphi$ of the required 2DNLPC structures for the tilting angle $\rho = 50°$.

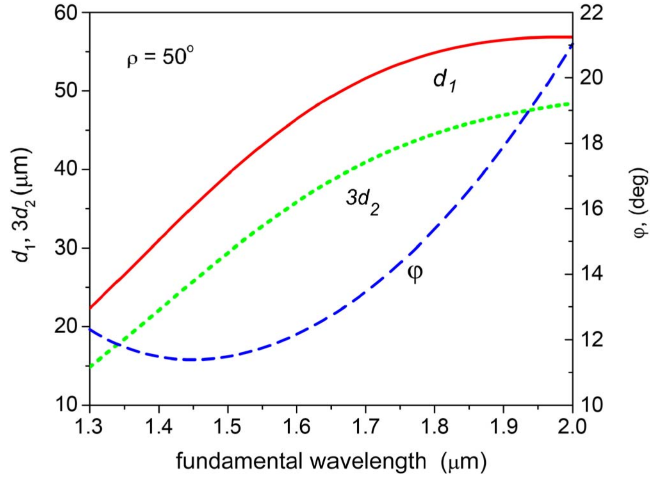


Fig. 2. (Color online) 2DNPC grating periods $d_1$ and $d_2$ and the angle $\varphi$ for GVM-compensated phase-matched THG.

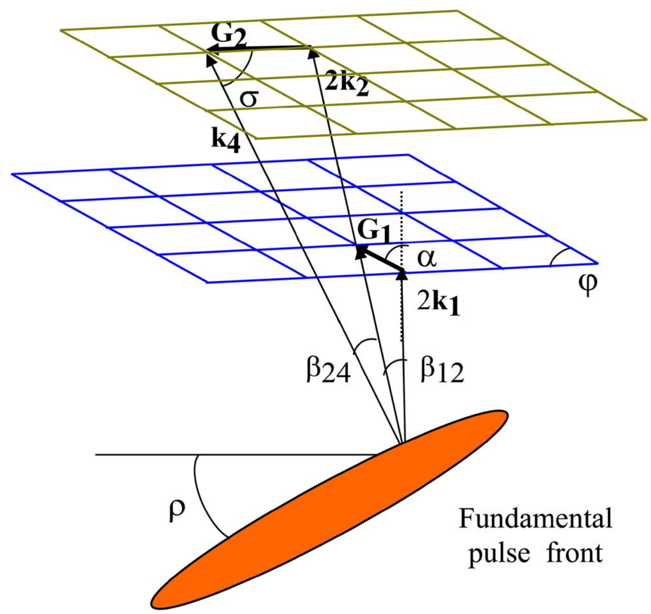


Fig. 3. (Color online) Geometry for FHG process with simultaneous double-phase matching and GVM compensation in 2DNPC. The grids show two reciprocal lattices.

Another important parametric process that can be realized in 2DNPC is FHG based on that two-step cascading: SHG and subsequent doubling of the generated SH wave.[19] Again, an asymmetric lattice for 2DNPC should be used to achieve simultaneous phase matching and GVM compensation. The corresponding phase-matching geometry is shown in Fig. 3. We use Eq. (2) again to find the noncollinearity angle for the first SHG process. For the noncollinearity of the second process ($2\omega + 2\omega = 4\omega$), first we calculate the tilting angle of the SH wavefront, $\rho' = \rho - \beta_{12}$, and then we apply the similar relation as above:

$$\tan\frac{\beta_{24}}{2}=\frac{v_4\tan\rho'-\sqrt{v_4^2\cos^{-2}\rho'-v_2^2}}{v_2+v_4}. \qquad (3)$$

With known angles $\beta_{12}$ and $\beta_{24}$, entrance angle $\alpha$, fundamental reciprocal vectors $G_1$ and $G_2$ of the 2DNLPC structure and the angle between them, $\varphi=\beta_{12}+\beta_{24}+\sigma-\alpha$, are all calculated by using simple trigonometrical relations. The $LiNbO_3$ structure parameters are found again from the relations $d_{1,2}=2\pi/(G_{1,2}\sin\varphi)$, and they are shown in Fig. 4 for tilting angle $\rho=50°$.

A very important practical issue is the acceptable tolerance for the pulse-front tilting for two parametric processes considered above. To find this tolerance, we first analyze the 2DNPC structures for other values of the tilting angle; after that we fix the lattice parameters and calculate the nonstationary lengths for each pair of waves $L_{v,12}$, $L_{v,13}$, and $L_{v,14}$, with the lengths defined as $L_{\nu,nm}=\tau/\nu_{nm}$, where $\tau$ is the duration of the fundamental pulse and $\nu_{nm}$ is the GVM parameter, $\nu_{nm}=|v_n^{-1}-v_m^{-1}|$. The results of these calculations show that when the tilting angle is chosen correctly all three processes (SHG, SFM, and FHG) are matched simultaneously with the GVM compensation. The tolerances for a deviation from the exact tilting conditions for the angle in a 2 mm long $LiNbO_3$ sample can be found as follows: $\Delta\rho_{SH}=6.8°$, $\Delta\rho_{TH}=2.3°$, and $\Delta\rho_{FH}=1.3°$.

In conclusion, we have suggested a novel application of two-dimensional quadratic nonlinear photonic crystals for group-velocity-matched multistep parametric generation with proper compensation for noncollinear processes by front tilting of the fundamental pulse. We have derived the explicit conditions for simultaneous phase- and group-velocity-matched third- and fourth-harmonic-generation processes in two-dimensional asymmetric hexagonally polled $LiNbO_3$ photonic structures.

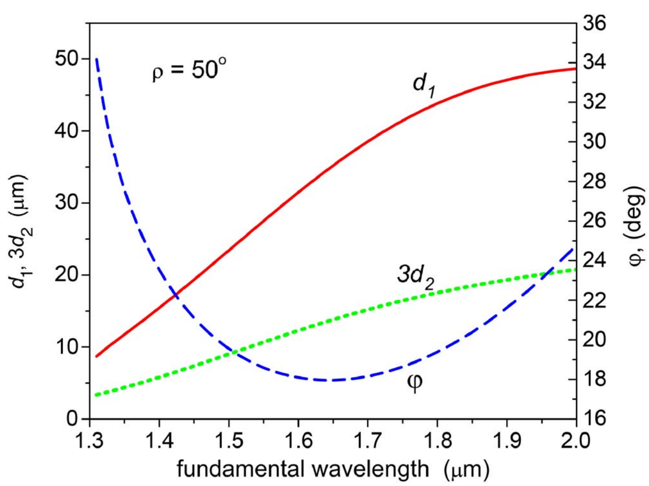


Fig. 4. (Color online) 2DNLC grating periods $d_1$ and $d_2$ and the angle $\varphi$ for GVM-compensated phase-matched FHG.

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