

# Self-induced transparency and self-induced darkening with a nonlinear frequency-doubling polarization interferometer

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**Abstract.** It is shown that the system polarizer–frequency-doubling crystal(s)–analyzer has a nonlinear transmission for the fundamental wave. The intensity-dependent transmission of this device is due to the nonlinear phase shift that the fundamental beam obtains in the nonlinear crystal as a result of cascaded second-order processes. Depending on the mutual orientation of the polarizer and the analyzer such effects as self-induced transparency and self-induced darkening can be realized.

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Elements or combination of elements that have nonlinear intensity-dependent transmittance or reflection are key factors for construction of different type of nonlinear devices as: optical limiters and sensor protectors [1], mode-locking devices [2], all optical switching devices [3] and all optical modulators [1]. Saturable absorbers are example for element with nonlinear transmission coefficient. They have limited lifetime and response time of the order of several picosecond [4]. Another device that was developed few years ago is so-called frequency-doubling nonlinear mirror [5]. It works in reflection mode and was successfully used as mode-locker for linear resonators [6–8]. Both first and second type of interactions for second-harmonic generation (SHG) were used in nonlinear mirrors of this type. In both cases the second-harmonic crystal was at exact phase-matched condition.

Here we introduce a nonlinear optical device of the new type that also consists of frequency-doubling crystal that is however not at exact phase-matched condition. The device constructed of polarizer, frequency-doubling crystal and analyzer, works in transmission mode. Its nonlinear transmission behavior is due to the nonlinear phase shift (NLPS) that the fundamental beam obtains in the nonlinear crystal as a result of cascade second-order processes [9, 10]. Its principle of operation differs from that of frequency-doubling nonlinear mirror [5].

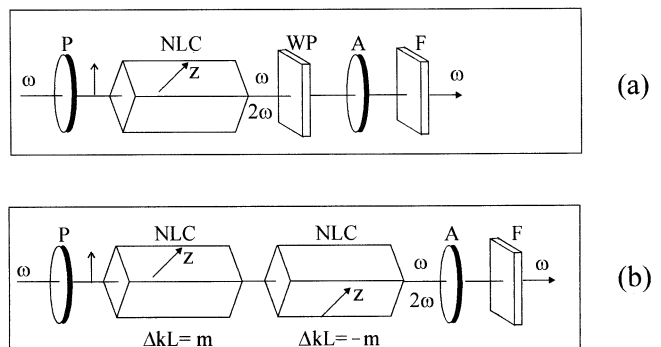
## 1 Principle of operation

Figure 1 illustrates the basic idea of the nonlinear frequency-doubling polarization interferometer (NFDPI). An intense linearly polarized light at frequency  $\omega$  is incident on the entrance face of crystal for SHG-type I. Plane of polarization of the input polarizer and the plane that consists of  $\mathbf{Z}$ -axis and wave vector  $\mathbf{k}$  confine an angle  $\alpha$ . As a result two waves (an “o” wave and an “e” wave) with amplitudes that depend on  $\alpha$  propagate through the crystal. It is important that only the “o” wave is converted into second-harmonic wave and only this wave obtains NLPS. At the output of the crystal, the “o” wave has phase shift:

$$\Delta\varphi_1^{(o)} = \Delta\varphi_{L,1}^{(o)} + \Delta\varphi_{NL,1}^{(o)}, \quad (1)$$

where linear part of the phase shift is  $(2\pi/\lambda)n_0L_1$  and  $\Delta\varphi_{NL,1}^{(o)}$  depends on the intensity of the “o” wave at the entrance of the crystal and on the normalized mismatch  $\Delta kL_1 = (k_2 - 2k_1)L_1$  of the process of SHG [9]. The “e” wave does not take part in the process of SHG and obtains only linear phase shift

$$\Delta\varphi_1^{(e)} = \frac{2\pi}{\lambda} n_{1e}(\theta_1) L_1, \quad (2)$$



**Fig. 1a,b.** The nonlinear frequency doubling interferometer: **a** with one nonlinear crystal; **b** with two nonlinear crystals. NLC–nonlinear crystal for SHG-type I, P–polarizer, WP–wave plates, A–analyzer, F–filter (harmonic stop)

where  $\theta_1$  is the detuning angle from the exact phase-match condition. As a result at the output of the crystal for SHG in Fig. 1a, we have change of the state of the polarization due to the existence of the phase difference

$$\Gamma_1 = \Delta\varphi_1^{(o)} - \Delta\varphi_1^{(e)} = 2\pi \frac{L_1}{\lambda} (n_{1o} - n_{1e}(\theta_1)) + \Delta\varphi_{NL,1}^{(o)}. \quad (3)$$

The linear term in (3) can be compensated by the wave plates (WP). At low input power, the whole system will have minimum or maximum transmission of the fundamental wave depending on the mutual orientation of the polarizer and the analyzer. With increasing of the input

$$\mathbf{W}_1 = \begin{bmatrix} t_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \exp\left(-i\left(\Delta\varphi_{NL,1}^{(o)} + \frac{2\pi}{\lambda} n_{1o} L_1\right)\right) & 0 \\ 0 & \exp\left(-i\frac{2\pi}{\lambda} n_{1e}(\theta_1) L_1\right) \end{bmatrix}, \quad (7)$$

$$\mathbf{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & t_2 \end{bmatrix} \begin{bmatrix} \exp\left(-i\frac{2\pi}{\lambda} n_{1e}(\theta_2) L_2\right) & 0 \\ 0 & \exp\left(-i\left(\Delta\varphi_{NL,2}^{(o)} + \frac{2\pi}{\lambda} n_{1o} L_2\right)\right) \end{bmatrix} \quad (8)$$

power  $\Delta\varphi_{NL,1}^{(o)}$  increases and when its value becomes  $\pi$ , the system switches from maximum to minimum transmission or vice versa. In order to reduce the switching power instead of wave plates is more convenient to use second, identical to the first one, crystal for SHG but with the plane ( $\mathbf{kZ}$ ) oriented at  $90^\circ$  with respect to the ( $\mathbf{kZ}$ ) plane of the first crystal. This arrangement is shown in Fig. 1b. In this case, the wave that was “o” in the first crystal for SHG propagates as “e” wave in the second crystal and the wave that was “e” wave in the first crystal is “o” wave in the second crystal. The reduction of the switching power is coming from the fact that the nonlinear phase shift is now the sum of the nonlinear phase shifts obtained by the “o” waves. To achieve the summation of the NLPS however the signs of the mismatches in the two crystals should be opposite [11]. Then at the output of the second SHG crystal, the phase difference between the two waves will be

$$\begin{aligned} \Gamma_2 &= \Delta\varphi_1^{(o)} - \Delta\varphi_2^{(o)} + \Delta\varphi_2^{(e)} - \Delta\varphi_1^{(e)} \\ &= \Delta\varphi_{NL,1}^{(o)} - \Delta\varphi_{NL,2}^{(o)} + \Delta\Phi, \end{aligned} \quad (4)$$

where

$$\Delta\Phi = \frac{2\pi}{\lambda} [(n_{1o} - n_{1e}(\theta_1)) L_1 - (n_{1o} - n_{1e}(\theta_2)) L_2]. \quad (5)$$

If one choose  $L_1 = L_2 = L$ , the linear phase shifts that these two waves obtain in the crystals compensate each other exactly when  $\Delta k = 0$  (i.e.  $\theta_1 = \theta_2 = 0$ ) and partially for small values of mismatches that have to be set for the two crystals. It is easy to show that when  $L_1 = L_2 = L$ ,

$$\Delta\Phi \approx \Delta k L \left( \frac{N_{o,\omega} - N_{e,\omega}}{N_{o,2\omega} - N_{e,2\omega}} \right). \quad (6)$$

Double crystal type of NFDPI as will be shown later can have considerable low switching intensity.

## 2 Jones matrix calculation of the transmission of NFDPI

The Jones matrix method is well-suited to problems involving a number of similar devices arranged in series. We used this method to calculate the output of the NFDPI shown in Fig. 1b. Deriving the transmittance of the system we take into account that the wave generating second harmonic not only gains NLPS but its amplitude changes periodically as a function of the input intensity.

Let us neglect the linear absorption and reflection and mark with  $t_1$  and  $t_2$ , the amplitude transmittance of the “o” waves in the first and the second crystal, respectively:  $t_j = a_o^{(o)}(L_j)/a_o^{(o)}(0)$ . The Jones matrix of each crystal will be

The common *matrix* of the two crystals will be

$$\mathbf{W}_0 = \mathbf{W}_1 \mathbf{W}_2 = \begin{bmatrix} \exp(-i\Gamma_m/2) & 0 \\ 0 & \exp(i\Gamma_m/2) \end{bmatrix} \exp\left(-i\frac{\Gamma_p}{2}\right), \quad (9)$$

where

$$\Gamma_m = \Delta\varphi_{NL,1}^{(o)} - \Delta\varphi_{NL,2}^{(o)} + \Delta\Phi + i \ln(t_1/t_2) \quad (10)$$

$$\begin{aligned} \Gamma_p &= \Delta\varphi_{NL,1}^{(o)} + \Delta\varphi_{NL,2}^{(o)} + \frac{2\pi}{\lambda} [(n_{1o} + n_{1e}(\theta_1)) L_1 \\ &\quad + (n_{1o} + n_{1e}(\theta_2)) L_2] + i \ln(t_1 t_2). \end{aligned} \quad (11)$$

Taking into account that input polarizer plane confines angle  $\alpha$  with ( $\mathbf{kZ}$ ) plane, we obtain for the transmittance

$$T_{11} = \frac{I_{out,11}}{I_{in}} = 1 - [\sin(2\alpha) \sin(\Gamma_m/2)]^2 \quad (12)$$

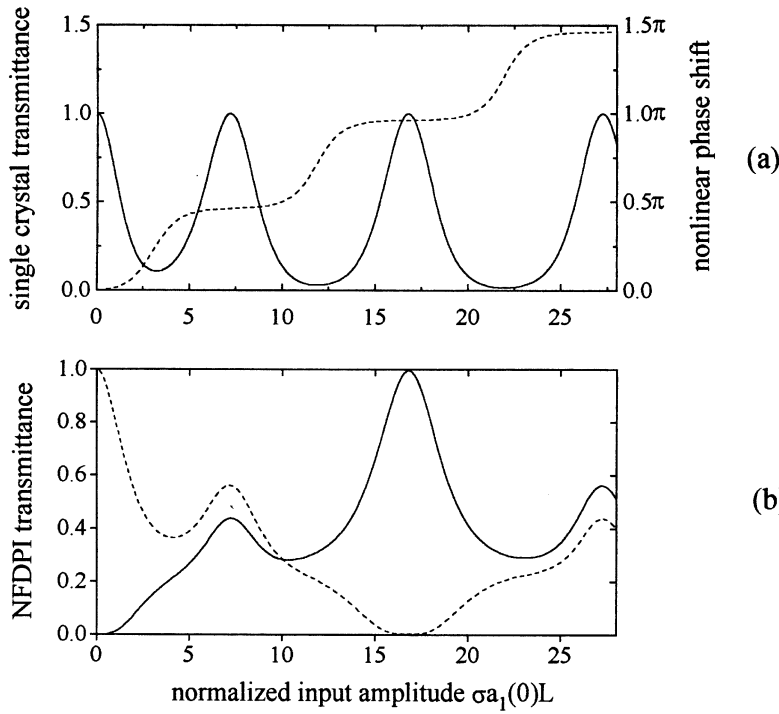
for parallel polarizer and analyzer planes and

$$T_{\perp} = \frac{I_{out,\perp}}{I_{in}} = [\sin(2\alpha) \sin(\Gamma_m/2)]^2 \quad (13)$$

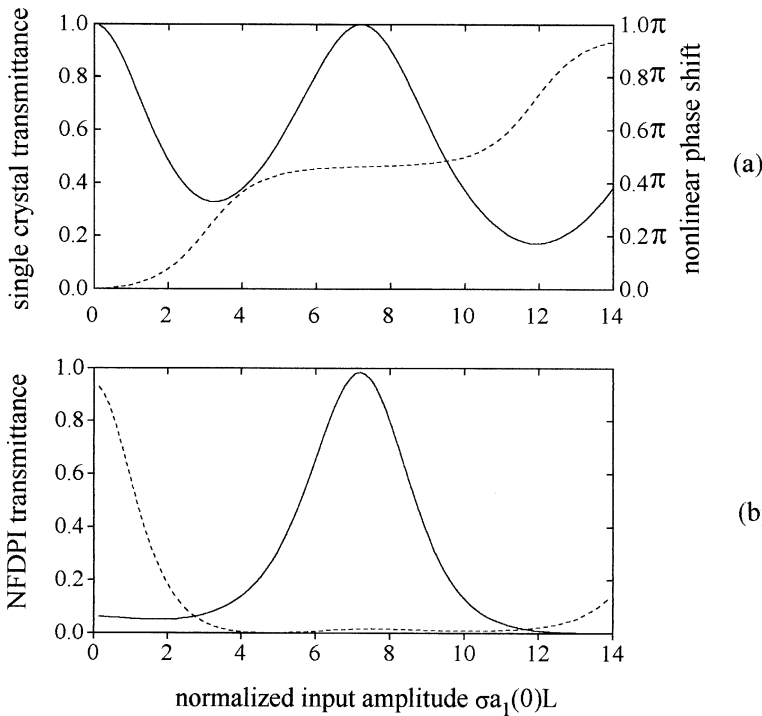
for perpendicular planes of the polarizer and the analyzer. Here  $I_{in}$  denote the wave intensity after the polarizer. Note that  $\Gamma_m$  is a complex variable.

## 3 Numerical analysis

For applying the expressions (12) and (13), we need to calculate  $t_1$ ,  $t_2$ ,  $\Delta\varphi_{NL,1}^{(o)}$  and  $\Delta\varphi_{NL,2}^{(o)}$ . These quantities depend on the input amplitude and on the mismatch  $\Delta k L$  of



(a) **Fig. 2.a** Nonlinear phase shift (*dashed line*) gained by the “o” wave and its intensity transmission (*solid line*) in one of the nonlinear crystals as a function of  $\sigma a_1(0)L$  – normalised input amplitude after the polarizer.  
 (b) Transmittance of the single-crystal NFDPI (shown in Fig. 1a) as a function of normalized input amplitude for parallel (*dashed line*) and perpendicular (*solid line*) planes of the polarizer and analyzer

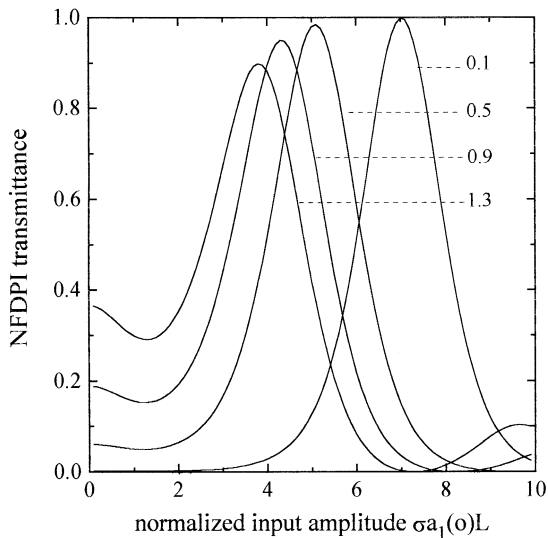


(a) **Fig. 3.a** Nonlinear phase shift (*dashed line*) gained by the “o” wave and its amplitude transmission (*solid line*) in one of the nonlinear crystals as a function of  $\sigma a_1(0)L$  normalized input amplitude after the polarizer.  
 (b) The transmittance of the NFDPI with two nonlinear crystals (shown in Fig. 1b) as a function of normalized input amplitude for parallel (*dashed line*) and perpendicular (*solid line*) planes of the polarizer and analyzer

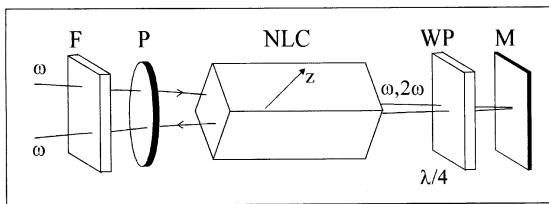
the SHG process. The value of  $\Delta\varphi_{\text{NL},1}^{(o)}$  and  $\Delta\varphi_{\text{NL},2}^{(o)}$  can be found analytically for relatively low input intensity  $I_{\text{in}}$  [11, 12]. The NLPS that can be calculated with this approach are not enough for obtaining substantial changes of the transmittance of the NFDPI. For arbitrary input intensity, the values of  $t_1$ ,  $t_2$ ,  $\Delta\varphi_{\text{NL},1}^{(o)}$  and  $\Delta\varphi_{\text{NL},2}^{(o)}$  can be found by numerical solution of reduced amplitude equations in slowly varying approximation with assumption of zero absorption for all interacting waves [9, 10, 12]. The

angle  $\alpha$  was chosen to be  $45^\circ$  and the length of the crystals, one and the same  $L_1 = L_2 = L$ . The mismatches  $\Delta k_j L$  in the two crystals are taken to have one and the same absolute value  $|\Delta k_1 L| = |\Delta k_2 L| = m$  but opposite in sign. With this input parameters  $t_1^2 = t_2^2$  and  $\Delta\varphi_{\text{NL},1}^{(o)} = -\Delta\varphi_{\text{NL},2}^{(o)}$ .

Figure 2a shows NLPS  $\Delta\varphi_{\text{NL},1}^{(o)}$  and the single-crystal transmittance of the “o” wave  $t_1^2$  as a function of the normalized input amplitude  $\sigma a_1(0)L$  (after the polarizer)



**Fig. 4.** The influence of the amount of the mismatch  $\Delta kL$  on the performance of the two crystal NFDPI with perpendicular planes of the polarizer and the analyzer ( $\Delta kL = 0.1; 0.5; 0.9; 1.3$ ). The horizontal axis is the amplitude of the “o” wave



**Fig. 5.** Folded-type NFDPI: NLC – nonlinear crystal for SHG – type I, P – polarizer, M – mirror, total reflector for the fundamental wave, F – filter (harmonic stop)

with  $m = 0.5$ . The critical amplitude for self-induced transparency of the NFDPI shown in Fig. 1a will be at such input parameters at which  $t_1^2$  is close to one and  $\Delta\varphi_{NL,1}^{(o)} = \pi$ . In Fig. 2b is shown the transmittance of the single-crystal NFDPI as a function of normalized input amplitude  $\sigma a_1(0)L$ . It is seen that self-induced transparency and self-induced darkening occur at  $\sigma a_1(0)L \approx 17$ . This parameter corresponds to  $35 \text{ GW/cm}^2$  if the crystal was BBO with length 10 mm.

In Fig. 3 are shown the same types of graphs as in Fig. 2, but for the Fig. 1b – two crystal NFDPI. The critical input amplitude for self-induced transparency and darkening is less because now is required only  $\pi/2$  NLPS in any of the crystals for SHG. The critical amplitude correspond to  $8 \text{ GW/cm}^2$  if two 10 mm long elements from BBO are used.

The role of the amount of the mismatch is illustrated in Fig. 4 only for perpendicular planes ( $\mathbf{P} \perp \mathbf{A}$ ). The higher is the value of  $\Delta kL$ , the lower is the contrast ratio of the device, the lower is the critical input amplitude.

## 4 Conclusion

We have presented the analysis of a new nonlinear optical device that works in transmission mode, has intensity-dependent transmission coefficient and have time response that depends only on the group velocity mismatch for any individual crystal. By appropriate choice of the crystal type and its length, the time response can be less than 1 ps.

The scheme shown in Fig. 1b can be simplified as shown in Fig. 5. This scheme is equivalent to Fig. 1b. The return by the mirror beam should not be retroreflected since the mismatch must have opposite signs for the two passes through the crystal.

Possible applications of NFDPI shown in Figs. 1 and 5 may include:

- Mode locking of lasers with ring resonators.
- All optical switching. The scheme proposed here as shown in Fig. 1b is equivalent to the schemes that use Mach Zehnder interferometer with two equivalent nonlinear media in the two arms [13]. The advantages of the “two crystals” NFDPI is that this scheme is single arm device and there is no need of special alignment for obtaining interference.
- Sensor protection. For this application the acceptance angle have to be large. This can be achieved by the use of nonlinear optical media prepared by the so-called quasi-phase-matching technique [1].

We have to point out that substantial reduction of the critical power of this type of nonlinear polarization interferometers can be achieved by use of two wavelength input. The results of these investigations are now prepared for publication.

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