

# Generation of Bessel beams by parametric frequency doubling in annular nonlinear periodic structures

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**Abstract:** We analyze the second-harmonic generation in two-dimensional photonic structures with radially periodic domains created by poling of a nonlinear quadratic crystal. We demonstrate that the parametric conversion of the Gaussian fundamental beam propagating along the axis of the annular structure leads to the axial emission of the second-harmonic field in the form of the radially polarized first-order Bessel beam.

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**OCIS codes:** (140.3300) Laser beam shaping (190.2620) Nonlinear optics; Frequency conversion

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## 1. Introduction

It is well-established that the efficient second-harmonic generation (SHG) depends critically on the phase-matching conditions usually achieved through crystal birefringence. For nonlinear crystals without or with a very small birefringence, the phase-matching conditions can be satisfied by means of the so-called quasi-phase-matching (QPM) technique [1] which relies on introducing an additional spatial periodicity of the quadratic response of a nonlinear medium. In typical situations, the QPM periodic structure is created by a series of parallel stripes with alternating signs of the quadratic nonlinear response. This is achieved, for example, by periodic poling of the ferroelectric crystals such as Lithium Niobate. Then the fundamental and second-harmonic beams propagate along (or at a small angle to) the direction of modulation.

Recently, a novel geometry based on the annular domain pattern has been suggested theoretically and studied experimentally. It has been shown that the annular geometry enables phase-matched parametric processes with larger acceptance angle, simultaneous frequency conversion into a number of beams propagating at different directions [2, 3], and second-harmonic beam shaping [4]. In these studies, the geometry of wave interaction was such that all beams propagate in the same plane which is also the plane of the structure's periodicity.

In the recent observation of SHG in Strontium Barium Niobate crystals with randomly distributed ferroelectric domains, the fundamental wave propagates *perpendicularly* to the plane of the nonlinearity modulation, and this results in the conical emission of the second-harmonic signal with its axis coinciding with the polar axis of the domains [5, 6, 7]. Random modulation of the second order nonlinearity in the transverse direction is a source of a pool of grating vectors which ensure the phase matching for all generated waves propagating along the cone with its apex angle  $2\alpha$  determined by the longitudinal phase-matching condition  $2k_1 = k_2 \cos \alpha$ , where  $k_1$  and  $k_2$  are the wave numbers of the fundamental and second-harmonic waves, respectively. In addition, because of the symmetry of the corresponding  $\chi^{(2)}$  tensor, the second-harmonic wave is radially polarized [5, 8].

In this paper, we discuss the quasi-phase-matched SHG processes in the annular geometry of periodically poled nonlinear crystal, as shown in Fig. 1. We demonstrate analytically that for the fundamental beam propagating along the symmetry axis of the structure, the transverse phase-matching condition leads to the emission of the second-harmonic wave in the form of a radially polarized Bessel beam. This method can be used for generating nondiffracting Bessel beams, and we discuss prospects for its experimental realization.

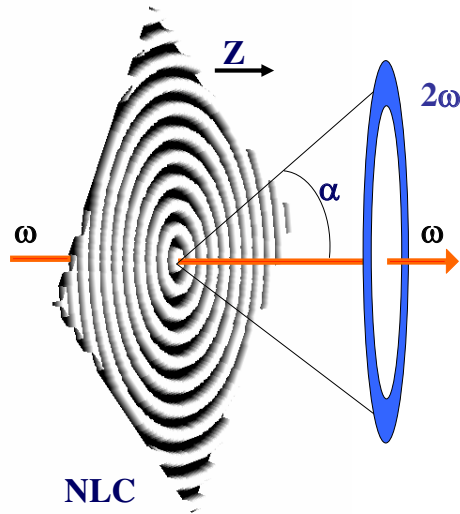


Fig. 1. Schematic of the parametric generation of the axial Bessel beam with the double frequency  $2\omega$ . NLC denotes a quadratic nonlinear crystal with poled domains of the radially symmetric periodic modulation of the second-order nonlinearity.

## 2. Transverse phase matching

We consider the annular second-harmonic interaction geometry in a quadratic nonlinear medium, as illustrated in Fig. 1. A nonlinear structure is created by a series of oppositely oriented ferroelectric domains in the form of the concentric rings of the constant width. This structure can be considered as a two-dimensional nonlinear photonic crystal [9]. It is illuminated by a strong pump wave with the frequency  $\omega$ . The resulting spatial modulation of the second-order nonlinearity coefficient  $d^{(2)}$  is radially periodic, and it can be presented in the form  $d^{(2)}(\rho) = d_0^{(2)}g(\rho)$ , where

$$g(\rho) = \text{sgn}[\cos(2\pi\rho/\Lambda + \delta)], \quad (1)$$

$\rho = (x^2 + y^2)^{1/2}$  is the transverse radial coordinate,  $\Lambda$  is the period of the annular modulation,  $\delta$  is the phase offset, and 'sgn' stands for the sign function. As a result of the second-order nonlinearity, the fundamental wave generates the second-harmonic beam. We assume that the pump beam propagates along the  $z$ -axis (which is also the symmetry axis of the periodic domain structure), and it can be presented in the form,

$$\mathbf{E}_0^\omega(\rho, z) = \frac{\mathbf{u}}{2} \left[ A(z)e^{-ik_1z} \exp\left(-\frac{\rho^2}{w_{01}^2}\right) + \text{c.c.} \right], \quad (2)$$

where  $\mathbf{u} = (u_x, u_y)$  is the polarization vector,  $k_1$  is the wave number of the fundamental beam, and we assume that the beam width  $w_{01}$  is much larger than the poling period  $\Lambda$ . In what follows, we consider a nonlinear crystal belonging to the symmetry groups  $4mm$  or  $3m$ .

The beam propagation in such a structure generates the nonlinear polarization of the medium which, due to the symmetry of the quadratic nonlinear-response tensor, has only one nonvanishing component, and it can be presented in the form,  $\mathbf{P}^{2\omega} = (0, 0, P_z^{2\omega})$ , where

$$P_z^{2\omega} = d_{31}E_x^\omega E_x^\omega + d_{32}E_y^\omega E_y^\omega. \quad (3)$$

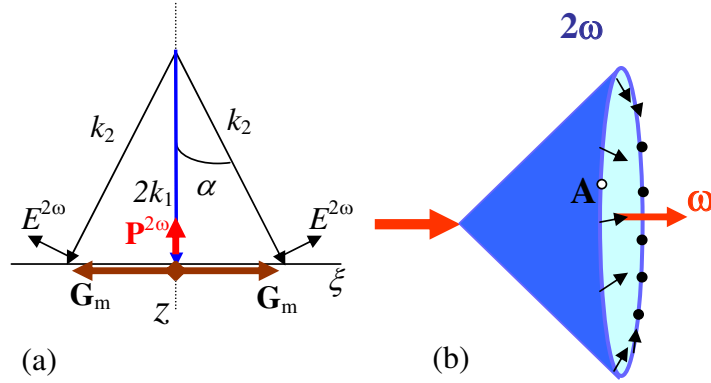


Fig. 2. (a). Phase matching diagram for the second-harmonic generation in the medium with the transverse modulation of the second-order nonlinearity. Notations are:  $G_m$  - quasi-phase matching vectors,  $\mathbf{P}_2$  - medium polarization at the doubled frequency. (b) Emitted cone of the radially polarized second-harmonic radiation, presented as a sum of infinite plane waves propagating at the angle  $\alpha$  with respect to the propagation axis  $z$ .

Here the fundamental field components are:  $E_x^\omega = E_0^\omega \cos \phi$  and  $E_y^\omega = E_0^\omega \sin \phi$ , where  $\phi$  is the angle between the polarization direction and the  $x$  axis. Note, that for the symmetry groups considered here  $d_{32} = d_{31}$  and, consequently

$$P_z^{2\omega} = d_{32} (E_0^\omega)^2. \quad (4)$$

Nonlinear polarization (3) becomes a source for generating the second-harmonic wave. Because of the orientation of this polarization vector along the  $z$ -axis, the second-harmonic wave can only be generated *non-collinearly* with the fundamental wave, and along the direction determined by the specific phase-matching condition, which can be written as

$$\mathbf{k}_2 - 2\mathbf{k}_1 = \mathbf{G}_m, \quad (5)$$

where  $\mathbf{G}_m$  is the QPM vector of the  $m$ -th order representing the radial modulation of the second-order nonlinearity, and defined as  $G_m = m(2\pi/\Lambda)$ , where  $m$  is integer. This phase-matching geometry is illustrated in Fig. 2(a).

Since we are dealing here with a non-collinear QPM process, we consider separately longitudinal and transverse phase-matching conditions [10]. The longitudinal phase-matching condition is  $2k_1 = k_2 \cos \alpha$ , where  $\alpha$  is the angle between the  $z$ -axis and the propagation direction of the second-harmonic wave [see Fig. 2(a)]. As this condition determines only the propagation angle of the second harmonics, this wave will be created via the simultaneous emission of many plane waves all located on a cone with the conical angle  $\alpha$  [see Fig. 2(b)]. Fig. 2(a) illustrates the process of simultaneous generation of two such plane waves located in the same plane, being both linearly polarized. For a thin medium, the amplitude of each of these components can be found from the expression

$$E^{2\omega} \propto P_z^{2\omega} \sin \alpha = \pm d_{32} (E_0^\omega)^2 \sin \alpha, \quad (6)$$

and, therefore, the effective nonlinearity for this parametric process is determined as  $d_{\text{eff}}^{(2)} = d_{32} \sin \alpha$ . As the electric field of each generated planar component is perpendicular to the cone surface, the emitted field is *radially polarized*.

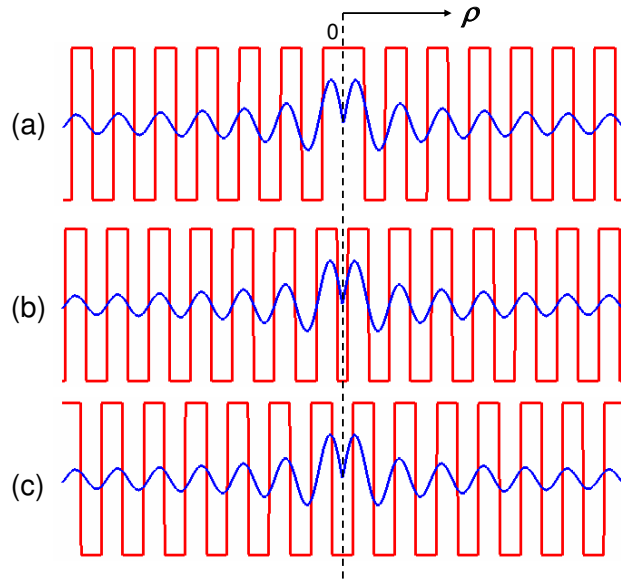


Fig. 3. Transverse structure of the second-harmonic field described by the first-order Bessel function (blue), and the corresponding annular domain grating (red) for three different values of the phase offset  $\delta$  defined in Eq. (1): (a)  $\delta = 0$ , (b)  $\delta = \pi/4$ , and (c)  $\delta = \pi/2$ .

The transverse phase-matching condition  $G_m = k_2 \sin \alpha$  determines the required periodicity of the QPM structure. Considering the particular example of Strontium Barium Niobate crystal and the fundamental wavelength of  $\lambda_1 = 1500\text{nm}$ , we determine the conical angle  $\alpha = 12.7^\circ$  and find  $G_m = 4.2\mu\text{m}^{-1}$ . For the first-order QPM process, this would require the period  $\Lambda_1 \approx 1.50\mu\text{m}$  of the domain structure. While, in principle, such a period could be fabricated with some advanced poling technologies [11, 12], one may relax this condition using a higher-order phase-matching process. For example, taking  $m = 3$  leads to  $\Lambda_3 = 4.5\mu\text{m}$  which can be achieved by patterned electrodes [13] or laser-induced domain nucleation [14].

### 3. Second-harmonic field

It is known that a superposition of an infinite number of plane waves with the wave vectors laying on a cone forms a Bessel beam [15, 16]. This corresponds exactly to the geometry shown in Fig. 2(b). In order to find an analytical form of the second-harmonic field, we integrate over all plane wave components contributing to the conical emission,

$$\mathbf{E}^{2\omega}(\rho, z) = S(z)e^{-ik_{2z}z} \int_0^{2\pi} \mathbf{u}_\rho(\phi) e^{-ik_{2\rho}\rho \cos(\phi-\varphi)} d\phi, \quad (7)$$

where  $k_{2z} = k_2 \cos \alpha$ ,  $k_{2\rho} = k_2 \sin \alpha$ ,  $S(z)$  is the amplitude of the second harmonic wave. The function  $\mathbf{u}_\rho(\phi) = \hat{x} \cos \phi + \hat{y} \sin \phi$  represents the radial component of the polarization vector,  $\phi$  is azimuthal angle and  $\varphi = \cos^{-1}(x/\rho)$  is the azimuthal coordinate of the observation point  $(x, y)$ . After integrating Eq. (7) we find that for a thin nonlinear medium the amplitude of the emitted second-harmonic field at the arbitrary location  $\mathbf{r} = (\rho, z)$  is given by

$$\mathbf{E}^{2\omega}(\rho, z) = 2\pi S(z)e^{-ik_{2z}z} [iJ_1(k_{2\rho}\rho \sin \alpha)\mathbf{u}_\rho - \tan \alpha J_0(k_{2\rho}\rho \sin \alpha)\mathbf{u}_z], \quad (8)$$

where  $\mathbf{u}_z$  represent the  $z$  component of the polarization vector.

The result (8) shows that the parametric generation in a radially symmetric, periodically poled  $\chi^{(2)}$  structure results in the emission of the second-harmonic field in the form of the radially polarized diffractionless Bessel beam. Such nondiffracting vector beam has been already discussed in the literature [16]. It is worth mentioning that while in the case discussed here the radial polarization of the beam is a natural consequence of the axial symmetry of the second harmonic generation process, in a conventional optics such beam can be created using polarizing axicon [17, 18].

In order to obtain a complete analytical formula for the amplitude of this beam we have to resort to the solution of the wave equation with the source term given by the polarization vector Eq. (3). While the rigorous approach would require a full vectorial analysis we will employ here a simplified scalar model due to Tewari *et al.* [10]. For the Gaussian pump beam as in Eq. (2) we search for the second harmonic field in the following form

$$E^{2\omega}(\rho, z) = \left[ \frac{\mathbf{u}_\rho}{2} S(z) J_1(k_{2\rho} \rho) \exp(-ik_{2z}z) + c.c. \right]. \quad (9)$$

After substituting Eq. (9) into the wave equation and following the procedure outlined in Ref. [10], we obtain (in the undepleted pump regime) the intensity of the radial component of the second-harmonic field in the following form

$$I^{2\omega}(\rho, L) = \frac{8\pi^2 d_{32}^2 (I^\omega)^2 L^2}{\epsilon_0 c \lambda_0^2 n_2 n_1^2} |\tan \alpha J_1(k_{2\rho} \sin \alpha) \mathcal{S}_{\text{TPM}}|^2, \quad (10)$$

where  $L$  is the crystal length,  $I^\omega = \frac{1}{2} |E_0^\omega|^2 \epsilon_0 c n_1$  is the intensity of the fundamental beam,  $c$  is velocity of light,  $\lambda_0$  is the fundamental wavelength, and  $n_1$  and  $n_2$  are the refractive indices of the fundamental and SH waves, respectively. The value  $\mathcal{S}_{\text{TPM}}$  denotes the so-called transverse phase matching (TPM) integral

$$\mathcal{S}_{\text{TPM}} = \frac{1}{T} \int_0^{w_m} J_1(r) \exp\left(-\frac{4r^2}{k_{2\rho}^2 w_{01}^2}\right) g\left(\frac{r}{k_{2\rho}}\right) dr, \quad (11)$$

with

$$T = \int_0^{w_m} r J_1^2(r) \exp\left(-\frac{2r^2}{k_{2\rho}^2 w_{01}^2}\right) dr \quad (12)$$

where the function  $g(r)$  is defined in Eq. (1), and it is assumed that  $w_m \gg w_{01}$ . The method of the TPM integral has been extensively used to study the frequency conversion of the fundamental Bessel beam [19, 20]. Here we use this approach to study the frequency conversion of the Gaussian beam in nonlinear media with the transverse patterning.

It is important to analyze the dependence of the efficiency of second-harmonic process on the TPM integral which characterizes an overlap between the Bessel function, i.e. the amplitude of the second-harmonic signal, and the function representing periodicity of the nonlinear medium. First, in Fig. 3 we show the overlap of the electrical field of the first-order Bessel beam  $J_1(k_{2\rho} \sin \alpha)$  inside the structure and the periodical change of the sign of the nonlinearity with the period defined from the transverse phase-matching condition  $\Lambda = 2\pi/(k_2 \sin \alpha)$ , for three different values of the phase offset  $\delta$  in Eq. (1). In the case of the optimal phase (b), the TPM integral (as shown in Fig. 4) has maximum, and there appears no shift between periodicity of the structure and the asymptotic representation of  $J_1(k_{2\rho} \sin \alpha)$ . In Fig. 4, we plot  $\mathcal{S}_{\text{TPM}}$  as a function of the poling period  $\Lambda$  for the first-order transverse quasi-phase matching for the parametric frequency doubling process. As follows from those results, the TPM integral and, consequently, the generated second-harmonic signal, depends strongly on the value of  $\Lambda$

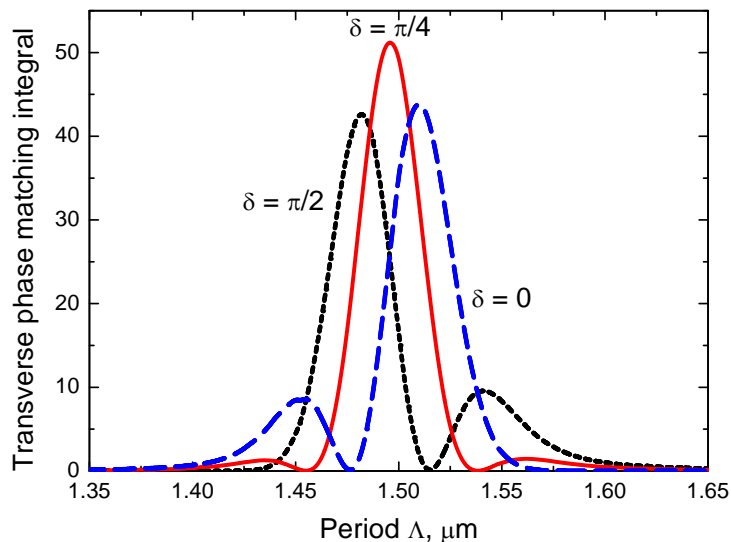


Fig. 4. Dependence of  $\mathcal{S}_{\text{TPM}}$  on the period  $\Lambda$  of the circular grating in Strontium Barium Niobate crystal for different values of the phase offset  $\delta$  and fixed angle  $\alpha$  defined from the condition  $2k_1 = k_2 \cos \alpha$ . The radius of the fundamental beam  $w_{01} = 40\mu\text{m}$ .

reaching its maximum for  $\Lambda = 1.5\mu\text{m}$  which coincides with an asymptotic value of the period of the first-order Bessel function that describes the spatial distribution of the generated second-harmonic field. This sensitivity on  $\Lambda$  and the phase offset  $\delta$  can be important to determine the allowed inaccuracy of the fabrication process of the annular periodic structure.

Finally, it is worth mentioning that in a real experimental situation a finite size of the pump beam may affect the spatial structure of the second-harmonic wave. One may expect that, similarly to the case of the Bessel beam generation using an axicon [21], the SH will reflect the first-order Bessel character only within a finite spatial region determined by the diameter of the pump and apex angle of the conical emission.

#### 4. Conclusion

We have studied the SHG in nonlinear quadratic crystals with an annular periodic domain structure. We have demonstrated that in such a structure the Gaussian fundamental beam propagating along the central axis of the radially symmetric structure will generate the second-harmonic field in the form of the radially polarized first-order Bessel beam. This process can be employed as a novel and efficient tool for generating optical Bessel beams of higher frequencies.

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