1 September 1997

ELSEVIER

Optics Communications 141 (1997) 173–179

OPTICS
COMMUNICATIONS

Intensity dependent change of the polarization state as a result of non-linear phase shift in type II frequency doubling crystals

I. Buchvarov [1], S. Saltiel, Ch. Iglev, K. Koynov

Quantum Electronics Department, Faculty of Physics, University of Sofia, 5 J. Bourchier Blvd., 1164 Sofia, Bulgaria

Received 23 January 1997; revised 26 March 1997; accepted 23 April 1997

Abstract

The effect of an intensity dependent change of the polarization state of the fundamental wave involved in imbalanced type II frequency doubling is considered. The effect is described and derived by us as analytical formulae expressed in terms of elementary functions. We show (i) that a type II second harmonic generation crystal can be considered as a wave plate with intensity dependent retardation and (ii) that the system "polarizer–type II frequency doubling crystal–analyser" has strong pulse shortening and self-induced transparency. © 1997 Elsevier Science B.V.

1. Introduction

Intensity dependent rotation of the polarization of the fundamental wave associated with type II second harmonics generation (SHG) at exact phase matching condition has been considered in Refs. [1,2]. The effect of the polarization rotation considered in Refs. [1,2] is connected with the different *intensity* dependent transmissions of the two fundamental waves, which relative phase remains constant. An all-optical transistor/switch based on this polarization rotation effect was proposed and demonstrated.

The change of the polarization state due to non-linear phase shift (NPS) in a crystal for type I SHG is predicted in Ref. [3]. There, by numerical analysis, is shown that devices based on this process may possess self-induced darkening and self-induced transparency effects. The effect of the change of the polarization state as a result of the non-linear phase shift due to $\chi^{(2)}:\chi^{(2)}$ cascading in type II nearly phase matched SHG crystals has not been described in the literature yet.

Most of the analysis of the NPS and of all-optical processing associated with it have been done numerically [4–11]. Some of the groups obtained expressions for the NPS that include Jacobean elliptic functions [12–14], that also have to be evaluated numerically. The analytical approximation of fixed intensity for the fundamental wave [15–17] correctly describes NPS only for low input intensities (second harmonic conversion coefficient less than 30%). The analytical formulae expressed in trigonometric functions proposed in Refs. [18,19] for the description of NPS only in type I SHG crystals are valid practically for the same range of input intensities as fixed intensity approximation. According to our knowledge there are no analytical formulae expressed in terms of elementary functions describing the amplitudes and NPS of the interacting waves at high input pump intensities. The existence of such formulae will be of great help for studying the phenomena connected with the cascade type NPS and for optimization of optical devices based on cascading non-linear optical effects.

In this paper we studied the effect of an intensity dependent change of the polarization state in quadratic media suitable for imbalanced type II frequency doubling. It is shown that the non-linear media can be considered as an induced wave plate. The analytical expressions show

[1] E-mail: ibuch@ani.phys.uni-sofia.bg.

PII S0030-4018(97)00212-5

that the retardation of this wave plate has a stepwise dependence on the input intensity. Additionally, it is shown that the system "polarizer–crystal for type II SHG–analyser" has intensity dependent transmission and pulse compression capability.

2. Analytical formulae for the NPS

In this section we derive analytical formulae expressed in terms of elementary functions for the intensities and the phases of the fundamental waves involved in type II interaction for SHG. The ratio of the input intensities of the two fundamental waves can take any value, i.e. taken into consideration includes *imbalanced* type II interaction as well. As it is shown in Refs. [20–23] this type of interaction is very attractive from the point of view of construction of low power all-optical switching devices. It is not necessary to study type I interaction separately, because it is equivalent to the type II case with symmetrical input [14].

We assume that the three interacting waves are linearly polarized plane waves. The stationary amplitude equations in the slowly varying envelope approximation assuming zero absorption for all interacting waves have been derived following the same approach as in Ref. [24]:

$$\frac{\mathrm{d}A_1}{\mathrm{d}z} = -\mathrm{i}\sigma A_3 A_2^* \exp(-\mathrm{i}\Delta kz),$$

$$\frac{\mathrm{d}A_2}{\mathrm{d}z} = -\mathrm{i}\sigma A_3 A_1^* \exp(-\mathrm{i}\Delta kz), \qquad (1)$$

$$\frac{\mathrm{d}A_3}{\mathrm{d}z} = -\mathrm{i}2\sigma A_1 A_2 \exp(\mathrm{i}\Delta kz),$$

where $\Delta k = k_3 - k_1 - k_2$ and σ is the non-linear coupling coefficient. The subscripts "1" and "2" denote the fundamental waves and the subscript "3" denotes the second harmonic wave. The complex amplitude $A_q(z)$ incorporates both the real amplitude and the phase of each wave $A_q(z) = a_q(z)\exp(\mathrm{i}\varphi_q(z))$, $q = 1,2,3$. Input values for the phases and the amplitudes of the two orthogonally polarized fundamental waves, are $\varphi_1(0)$, $\varphi_2(0)$, $a_1(0)$ and $a_2(0)$. Let us consider that wave "1" input intensity is higher than input intensity of wave "2" ($a_1^2(0) \geq a_2^2(0)$), then the ratio of the intensities $r = a_2^2(0)/a_1^2(0) \leq 1$.

For the case of $a_3(0) = 0$ the equation system (1) has the following invariants:

$$2a_j^2 + a_3^2 = 2a_j^2(0), \quad (j = 1,2) \qquad (2)$$

$$\sigma a_1 a_2 a_3 \cos\Phi + \tfrac{1}{4}\Delta k a_3^2 = 0, \qquad (3)$$

where $\Phi = \varphi_3 - \varphi_2 - \varphi_1 - \Delta kz = 0$.

It is well known that the solution of the system (1) for the amplitudes of the fundamental waves is expressed by the Jacobian elliptic sinus $\mathrm{sn}((z/æ)|m)$ [24]:

$$a_j^2(z) = a_j^2(0) - u æ^2 \,\mathrm{sn}^2\left(\frac{z}{æ}\middle| m\right) \quad (j = 1,2), \qquad (4)$$

where the parameters are

$$æ^2 = \frac{v - \sqrt{v^2 - 4u}}{2u}, \quad v = 2\sigma^2 a_1^2(0)(1 + r) + \frac{\Delta k^2}{4},$$

$$u = 4\sigma^4 a_1^4(0)\, r, \quad m = uæ^4.$$

In order to obtain an analytical formula for the NPS expressed in terms of elementary functions, the square of the elliptic sinus must be replaced by a suitable approximation. For the case of $\Delta k \neq 0$ the following approximation for $\mathrm{sn}^2((z/æ)|m)$ can be used:

$$\mathrm{sn}^2\left(\frac{z}{æ}\middle| m\right) \approx \sin^2\left(\frac{\pi}{2}\frac{z}{æK(m)}\right) + m^2 F \sin^2\left(\pi\frac{z}{æK(m)}\right), \qquad (5)$$

$K(m)$ is the complete elliptic integral of the first kind and it can be calculated with good accuracy by [25]:

$$K(m) = \left[p_0 + p_1(1-m) + p_2(1-m)^2\right] - \left[q_0 + q_1(1-m) + q_2(1-m)^2\right]\ln(1-m) + \varepsilon(m),$$

$p_0 = 1.3862944$, $p_1 = 0.1119723$, $p_2 = 0.0725296$;

$q_0 = 0.5$, $q_1 = 0.1213478$, $q_2 = 0.0288729$;

$\varepsilon(m) \leq 3 \times 10^{-5}$.

F is a coefficient that depends on the intensity ratio r. Its value will be discussed later.

The validity of the approximation (5) was verified for $F = 0.25$ and different values of m. For $m \leq 0.85$ the deviation from the exact $\mathrm{sn}^2((z/æ)|m)$ function does not exceed 5% independently of the value of ΔkL. The values of m up to 0.85 correspond to the ratios between the intensities of the two fundamental waves $r \leq 0.85$.

As a result for the intensity of the fundamental waves we have:

$$a_j^2(z) = a_j^2(0)\left\{1 - N_j\left[\sin^2\left(\frac{\pi}{2}\frac{z}{l_{\mathrm{coh}}}\right) + m^2 F \sin^2\left(\pi\frac{z}{l_{\mathrm{coh}}}\right)\right]\right\}, \qquad (6)$$

where $N_1 = \sqrt{mr}$ and $N_2 = \sqrt{m/r}$. The depletion and the reconstruction of the fundamental waves occur with period $2l_{\mathrm{coh}}$, where $l_{\mathrm{coh}} = æK(m)$. The coherence length depends not only on the phase mismatch Δk, but also on input intensities $a_1(0)$ and $a_2(0)$.

The imaginary part of the system (1) gives equations for the phases of the two fundamental waves

$$a_j^2 \frac{d\varphi_j}{dz} = -\sigma a_1 a_2 a_3 \cos\Phi \quad (j=1,2). \tag{7}$$

Now, with the use of the invariants (2) and (3) one obtains

$$\frac{d\varphi_j}{dz} = \frac{\Delta k}{2}\left(\frac{a_j^2(0)}{a_j^2} - 1\right). \tag{8}$$

Using (6) and applying the substitution $\xi = \tan(\pi z/2l_{coh})$ we find for the output phase of the two fundamental waves at the output of the non-linear media

$$\varphi_j(L) = \varphi_j(0) - \frac{\Delta kL}{2} + \frac{\Delta k l_{coh} c_j}{\pi}\int_0^p \frac{(\xi^2+1)\,d\xi}{\xi^4 + b_j\xi^2 + c_j}, \tag{9}$$

where

$$p = \tan(\pi L/2l_{coh}), \quad b_j = \frac{2 - N_j(1+4m^2F)}{1-N_j},$$

$$c_j = (1-N_j)^{-1}.$$

L is the length of the non-linear media.

The solution of (9) depends on the values of b_j and c_j:

(a) when $b_j^2 > 4c_j$

$$\varphi_j(L) = \varphi_j(0) - \frac{\Delta kL}{2} + \frac{\Delta k l_{coh} c_j}{\pi\left(K_j^{(2)} - K_j^{(1)}\right)} \times \left[\frac{K_j^{(2)}-1}{\sqrt{K_j^{(2)}}}\arctan\frac{p}{\sqrt{K_j^{(2)}}} - \frac{K_j^{(1)}-1}{\sqrt{K_j^{(1)}}}\arctan\frac{p}{\sqrt{K_j^{(1)}}}\right]; \tag{10a}$$

(b) when $b_j^2 < 4c_j$

$$\varphi_j(L) = \varphi_j(0) - \frac{\Delta kL}{2} + \frac{\Delta k l_{coh} c_j}{\pi} \times \left\{\frac{f_j}{2}\ln\left|\frac{p^2 + d_j p + \sqrt{c_j}}{p^2 - d_j p + \sqrt{c_j}}\right| + \frac{\left(1/\sqrt{c_j}\right) - d_j f_j}{2\sqrt{\sqrt{c_j} - d_j^2/4}} \times \left[\arctan\frac{2p + d_j}{2\sqrt{\sqrt{c_j} - d_j^2/4}} + \arctan\frac{2p - d_j}{2\sqrt{\sqrt{c_j} - d_j^2/4}}\right]\right\}. \tag{10b}$$

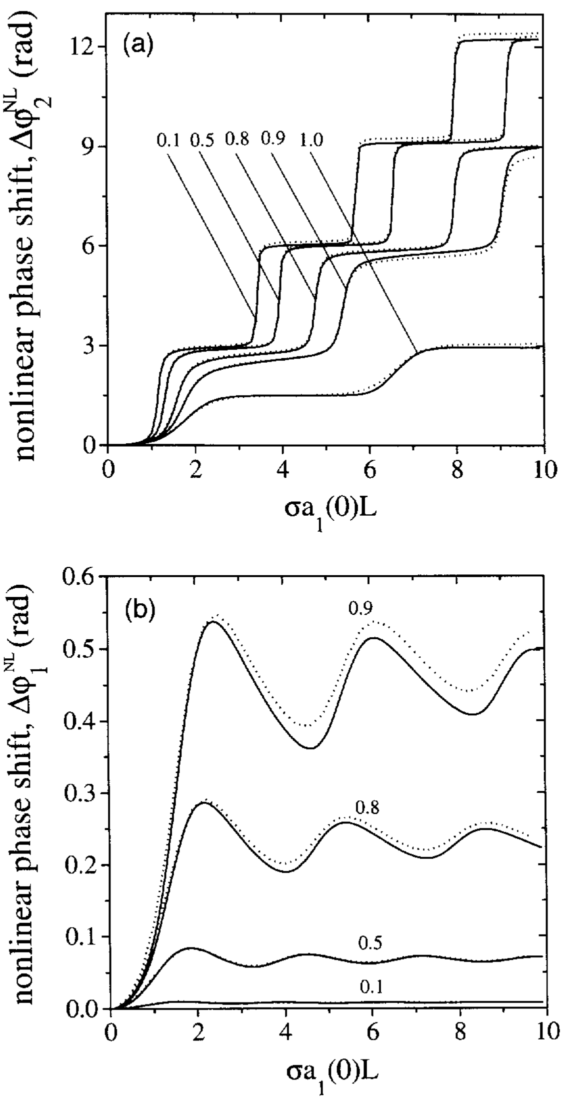

Fig. 1. Non-linear phase shift $\Delta\varphi_2^{(NL)}$ of the weak (a) and $\Delta\varphi_1^{(NL)}$ of the strong (b) wave versus normalized input amplitude $\sigma a_1(0)L$ of the strong wave for $\Delta kL = 0.3$ rad and different input ratios $r = a_2^2(0)/a_1^2(0)$. The solid line is the analytical solution (formulae (10a), (10b) derived in this work and F is defined by (11)). The dashed line represents the direct numerical solution of the system (1).

In (10) the following notations have been used

$$K_j^{(1)} = \tfrac{1}{2}\left(b_j - \sqrt{b_j^2 - 4c_j}\right), \quad K_j^{(2)} = \tfrac{1}{2}\left(b_j + \sqrt{b_j^2 - 4c_j}\right),$$

$$d_j = \sqrt{2\sqrt{c_j} - b_j}, \quad f_j = \frac{1 - \sqrt{c_j}}{2d_j\sqrt{c_j}}.$$

The results of the comparison of the NPS, $\Delta\varphi_j^{NL} = \varphi_j(L) - \varphi_j(0)$, for both waves obtained by direct numerical solution of the system (1) and by the use of the formulae (10a), (10b) are presented in Fig. 1a and 1b. Good agreement between the numerical and the analytical approach can be seen from the figures. The accuracy of the analytical data in comparison with the exact numerical data depends on the parameter F and in the case we can use expression

$$F = 0.66(r^2 + 1) - 1.064r \quad (r \in [0.1, 1]) \tag{11}$$

is less than 5%. If one takes F to be constant equal to 1/4 the accuracy is not more than 20%. We have to point out that the NPS of the weak wave "2" (shown in Fig. 1a) is the parameter of interest for most already proposed all-optically switching devices, because the weaker wave gets higher NPS [16,20–23]. The proposed formulae can easily be extended for a description of the process of NPS of the waves involved in sum frequency mixing interactions.

We would like to note that the middle of each plateau of the curves shown in Fig. 1a are the points of full reconstruction of the pump intensities at the output of the non-linear media. The first point of the full intensity reconstruction corresponds to the fundamental intensities for which $L = 2l_{coh}$. Next, the points of full intensity reconstruction occur at higher intensities for which $L = 2nl_{coh}$ (n integer).

The formulae (10a), (10b) were used to study the effect of an induced change of the polarization state in SHG crystals with type II interaction as presented in the next section.

3. Intensity dependent change of the polarization state

Let us consider a crystal for SHG oriented such that the normal N to the plane formed by the wave vector $\boldsymbol{k}$ and the crystal axis is at angle α with respect to the input polarization (Fig. 2). If $\alpha = 0$ there is only wave "1" in the crystal, if $\alpha = 45°$ the amplitudes of the orthogonally polarized waves "1" and "2" are equal. After passing through the crystal the waves "1" and "2" collect both

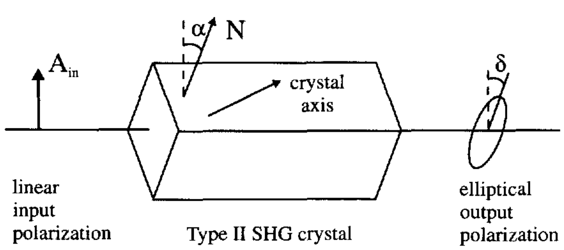

Fig. 2. Schematic drawing of the considered arrangement for obtaining an intensity dependent change of the fundamental wave polarization state.

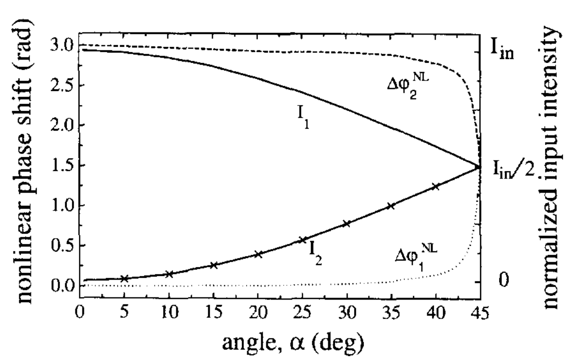

Fig. 3. Non-linear phase shift of the weak (dashed line) and the strong (dotted line) wave as a function of the input angle α at the point of first full reconstruction of the fundamental waves. Input intensities of the strong (solid line) and the weak (marked solid line) waves are also shown. Normalized mismatch $\Delta kL = 0.3$ rad.

linear $\Delta\varphi^{LIN}$ and non-linear $\Delta\varphi^{NL}$ phase shifts. The polarization state after the crystal will depend on the transmittance of the two waves and on the sum of the two phase differences

$$\Gamma = \Gamma_{LIN} + \Gamma_{NL} = \Delta\varphi_2^{LIN} - \Delta\varphi_1^{LIN} + \Delta\varphi_2^{NL} - \Delta\varphi_1^{NL}.$$

Γ_{LIN} is easy to compensate by inserting an additional phase corrector or by a proper choice of the non-linear media length making it $\Gamma_{LIN} = 2n\pi$ (n integer). Then the change of the state of the polarization will depend only on Γ_{NL} that can be calculated with formulae (10).

In general, the output polarization will be elliptical with the large semiaxis rotated at an angle δ with respect to the input polarization

$$\delta = \alpha - \tfrac{1}{2}\arctan\frac{\rho_1\rho_2\sin2\alpha}{\rho_1^2\cos^2\alpha - \rho_2^2\sin^2\alpha}\cos\Gamma. \tag{12}$$

In this formula ρ_1 and ρ_2 are amplitude transmission coefficients for the two fundamental waves $\rho_j = a_j(L)/a_j(0)$. The amplitudes $a_j(L)$ have to be calculated with expression (6). The semiaxes "a" and "b" are defined by

$$a^2 = \rho_1^2\cos^2\alpha\cos^2(\alpha-\delta) + \rho_2^2\sin^2\alpha\sin^2(\alpha-\delta) + \tfrac{1}{2}\rho_1\rho_2\sin2\alpha\sin(2\alpha-2\delta)\cos\Gamma,$$

$$b^2 = \rho_1^2\cos^2\alpha\sin^2(\alpha-\delta) + \rho_2^2\sin^2\alpha\cos^2(\alpha-\delta) - \tfrac{1}{2}\rho_1\rho_2\sin2\alpha\sin(2\alpha-2\delta)\cos\Gamma. \tag{13}$$

For input intensities that yield $\Gamma_{NL} \le \pi$ the output light is left elliptical polarized if $\Delta k\ \tan2\alpha > 0$ and right elliptical polarized if $\Delta k\ \tan2\alpha < 0$. Since the transmission coefficients ρ_1 and ρ_2 and Γ_{NL} strongly depend on

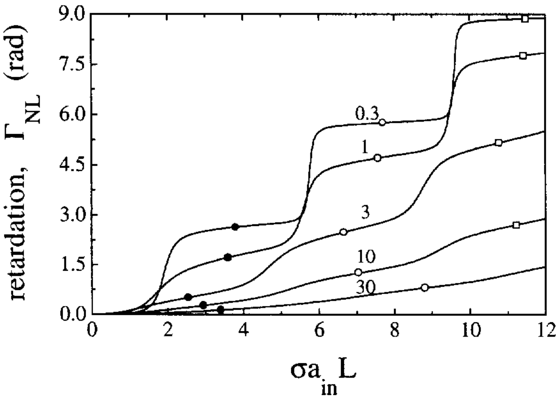

Fig. 4. Non-linear retardation Γ_{NL} of the plate for type II SHG crystal as a function of normalized input intensity $\sigma a_{in} L = \sigma L \sqrt{a_1^2(0) + a_2^2(0)}$ for input angle $\alpha = 40°$ and different values of the normalized mismatch ΔkL. Full circles are points of the first full reconstruction of the amplitudes of the fundamental waves at the output of the crystal. Open circles are points of the second full reconstruction and open squares are points of the third full reconstruction.

input intensities, the rotation angle δ is intensity dependent as well.

The possibility of the effective change of the polarization state of the input fundamental wave is a result of the large difference obtained from the NPS of the weaker and the stronger wave at relatively small disbalance of the input amplitudes of these two waves. This is illustrated in Fig. 3 where we show the dependence of $\Delta\varphi_1^{NL}$ and $\Delta\varphi_2^{NL}$ as a function of the angle α at the point of the first full reconstruction of the "1" and "2" waves ($L = 2l_{coh}$). It is seen that there is no correspondence between the disbalance of the input intensities and the disbalance of the obtained NPS. For angles α below 40° ($r \sim 0.7$) the phase difference $\Gamma_{NL} = \Delta\varphi_2^{NL} - \Delta\varphi_1^{NL}$ is practically constant and equals to 3 rad.

The crystal for SHG for type II interaction oriented at $\alpha = 40°–43°$ can be considered as a wave plate with a retardation effect that depends on the input intensity and the phase mismatch. If we are interested in the pure effect of the change of the polarization state of the fundamental wave we have to consider the condition for which the intensity of this wave is reconstructed, i.e. $L = 2nl_{coh}$ (n integer).

With a type II crystal for SHG, at the first point of full reconstruction of the fundamental waves, a retardation can be obtained in the effect up to π. This is illustrated in Fig. 4, where the phase difference $\Gamma_{NL} = \Delta\varphi_2^{NL} - \Delta\varphi_1^{NL}$ is shown as a function of the normalized input amplitude. The points of full reconstruction of the fundamental wave are marked. With a proper value of the mismatch Δk and the input intensity, the value of the retardation can be made to be equal to any value between 0 and π, this

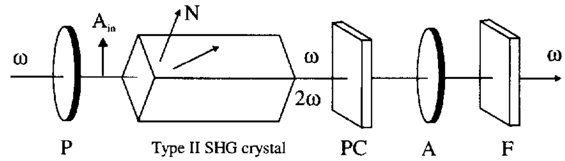

Fig. 5. Non-linear frequency doubling polarization interferometer with type II SHG crystal. P – polarizer, PC – phase corrector, A – analyser, F – filter (harmonic stop).

means that the SHG crystal has the behaviour of a wave plate with variable retardation and can be called "frequency-doubling wave plate".

Additional analytical investigations of the output polarization state for small values of the mismatch Δk ($\Delta k = 0.3$) showed that the eccentricity is in the range 0.9–1, i.e. the output wave remains predominantly linearly polarized [26].

4. Self-induced transparency and pulse shortening

On the basis of what has been considered in the previous section by induced change of the polarization state we propose a modification of the non-linear frequency doubling polarization interferometer (NFDPI) as depicted in Fig. 5. In contrast to the device proposed in Ref. [3], here the crystal is for type II interaction for SHG. The polarizer P and the analyser A have crossed or parallel polarization planes. The linear phase shift is supposed to be compensated with the phase corrector PC, $\Gamma = 2n\pi$, as discussed in Section 3. Then $\Gamma = \Gamma_{NL}$.

The two orthogonally polarized fundamental waves interfere at the analyzer plane. The result of this interference is called by us transmission of the NFDPI and depends on the induced phase shifts $\Delta\varphi_1^{NL}$ and $\Delta\varphi_2^{NL}$.

With formulae (10a), (10b) obtained in Section 2 we calculated the phase difference $\Gamma = \Gamma_{NL} = (\varphi_2(L) -$

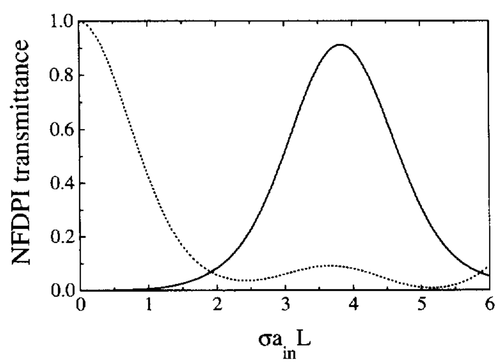

Fig. 6. Transmittance of the system shown in Fig. 5 versus normalized input amplitude $\sigma a_{in} L$ for parallel (dashed line) and perpendicular (solid line) planes of the polarizer and the analyser.

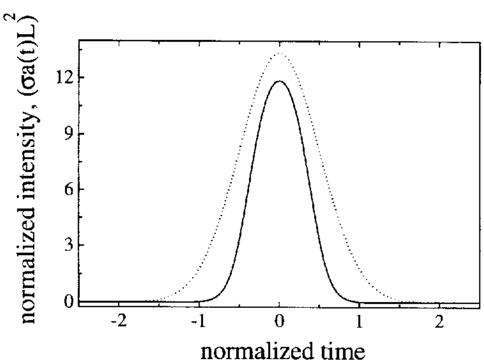

Fig. 7. Single pass pulse shortening of Gaussian pulse with the system shown in Fig. 5. Dashed line – input pulse, solid line – transmitted pulse. Input angle $\alpha = 40°$. Normalized mismatch $\Delta kL = 0.3$ rad.

$\varphi_2(0)) - (\varphi_1(L) - \varphi_1(0))$, which is necessary for the calculation of the transmission $T = |A_{out}|^2/|A_{in}|^2$ of the NFDPI,

$$T_{\perp} = \tfrac{1}{4}(\rho_o^2 + \rho_e^2 - 2\rho_o\rho_e \cos\Gamma)\sin^2(2\alpha),$$

$$T_{\parallel} = \rho_o^2\cos^4\alpha + \rho_e^2\sin^4\alpha + \tfrac{1}{2}\rho_o\rho_e\sin^2(2\alpha)\cos\Gamma. \quad (14)$$

In Fig. 6 we plot the transmission of the system shown in Fig. 5 for $\alpha = 40°$ and $\Delta kL = 0.3$. It is seen that, depending on the mutual arrangement of the polarizers, the system "polarizer–type II SHG crystal–analyser" has strong self-induced darkening and self-induced transparency effects. The critical normalized amplitude for self-induced transparency is almost twice less in comparison to the correspondent critical power for the similar device that uses type I SHG crystal [3]. Additional reduction of the critical amplitude can be obtained for higher values of ΔkL, but this will result in less contrast.

The device shown in Fig. 5 has also the capability to shorten input pulses. When the NFDPI is arranged for a self-induced transparency regime strong shortening of the pulses can be obtained (Fig. 7). It is seen that a single pass traverse through the system results in 33% compression of the input Gaussian pulse. This result is obtained under assumption that there is no non-stationary effects inside the non-linear media. The length of the media is short enough, so the three pulses (two fundamental and one generated) remain approximately overlapped.

It is clear that the two capabilities of the NFDPI, the pulse shortening and self-induced transparency effect at relatively low power (we calculated 60 MW/cm^2 for 10 mm long KTP crystal), make this device suitable for mode-locking in lasers. At the same time we understand that there are restriction effects that can limit the application of this device as mode locker in femtosecond lasers: the group velocity mismatch [27] and the Liot effect.

5. Conclusion

In summary here we showed that near phase matched type II SHG crystals can be used as "frequency-doubling wave plates" with intensity dependent retardation. The devices that employ such kind of wave plates show strong self-transparency, self-darkening and pulse compression effects. Possible applications of such kind of devices may be mode-locking, all-optical switching and sensor protections.

The system "polarizer–type II SHG crystal–analyser" described here can be realized in a way so that light traverses twice the SHG crystal. The system "polarizer–SHG crystal–mirror" will have the same properties but at lower input intensities.

Acknowledgements

We appreciate very much the participation of S. Kumilov and P. Tzankov in some of the stages of this work. We also would like to express our acknowledgments to the Bulgarian Science Foundation for support under contracts F 601 and F405.

References

[1] L. Lefort, A. Barthelemy, Optics Lett. 20 (1995) 1749.
[2] L. Lefort, A. Barthelemy, Electron. Lett. 31 (1995) 910.
[3] S. Saltiel, K. Koynov, I. Buchvarov, Appl. Phys. B 63 (1996).
[4] H.J. Bakker, P.C.M. Planken, L. Kuipers, A. Lagendijk, Phys. Rev. A 42 (1990) 4085.
[5] R. DeSalvo, D.J. Hagan, M. Sheik-Bahae, G. Stegeman, E.W. Van Stryland, H. Vanherzeele, Optics Lett. 17 (1992) 28.
[6] G.I. Stegeman, M. Sheik-Bahae, E.W. Van Stryland, G. Assanto, Optics Lett. 18 (1993) 13.
[7] D.C. Hutchings, J.S. Aitchison, C.N. Ironside, Optics Lett. 18 (1993) 793.
[8] A.L. Belostotsky, A.S. Leonov, A.V. Meleshko, Optics Lett. 19 (1994) 856.
[9] A.V. Smith, M.S. Bowers, J. Opt. Soc. Am. B 12 (1995) 49.
[10] G. Assanto, G. Stegeman, M. Sheik-Bahae, E. Van Stryland, Appl. Phys. Lett. 62 (1993) 1323.
[11] G. Assanto, G. Stegeman, M. Sheik-Bahae, E. Van Stryland, IEEE J. Quantum Electron. 31 (1995) 673.
[12] C.N. Ironside, J.S. Aitchison, J.M. Arnold, IEEE J. Quantum Electron. 29 (1993) 2650.
[13] P.S. Russell, Electron. Lett. 29 (1993) 1228.
[14] A. Kobyakov, U. Peschel, F. Lederer, Optics Comm. 124 (1996) 184.
[15] Z. Tagiev, A. Chirkin, Zh. Eksp. Teor. Fiz. 73 (1977) 1271.
[16] S. Saltiel, K. Koynov, I. Buchvarov, Bulg. J. Phys. 22 (1995) 39.

[17] S. Saltiel, K. Koynov, I. Buchvarov, Appl. Phys. B 62 (1996) 39.
[18] N.P. Balashenkov, S.V. Gagarskii, M.V. Inochkin, Opt. Spectrosc. 66 (1989) 806.
[19] A. Re, C. Sibilia, E. Fazio, M. Bertolotti, J. Mod. Optics 42 (1995) 823.
[20] G. Assanto, I. Torelli, Optics Comm. 119 (1995) 143.
[21] G. Assanto, Optics Lett. 20 (1995) 1595.
[22] G. Assanto, Z. Wang, D.J. Hagan, E.W. VanStryland, Appl. Phys. Lett. 67 (1995) 2120.
[23] G.I. Stegeman, D.J. Hagan, L. Torner, Opt. Quantum Electronics 28 (1996) 1691.
[24] J.A. Amstrong, N. Blombergen, J. Ducuing, P.S. Pershan, Phys. Rev. 127 (1962) 1918.
[25] M. Abramovich and I. Stegun, Eds., Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables, National Bureau of Standarts, Appl. Math. series 55, 1964.
[26] S. Saltiel, I. Buchvarov, K. Koynov, P. Tzankov, Ch. Iglev, Intensity induced polarization rotation due to cascading in imbalanced type II nearly phase matched frequency doubling, in: Proceedings of the NATO Advanced Study Institute Advanced electronic technologies and systems based on low-dimensional quantum devices, Ed. M. Balkanski (Kluwer Academic Publishers, 1996).
[27] I. Buchvarov, G. Christov, S.M. Saltiel, Optics Comm. 107 (1994) 281.