Nonlinear phase shift resulting from two-color multistep cascading

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We propose a novel type of cascading parametric interaction for generating a nonlinear phase shift in dielectric media with a quadratic nonlinear response based on two-frequency wave mixing of the fundamental and second-harmonic waves. Self-phase modulation of the fundamental wave results from a cascading process consisting of four second-order subprocesses, the direct and reverse subprocesses of Type I second-harmonic generation (SHG) and the direct and reverse subprocesses of Type II SHG. It is found analytically and numerically that the fundamental wave passing through a quadratic medium, tuned for simultaneous near phase matching for these two processes, collects 60% more nonlinear phase shift than does the corresponding two-step cascading. We also obtain the conditions for stationary waves (nonlinear modes) supported by such multistep cascading processes. © 2000 Optical Society of America [S0740-3224(00)01405-3]

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1. INTRODUCTION

It is well recognized that two-step second-order [or $\chi^{(2)}:\chi^{(2)}$] cascading provides an efficient way to generate an effective nonlinear phase shift for all-optical switching devices.[1–3] $\chi^{(2)}:\chi^{(2)}$ cascading allows all-optical switching to be achieved at pump levels substantially lower than those in centrosymmetric media with the highest known cubic nonlinearity.[4] A further search for methods to reduce the switching intensity is crucial for future applications of all-optical switching devices based on $\chi^{(2)}:\chi^{(2)}$ cascading. The switching intensity is usually related to the intensity necessary for achieving a nonlinear phase shift (NPS) of π or $\pi/2$, depending on the type of the device. The larger the efficiency of the NPS generation, the lower the switching intensity is.

Some methods for enhancement of the NPS by second-order cascading in quadratic nonlinear media were suggested recently.[5–8] In Ref. 5, the use of a nonlinear frequency-doubling mirror, for which an increase in the NPS comes from a two-way pass of the fundamental beam through a nonlinear crystal, was proposed. In Refs. 6 and 7 use of aperiodic quasi-phase-matching structures was suggested. A different approach was suggested in Ref. 8, which reported achievement of NPS enhancement by multistep cascading (MSC) of several second-order processes. The third-harmonic MSC scheme suggested in Ref. 8 involves two nearly phase-matched upconversion $\chi^{(2)}$ processes: second-harmonic generation (SHG) by a Type I process and third-harmonic generation by sum-frequency mixing of the fundamental and the second-harmonic waves. Self-phase modulation of the fundamental wave is a result of cascading of three or four second-order subprocesses: (i) $\omega + \omega = 2\omega$, $\omega + 2\omega = 3\omega$, and $3\omega - 2\omega = \omega$ or (ii) $\omega + \omega = 2\omega$, $\omega + 2\omega = 3\omega$, $3\omega - \omega = 2\omega$, and $2\omega - \omega = \omega$. However, this method requires that the nonlinear media be transparent for all three waves (1ω, 2ω, 3ω).

Another type of efficient interaction that is based on two simultaneously nearly phase-matched $\chi^{(2)}$ processes was proposed by Assanto *et al.*[9] This device cannot work in a single-input-wave regime, and it requires accurate control of the phase difference between the input waves. The parameters of the switching device suggested by Assanto *et al.*[9] must be compared with those that result from the interaction that uses generation of a NPS by Type II SHG,[10,11] which is also a two-input device.

In this paper we analyze a novel MSC scheme that comprises frequency conversion and multistep cascading in a single-input-wave regime. The MSC effect is a result of simultaneous action of one upconversion and one downconversion $\chi^{(2)}$ process. The former process is Type I SHG with input fundamental wave A, which generates second-harmonic wave S, i.e., $AA{-}S$. The latter process is the generation of orthogonal component B at the fundamental frequency by the complimentary process, $SA{-}B$. We show that this kind of twocolor MSC has interesting properties. In particular, it can yield an increase in the NPS of the fundamental wave and, as a result, reduction of the switching intensity. Moreover, under certain conditions one can achieve a linear dependence of the NPS on the input fundamental field. The optimal mismatches of the two processes are found. A similar scheme, for the properly chosen phase mismatches of the two processes, leads to efficient generation of the orthogonal polarization of the input fundamental beam.[12] We also show that MSC can support phase-locked stationary states of parametric-wave interaction in the form of stable nonlinear modes.

2. CONCEPT OF TWO-COLOR MULTISTEP CASCADING

We assume that linearly polarized fundamental wave A enters a quadratic nonlinear medium that is transparent at both the fundamental and the second-harmonic frequencies. By the appropriate choices of the fundamental wavelength and the quasi-phase-matched grating period,[13,14] simultaneous phase matching for Type I and Type II SHG can be achieved. The MSC process starts with the generation of second-harmonic wave S by the Type I SHG process, i.e., AA–S. By difference-frequency mixing, fundamental wave B with polarization orthogonal to that of wave A is then generated. Waves S and B are involved in a number of chains (cascades) of parametric interactions that reconstruct depleted fundamental wave A. The most important MSC chains are the following (see also Ref. 15):

(i) AA–S, SA–A;
(ii) AA–S, SA–B, SB–A;
(iii) AA–S, SA–B, AB–S, SA–A;
(iv) AA–S, SA–B, AB–S, SB–A,

where chains (i), (ii), and (iii)–(iv) are two-, three-, and four-step $\chi^{(2)}$ cascading processes that mimic $\chi^{(3)}$, $\chi^{(5)}$, and $\chi^{(7)}$ self-action effects, respectively, in a quadratic nonlinear medium. As shown below, higher-order self-action effects have a strong influence on the process of generation of the NPS by the fundamental wave.

In the slowly varying envelope approximation the reduced amplitude equations for the simultaneous action of Type I and Type II processes for SHG with linearly polarized plane waves in lossless quadratic media can be written as follows (see details in Ref. 12):

$$\frac{\mathrm{d}S}{\mathrm{d}z} = -i\sigma_1 A^2 \exp(i\Delta k_1 z) - 2i\sigma_2 AB \exp(i\Delta k_2 z), \quad (1.1)$$

$$\frac{\mathrm{d}A}{\mathrm{d}z} = -i\sigma_1 SA^* \exp(-i\Delta k_1 z) - i\sigma_2 SB^* \exp(-i\Delta k_2 z), \quad (1.2)$$

$$\frac{\mathrm{d}B}{\mathrm{d}z} = -i\sigma_2 SA^* \exp(-i\Delta k_2 z), \quad (1.3)$$

where S, A, and B are the complex amplitudes of the second-harmonic wave and of the two fundamental orthogonally polarized waves, respectively; Δk_1 and Δk_2 are the phase mismatches for the two processes. Nonlinear coupling coefficients σ_1 and σ_2 are proportional to the appropriate $\chi^{(2)}$ tensor components. Other possible interactions such as higher-harmonic generation and BB–S interaction are neglected, e.g., because the $\chi^{(2)}$ tensor component that is responsible for the process is negligible or because of the large value of the corresponding phase mismatch.

3. SINGLE-INPUT-WAVE REGIME

First we consider the case when only one wave, fundamental wave A, enters a quadratic crystal, and our primary goal is to find the NPS of this wave at the output of the crystal. Equations (1) cannot be integrated analytically, and their solution can be obtained by numerical integration only. An approximate analytical solution for the NPS that will help us to discuss the physics of MSC can be obtained by use of the approximation of negligible pump depletion of wave A as described below.

A. Approximate Analytical Results

Using the substitutions $S = \tilde{S}\exp(i\Delta k_1 z)$ and $B = \tilde{B}\exp(i\Delta k z)$, where $\Delta k = \Delta k_1 - \Delta k_2$, we can transform Eqs. (1) into the following equations:

$$\frac{\mathrm{d}\tilde{S}}{\mathrm{d}z} + i\Delta k_1\tilde{S} = -i\sigma_1 A^2 - i2\sigma_2\tilde{B}A, \quad (2.1)$$

$$\frac{\mathrm{d}A}{\mathrm{d}z} = -i\sigma_1\tilde{S}A^* - i\sigma_2\tilde{B}^*\tilde{S}, \quad (2.2)$$

$$\frac{\mathrm{d}\tilde{B}}{\mathrm{d}z} + i\Delta k\tilde{B} = -i\sigma_2\tilde{S}A^*. \quad (2.3)$$

To employ the perturbation-theory approach, we first assume that A does not depend on z. This allows us to find the amplitudes of second-harmonic wave S and fundamental wave B in an explicit analytical form. Indeed, with the assumption that $|\Delta kL| \gg 1$, we have $\tilde{B} = -\sigma_2\tilde{S}A^*/\Delta k$. Then, integrating Eq. (2.1), we obtain

$$\tilde{S} = -\sigma_1 A^2[1 - \exp(-iQz)]/Q, \quad (3)$$

$$\tilde{B} = \sigma_1\sigma_2|A|^2 A[1 - \exp(-iQz)]/Q\Delta k, \quad (4)$$

where $Q = \Delta k_1 - 2{\sigma_2}^2|A|^2/\Delta k$.

The result [Eq. (4)] for $\tilde{B}$ indicates that the generation of the orthogonal polarization wave can be understood to be a result of the effective four-wave mixing process that corresponds to the cascading process AAA–B.[12] If now, in the second-order approximation, the amplitude of wave A is taken as $A = a\exp[i\varphi_A(z)]$, we obtain for the nonlinear phase shift $\Delta\varphi_A = \varphi_A(L) - \varphi_A(0)$ the following result:

$$\Delta\varphi_A = \Delta\varphi_A^{\mathrm{TSC}} + \Delta\varphi_A^{\mathrm{MSC}}, \quad (5)$$

where TSC means two-step cascading and

$$\Delta\varphi_A^{\mathrm{TSC}} = \frac{{\sigma_1}^2 a^2 L}{Q}\left[1 - \frac{\sin(Qz)}{Qz}\right], \quad (6)$$

$$\Delta\varphi_A^{\mathrm{MSC}} = \frac{2{\sigma_1}^2{\sigma_2}^2 a^4 L}{Q^2\Delta k}\left[1 - \frac{\sin(Qz)}{Qz}\right]. \quad (7)$$

Results (3)–(7) show (this has been confirmed by numerical integration) that the amplitudes and the NPS of the three waves are, respectively, symmetric and antisymmetric functions with respect to vector $\mathbf{v} = (\Delta k_1, \Delta k_2)$. For example, $a(\Delta k_1, \Delta k_2) = a(-\Delta k_1, -\Delta k_2)$ and $\Delta\varphi_A(\Delta k_1, \Delta k_2) = -\Delta\varphi_A(-\Delta k_1, -\Delta k_2)$. This is an important finding for all-optical switching devices that employ two nonlinear media, for which the requirement is that the NPS in the two media have opposite signs.

Two terms in Eq. (5) have a clear physical meaning. The first term (proportional to a^2) is the NPS that is due to the $\chi^{(2)}$-induced cascaded cubic nonlinearity, and the second term, which is proportional to a^4, should be attributed to the $\chi^{(2)}$-induced fifth-order cascaded nonlinearity. This fifth-order nonlinearity is a direct manifestation of the MSC effect in this double phase-matched process. We can confirm such an interpretation by considering the double-cascading limit of the system of Eqs. (2). Indeed, when simultaneously $|\Delta k L| \gg 1$ and $|\Delta k_1 L| \gg 1$, we have $|QL| \gg 1$ and $\widetilde{S} = -\sigma_1 A^2/Q$. In this limit, Eq. (2.2) includes an effective cubic–quintic nonlinearity

$$\frac{\mathrm{d}A}{\mathrm{d}z} - i\frac{{\sigma_1}^2}{Q}|A|^2 A - i\frac{{\sigma_1}^2{\sigma_2}^2}{Q^2\Delta k}|A|^4 A = 0, \tag{8}$$

and the double-cascading limit for the NPS yields

$$\Delta\varphi_A \approx \frac{{\sigma_1}^2 a^2}{Q}\left(1 + \frac{2{\sigma_2}^2 a^2}{Q\Delta k}\right)L, \tag{9}$$

which is in accordance with formula (5).

From Eq. (8) it can be seen that, in this limit, the MSC process can be modeled by an effective nonlinear model with competing cubic and quintic nonlinearity whose magnitudes and signs are controlled by the two phase-matching parameters Δk_1 and Δk_2. This kind of nonlinear medium is known to support solitary waves.[16,17] The centrosymmetric crystals with the third-order TSC processes also allow the sign and the relative contribution of the fifth-order effect to be controlled.[18–20]

The validity of the approximate analytical solutions, derived in the limit $|\Delta k_2| \gg |\Delta k_1|$, has been verified by a comparison with the numerical solution of the full system of Eqs. (1). The result of this comparison is shown in Fig. 1 for $\sigma_1 = \sigma_2 = \sigma$. The approximate solutions can be used up to power levels $\sigma a L \approx 1$. Result (5) for the NPS owing to MSC provides a better approximation then the previously published approximate analytical solution.[8] Similar analytical expressions for the NPS of fundamental wave A can be derived for the other cases, $|\Delta k_1| \gg |\Delta k_2|$ and $|\Delta k_1|, |\Delta k_2| \gg |\Delta k|$. They show also that, at low input powers of fundamental wave A (i.e., when $\sigma a \leqslant |\Delta k_1|, |\Delta k_2|$) the contributions of TSC and higher-order MSC can be separated. The analytical results obtained above can be used to describe the NPS for passive mode-locking applications, for which quadratic media with second-order cascading are employed for Kerr lens mode locking or in nonlinear mirror configurations.[3,21,22] However, the NPS's of the amount of π or $\pi/2$ that are required for all-optical switching applications can be obtained at the input powers for fundamental wave A that cannot be described by the proposed analytical approach. Therefore, for the next steps of our analysis we employ numerical simulations.

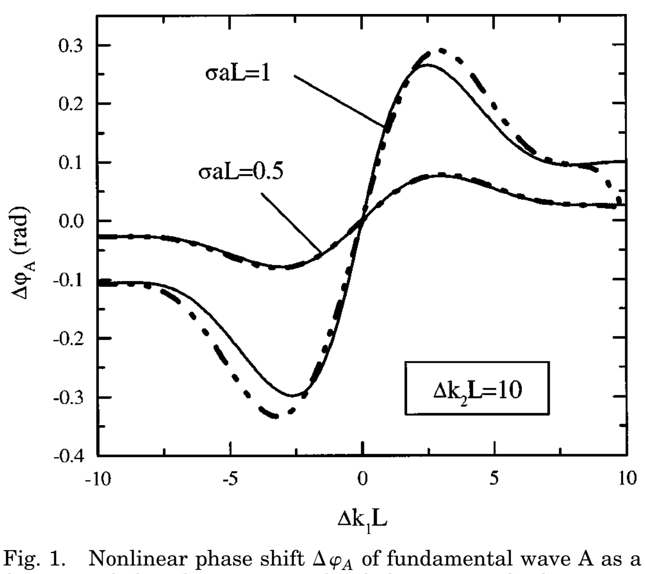

Fig. 1. Nonlinear phase shift $\Delta\varphi_A$ of fundamental wave A as a function of the phase mismatch of the Type I SHG process. Dashed–dotted–dotted curves are analytical results [Eq. (5)]; solid curves are numerical solutions of the system of Eqs. (1).

B. Numerical Results

The behavior of intensity transmission $a^2/a^2(0)$ and the phase shifts at arbitrary input power and mismatches Δk_1 and Δk_2 were obtained by solution of Eqs. (1) numerically. For simplicity, we take $\sigma_1 = \sigma_2 = \sigma$. Numerical analysis of the NPS of the fundamental wave as a function of the pair of mismatch parameters $(\Delta k_1, \Delta k_2)$ reveals that, at certain values of Δk_1 and Δk_2, a strong self-action effect combined with almost 100% transmission of the fundamental beam is possible. Figure 2 shows typical examples of such dependencies, where a NPS with magnitude π [Fig. 2(a)] or $\pi/2$ [Fig. 2(b)] is obtained for an input normalized intensity $(\sigma_1 a L)^2$ equal to 29 or 11.5, respectively. In the same figure we show, for comparison, the phase shift that can be achieved from second-order TSC employing the Type I SHG process. The value of the Type I SHG phase mismatch for this comparison was chosen such that the first point of the fundamental reconstruction is at the same input intensity as for the corresponding two-color MSC scheme. It is clearly seen that the presence of the second phase-matched interaction permits a substantial increase of the collected NPS: $\Delta\varphi_A^{\mathrm{MSC}}/\Delta\varphi_A^{\mathrm{TSC}} = 1.58$ for the conditions suitable for collecting the NPS value $\pi/2$ and $\Delta\varphi_A^{\mathrm{MSC}}/\Delta\varphi_A^{\mathrm{TSC}} = 1.64$ for the conditions suitable for collecting the NPS value π.

For some of the mismatches that are favorable for collecting $\pi/2$ NPS it is possible to obtain the dependence of the NPS as a function of the input amplitude that is close to linear, in combination with almost no depletion of the fundamental wave [see Fig. 2(b)]. For such values of the phase mismatch, the resultant NPS for fundamental wave A is due to a strong energy exchange between fundamental wave B and second-harmonic wave S. This property of MSC schemes is in a sharp contrast to the same dependence when only TSC of second-order processes is employed. In the latter case, the dependence of NPS on input amplitude has a stepwise character.[1,2]

From the point of view of practical realizations it is useful to compare the switching intensities of the devices that are based on the MSC interactions and those that use a single Type I SHG process. The two-color MSC scheme investigated here has lower switching intensities than that reported in Ref. 8 for the third-harmonic MSC scheme. However, for calculating the gain in terms of the switching intensity with respect to all-optical switching devices based on the Type I SHG process, one has to introduce some simple criteria. In this paper, as a criterion for such a comparison, we take the transmission of the Mach–Zehnder interferometer (MZI) with one nonlinear medium (π NPS is required) and with two nonlinear media ($\pi/2$ NPS is required). If we chose for the ON position a MZI transmission of 95% or more, the reduction of the switching intensity as a result of the use of the MSC is 2.1 times for conditions suitable for collecting $\pi/2$ NPS and 2.7 times for conditions suitable for collecting π NPS. In the research reported in Ref. 8 the criterion of 99% transmission was used, and the switching intensity for type I SHG was larger than for 95% transmission. In contrast, the MSC switching intensity is almost independent of the transmission criterion that is applied.

Figure 3 shows the values of the suitable mismatch parameters (Δk_1, Δk_2) for which the fundamental intensity reconstruction is more than 90% and the obtained NPS is close to π (darker-colored curves) and to $\pi/2$ (lighter-colored curves). This figure is plotted for normalized switching intensities $(\sigma a L)^2 = 11.5$ for a MZI with two nonlinear media and $(\sigma a L)^2 = 32.5$ for an asymmetric

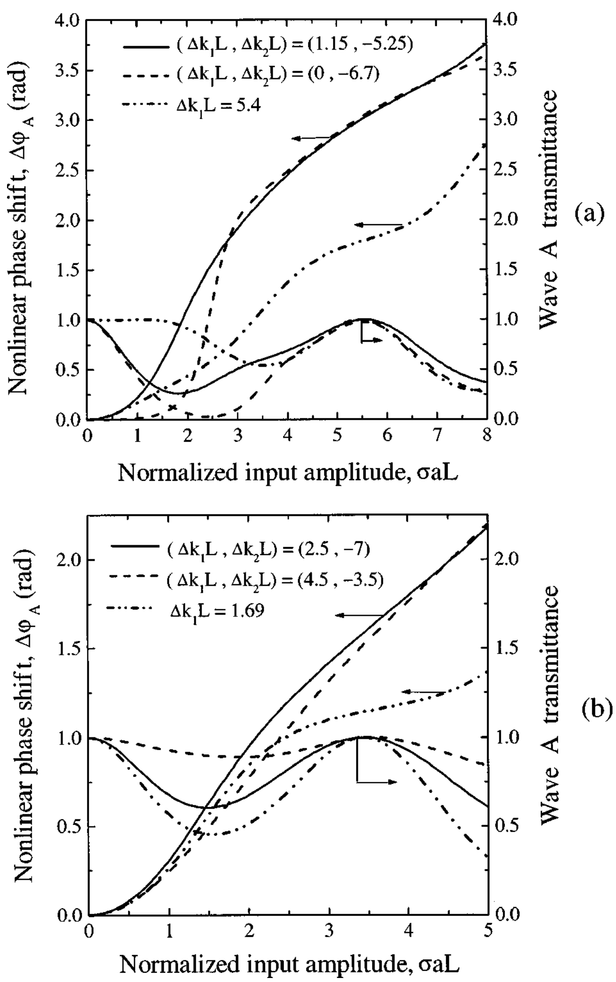

Fig. 2. Nonlinear phase shift and the intensity transmittance of fundamental wave A as a function of its normalized input amplitude for the two-color MSC scheme (solid and dashed curves) and for Type I SHG (dashed–dotted–dotted curve). The chosen phase-mismatch parameters are for collection of (a) π NPS or (b) $\pi/2$ NPS at the point of full reconstruction of the intensity of fundamental wave A.

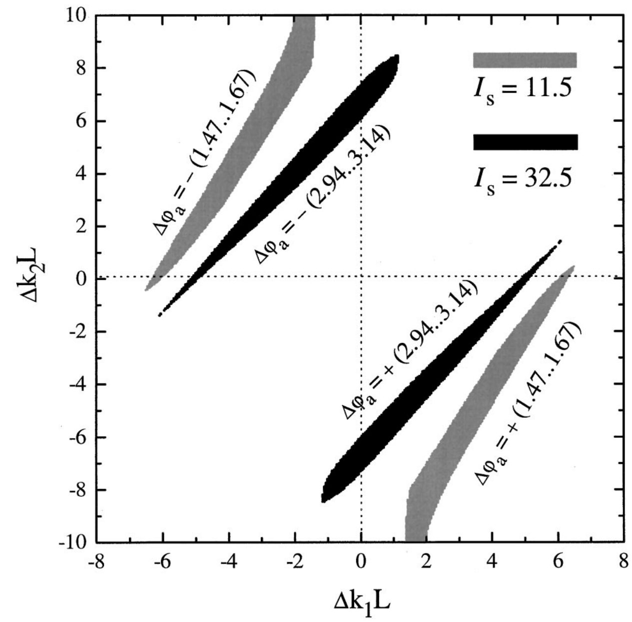

Fig. 3. Values of the phase mismatches for achieving π NPS (darker-colored curves) and $\pi/2$ NPS (lighter-colored curves). The switching intensities $I_s = (\sigma a L)^2$ are indicated as well. Only data for which the transmittance of the fundamental wave is more than 0.9 are shown.

MZI with one nonlinear medium. For all points shown in Fig. 3 the transmission of the MZI is more than 90%. As is clearly seen, the tuning for the mismatch parameters (Δk_1, Δk_2) is not critical. Moreover, the bandwidth is of the same order as that for the SHG process in the chosen medium.

For an experimental realization of simultaneous phase matching of Type I and Type II SHG processes for SHG we can suggest several methods that were discussed previously in Refs. 12 and 15. The first method uses angle phase matching for the one of the processes and quasi-phase matching for the second process.[14] The second approach is based on the use of two commensurate periods of the quasi-phase-matched periodic grating. For example, one can employ first-order quasi-phase matching for one parametric process and third-order quasi-phase matching for the other parametric process. Our calculations were made for the nonlinear crystal $LiNbO_3$, and they show that, for the fundamental wavelength of 1.55 μm, simultaneous phase matching of the Type I and Type II SHG processes can be achieved in a single quasi-phase-matched grating with period $\Lambda = 30.5\,\mu$m. The other method for achieving the conditions for double phase matching is based on the idea of a quasi-periodic grating. As was recently shown experimentally[23] and numerically,[24] Fibonacci optical superlattices provide an effective way to achieve phase matching at several incommensurate periods to permit multifrequency harmonic generation in a single structure.

4. NONLINEAR MODES AND STATIONARY PROPAGATION

In this section we consider the situation in which all three waves, the two fundamental waves and the second-harmonic wave, are nonzero at the input of the crystal. Using, as an example, the MSC system investigated here, we demonstrate that such double phase-matched parametric processes support the propagation of phase-locked stationary states in the form of nonlinear modes.

Let us rescale the system of Eqs. (1) by introducing normalized stationary amplitudes A_n, B_n, S_n and h_A, h_B, h_S—the rates of collection of nonlinear phase shifts by the three waves—as follows:

$$\sigma_1 AL = A_n \exp[i(\varphi_A + h_A \xi)],$$

$$\sigma_1 BL = B_n \exp[i(\varphi_A + h_B \xi)],$$

$$\sigma_1 SL = S_n \exp[i(2\varphi_A + h_S \xi)], \qquad (10)$$

where φ_A is the input phase of wave A and $\xi = z/L$. If h_A, h_B, and h_S are connected to the phase mismatches by the relations

$$h_S - 2h_A = \Delta k_1 L, \qquad h_S - h_A - h_B = \Delta k_2 L, \quad (11)$$

Eqs. (1) are transformed into

$$i\frac{dS_n}{d\xi} - h_S S_n - A_n^2 - 2\beta B_n A_n = 0, \qquad (12.1)$$

$$i\frac{dA_n}{d\xi} - h_A A_n - S_n A_n^* - \beta S_{st} B_n^* = 0, \qquad (12.2)$$

$$i\frac{dB_n}{d\xi} - h_B B_n - \beta S_{st} A_n^* = 0, \qquad (12.3)$$

where $\beta = \sigma_2/\sigma_1$.

For the situation when the amplitudes A_n, B_n, and S_n correspond to the stationary values A_{st}, B_{st}, and S_{st}, respectively, the derivatives in Eqs. (12) vanish, and we find directly the expressions for h_A, h_B, and h_S and the two phase mismatches that will ensure the stationary propagation of all three coupled waves:

$$\Delta k_1 L = \left(1 + 2\beta\frac{B_{st}}{A_{st}}\right)\frac{S_{st}^2 - A_{st}^2}{S_{st}} + S_{st}, \qquad (13.1)$$

$$\Delta k_2 L = \left(1 + \beta\frac{B_{st}}{A_{st}}\right)\frac{S_{st}^2 - A_{st}^2}{S_{st}} + \frac{\beta A_{st}}{S_{st} B_{st}}(S_{st}^2 - B_{st}^2). \qquad (13.2)$$

The meaning of the results of Eqs. (13) is that, for each set of the input amplitudes (A_{st}, B_{st}, S_{st}), they define the wave-vector mismatches $\Delta k_1 L$ and $\Delta k_2 L$ of the Type I and Type II SHG processes when there is no energy exchange between the waves. For example, for the stationary propagation of the waves A, B, and S, with the normalized amplitudes $A_{st} = B_{st} = S_{st} = 1$ and also $\beta = 1$, the mismatches must take the values $\Delta k_1 L = 1$ and $\Delta k_2 L = 0$. There is a simple connection between the signs of the amplitudes (phase of the input waves) and the stationary-phase mismatches: a simultaneous change of the signs of amplitudes A_{st} and B_{st} does not change the magnitudes and signs of Δk_1 and Δk_2; a change of the sign of S_{st} leads to a change of the signs of both Δk_1 and Δk_2. Figure 4 shows the dependence of the phase-mismatch parameters and the nonlinear phase shift $\Delta\varphi_A = h_A$ on the input amplitude of fundamental wave A.

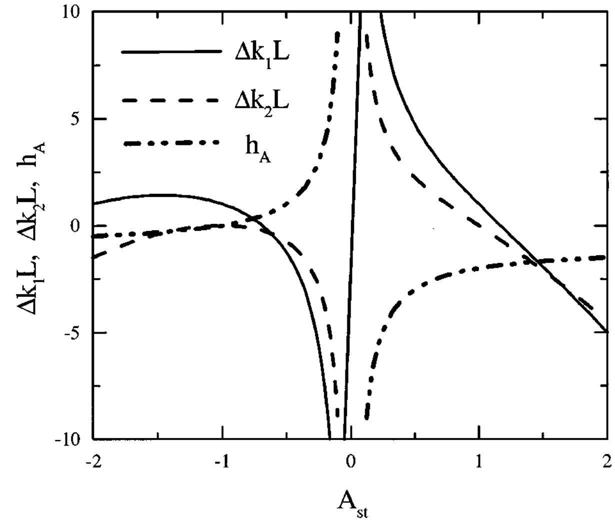

Fig. 4. Dependence of the NPS of fundamental wave A and phase mismatches $\Delta k_1 L$ and $\Delta k_2 L$ on the input amplitude of fundamental wave A in a regime of stationary-wave propagation. The normalized input amplitudes of waves B and S are $B_{st} = 1$ and $S_{st} = 1$, respectively ($\sigma_1 = \sigma_2 = \sigma$).

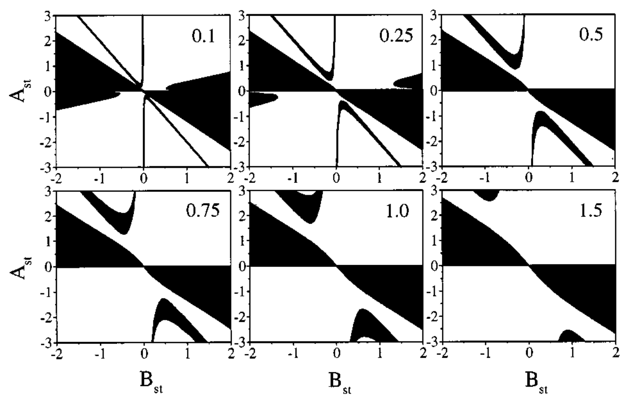

Fig. 5. Conditions for stable propagation of phase-locked stationary waves. The numbers indicate the values of S_{st}, the normalized input amplitude of the second-harmonic wave ($\beta = \sigma_2/\sigma_1 = 1$). Black and white areas correspond to unstable and stable stationary modes, respectively.

The stability of these stationary states is a special issue, and it is closely connected to the modulational instability analysis required for the soliton propagation. The analysis of the stability conditions reveals that, although any sets of input amplitudes can lead to formation of stationary waves in the MSC nonlinear medium, not all of them correspond to stable stationary waves. Figure 5 summarizes the stability conditions in terms of the input amplitudes of the three waves. A change of the sign of the amplitude of wave S (or simultaneous change of the signs of the amplitudes A_{st} and B_{st}) does not lead to a change in the stability conditions [see Eqs. (A7)–(A9) below]. It is also important to note that, when the seeding of the second-harmonic wave is higher than 0.4, polarization components A and B, when they are in phase at the input, will always propagate as stable stationary modes, independently of the ratio of their amplitudes. Details of the stability analysis are briefly outlined in Appendix A.

5. CONCLUSIONS

We have shown that multistep cascading parametric-wave interaction, which results in the simultaneous action of two nearly phase-matched Type I and Type II SHG processes, provides efficient enhancement of the nonlinear phase shift collected by the input fundamental wave. Correspondingly, the switching intensities are strongly reduced. The advantage of this single-input two-color MSC interaction, with respect to the third-harmonic MSC considered in Ref. 8, is that it does not require that the medium be transparent in the whole spectral window (ω–3ω). This kind of double-phase-matched MSC interaction can be used to lower the input power required for all-optical switching. In addition, as was recently shown in Refs. 15, 25 and 26, the MSC interactions can support multicomponent solitary waves with a number of interesting properties. In particular, in Ref. 26 it was shown theoretically that nonlinear parametric interactions with double phase matching can permit soliton bistability and switching, similarly to the previously known case of nondegenerate three-wave mixing.[27]

APPENDIX A: STABILITY OF NONLINEAR MODES

To analyze the stability of the stationary waves (or nonlinear modes) considered in Section 4, we introduce the mode perturbations in the form

$$A_n = A_{st} + (a_1 + ia_2)\exp(\lambda\xi), \tag{A1}$$

$$B_n = B_{st} + (b_1 + ib_2)\exp(\lambda\xi), \tag{A2}$$

$$S_n = S_{st} + (s_1 + is_2)\exp(\lambda\xi), \tag{A3}$$

assuming that $a_1, a_2 \ll A_{st}$, $b_1, b_2 \ll B_{st}$, and $s_1, s_2 \ll S_{st}$. Substituting these expansions into Eq. (12) and linearizing the resultant equations for a_1, a_2, b_1, b_2, s_1, and s_2, we find that the vectors $\mathbf{r} = (a_1, b_1, s_1)$ and $\mathbf{m} = (a_2, b_2, s_2)$ satisfy the following linear system of equations:

$$\hat{L}_{(-)}\mathbf{m} = \lambda\hat{I}\mathbf{r}, \tag{A4.1}$$

$$\hat{L}_{(+)}\mathbf{r} = -\lambda\hat{I}\mathbf{m}, \tag{A4.2}$$

where $\hat{I}$ is a (3×3) unit matrix and $\hat{L}_{(-)}$ and $\hat{L}_{(+)}$ are (3×3) matrices with the following elements:

$$\hat{L}_{(\pm)} = \begin{bmatrix} h_a \pm S_{st} & \pm\beta S_{st} & A_{st} + \beta B_{st} \\ \pm\beta S_{st} & h_b & \beta A_{st} \\ 2A_{st} + 2\beta B_{st} & 2\beta A_{st} & h_s \end{bmatrix}. \tag{A5}$$

Not all equations of the system of Eqs. (A4) are independent. Indeed, considering Eq. (A4.1) separately, we find that $\det|\hat{L}_{(-)}| = 0$. By manipulating Eq. (A4.1) it is easy to express a_2 and s_2 as functions of four other variables, a_1, b_1, s_1, and b_2, and also to verify that the real part of the perturbations obeys the equality

$$A_{st}a_1 + B_{st}b_1 + S_{st}s_1 = 0. \tag{A6}$$

A nontrivial solution of the new system of Eqs. (A6) and (A4.2) for a_1, b_1, s_1, and b_2 exists, provided that the following condition holds:

$$\lambda^4 + C_2\lambda^2 + C_0 = 0, \tag{A7}$$

where the coefficients are functions of A_{st}, B_{st}, S_{st} and the ratio β only, i.e.,

$$\begin{aligned} C_2 = {} & \frac{A_{st}^2B_{st}^2}{S_{st}^2}\left(4\beta^2 + 4\beta\frac{A_{st}}{B_{st}} + \frac{A_{st}^2}{B_{st}^2}\right) \\ & + S_{st}^2\beta^2\left(\frac{2B_{st}}{\beta A_{st}} + \frac{B_{st}^2}{A_{st}^2} + \frac{A_{st}^2}{B_{st}^2}\right) \\ & + 2\beta^2(2B_{st}^2 + 2A_{st}^2 - S_{st}^2) + 4A_{st}^2 + 8\beta A_{st}B_{st}, \end{aligned} \tag{A8}$$

$$\begin{aligned} C_0 = {} & \frac{A_{st}^4S_{st}^2}{B_{st}^2}\beta^2\left(4 + 8\beta\frac{B_{st}}{A_{st}} + \frac{A_{st}^2}{S_{st}^2}\right) \\ & + A_{st}^2\beta^2(6A_{st}^2 + 4S_{st}^2 - 3B_{st}^2) - 2\beta A_{st}^3B_{st} \\ & + 8\beta^3A_{st}B_{st}(A_{st}^2 + S_{st}^2). \end{aligned} \tag{A9}$$

The magnitudes of these coefficients do not depend on the sign of S_{st}. The same is true for the operation of simultaneous changes of the signs of amplitudes A_{st} and B_{st}.

If all four roots of Eq. (A7) are imaginary, the stationary waves are stable. The stability results based on the analysis outlined above are summarized in Fig. 5 and are described in the main text.

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