Appl. Phys. B 63, 39–42 (1996)

Applied Physics B Lasers and Optics

Analytical formulae for the optimization of the process of low-power phase modulation in a quadratic nonlinear medium

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Received: 26 September 1995/Accepted: 30 November 1995

Abstract. We show that using the approximation of fixed intensity analytical formulae, describing the process of induced phase modulation for the beams involved in second-order nonlinear optical processes can be derived. Expressions that allow the optimization of the phase shifts experienced by the fundamental and generated waves are presented for nonlinear quadratic processes, second-harmonic generation and sum-frequency mixing. In the case of seeding at the generated wavelength, the phase shift of the fundamental wave is due to two interactions: (*i*) a cubic one, based on coupled second-order processes (cascade cubic nonlinearity) and (*ii*) single quadratic interaction with participation of the seeding wave. By comparison with the exact numerical solution, we defined the input parameters of the beams for which this analytical approach is valid. It is shown that phase shifts exceeding $\pi/2$ can be correctly predicted using the expressions obtained.

PACS: 42.65

The effect of strong self- and cross-phase modulation of pump waves in the second-order nonlinear processes has been studied extensively in recent years for the case of Second-Harmonic Generation (SHG) [1–3], sum-frequency mixing [4–8] and parametric processes [1, 9]. The main reason for this interest is connected with the prospects of using this type of phase modulation for construction of all-optical switching and processing devices [10–12]. As noted in [11], the presence of weak coherent seeding at the wavelength of the generated wave offer new ways to control both amplitude and phase modulation of the beams at the output of the nonlinear media. Additionally, the attention to this effect arises from the fact that for many experiments it is necessary to control all the parameters involved in the nonlinear process waves. And finally, as suggested in [2], this effect of self- and cross-phase modulation of the beams in a quadratic nonlinear medium can be used for measurement of second-order susceptibilities.

For theoritical description of the effect of self- and cross-phase modulations of the beams in a non-centrosymmetric medium, most of the groups have used numerical approach [1–6, 10, 11] or expressions that include Jacobi integrals [12–14], that also have to be solved numerically. In [2, 4, 9], an analytical expression that uses approximation valid for low conversion coefficients and for high values of wave vector mismatch have been used. As it is seen in Fig. 1, this analytical approach (shown with dashed line) cannot be used for description of the fundamental waves phase modulation at low values of the mismatch. Approximations, assuming no depletion for the fundamental waves, are considered in [3, 8, 15] for the case of no generated wave seeding. The case of non-zero input for the generated wave have been treated only numerically [6, 11, 16].

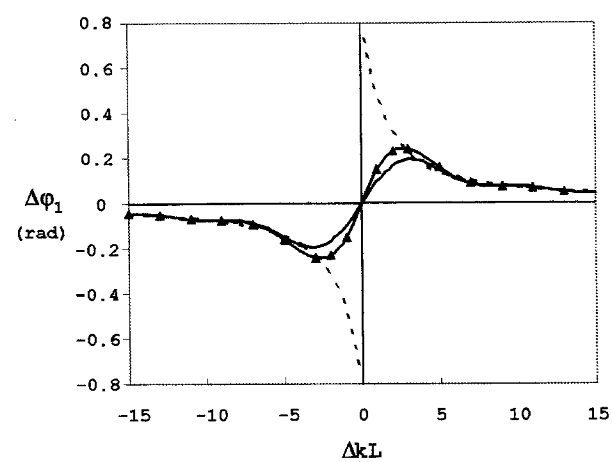

Fig. 1. Output phase shift of wave "1" as a function of the phase mismatch ΔkL for the case of zero seeding ($a_{30} = 0$), ratio of the intensities $I_1/I_2 = 0.2$ and ratio of the length of the crystal to the nonlinear interacting length $L/L_{NL} = \sigma|a_{20}|L = 0.6$. *Dashed line* is the analytical solution obtained with approximation used in [2, 4]. *Solid line marked with triangles* is the numerical solution of system (2). *Solid line* alone represents the analytical solution (9) derived in this work.

The approach of fixed intensity approximation in nonlinear optics was developed by Tagiev and Chirkin [17] for description of type I SHG process. This approximation for description of nonlinear optical wave interactions suggests no depletion for the intensity of the fundamental waves but a possible change of their phases. It is shown there that fixed intensity approximation is valid for higher power levels than the approximation of fixed amplitude. Here, we extend the approach of fixed intensity approximation over the cases of type II SHG and sum-frequency mixing. The case of non-zero input for the generated wave is considered too. Analytical formulae that describe the phase shifts experienced by all three interacting waves are presented

1. Theoritical analysis

We start our investigation of the process of sum-frequency mixing $\omega_3 = \omega_2 + \omega_1$ in nonlinear crystal (NLC) with length L. We assume that the three interacting fields are linearly polarized plane waves. The total electric field can be written as

$$\mathbf{E}(z,t) = \frac{1}{2}\sum \mathbf{e}_j A_j(z) \exp\left[\mathrm{i}(\omega_j t - k_j z)\right] + \text{c.c.}, \tag{1}$$

where $j = 1, 2, 3$; $A_j(z)$ are the complex amplitude which incorporate both the real amplitude and the phase of the "j" wave: $A_j(z) = a_j(z)\exp[\mathrm{i}\varphi_j(z)]$; $k_j = (2\pi/\lambda_j)n_j$ and ω_j the corresponding propagating constants and frequencies; $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ the polarization unit vectors of the three waves. In our investigations only second-order nonlinear processes are taken into account. The reduced amplitude equations in the slowly varying envelope approximation, with assumption of zero absorption for all interacting waves, have the following form:

$$\begin{aligned} \frac{dA_1}{dz} &= -\mathrm{i}\sigma_1 A_3 A_2^* \exp(-\mathrm{i}\Delta kz), \\ \frac{dA_2}{dz} &= -\mathrm{i}\sigma_2 A_3 A_1^* \exp(-\mathrm{i}\Delta kz), \\ \frac{dA_3}{dz} &= -\mathrm{i}\sigma_3 A_1 A_2 \exp(\mathrm{i}\Delta kz), \end{aligned} \tag{2}$$

where the subscripts "1" and "2" denote the fundamental waves and the subscript "3" denotes the generated wave. The wave vector mismatch is $\Delta K = k_3 - k_2 - k_1$. Nonlinear coupling coefficients $\sigma_1, \sigma_2, \sigma_3$ include the convolution of second-order susceptibility tensor $\mathbf{d}^{(2)} = \chi^{(2)}/2$ with the polarization unit vectors

$$\sigma_j = \frac{2\pi}{\lambda_j n_j} d_{\text{eff}} \quad (\mathrm{j} = 1, 2, 3)$$

with

$$d_{\text{eff}} = (\mathbf{e}_1 \cdot \mathbf{d}^{(2)} : \mathbf{e}_2\mathbf{e}_3) = (\mathbf{e}_2 \cdot \mathbf{d}^{(2)} : \mathbf{e}_1\mathbf{e}_3) = (\mathbf{e}_3 \cdot \mathbf{d}^{(2)} : \mathbf{e}_1\mathbf{e}_2).$$

The same equations describe the process of type II SHG. For this case, A_3 is the second-harmonic field complex amplitude, and A_1, A_2 are the complex amplitudes of the two ortogonally polarized input waves.

Following the method used in [15], we obtain for the complex amplitude of the generated wave:

$$A_3(z) = \frac{\sigma_3 a_{10} a_{20}}{\Lambda} e^{\mathrm{i}(\frac{\Delta kz}{2} + \phi_{30})} \times \left[R\Lambda \cos\left(\frac{\Lambda z}{2}\right) - \mathrm{i}(2e^{\mathrm{i}\Delta\varphi_0} + R\Delta k)\sin\left(\frac{\Lambda z}{2}\right)\right], \tag{3}$$

where the following notation have been used: $a_{10}, a_{20}, a_{30}, \varphi_{10}, \varphi_{20}, \varphi_{30}$ are input amplitudes and phases of the three beams at $z = 0$, $\Delta\varphi_0 = \varphi_{10} + \varphi_{20} - \varphi_{30}$, $\Lambda = \sqrt{\Delta k^2 + 4S}$, $S = \sigma_3(\sigma_2 a_{10}^2 + \sigma_1 a_{20}^2)$, and $R = a_{30}/(\sigma_3 a_{10} a_{20})$.

For the case of $a_{30} = 0$ the amplitude and the phase are

$$a_3(z) = \sigma_3 a_{10} a_{20} z \,\text{sinc}(\tfrac{1}{2}\Lambda z), \tag{4}$$

$$\varphi_3(z) = \varphi_{10} + \varphi_{20} - \frac{\pi}{2} + \frac{\Delta kz}{2} \tag{5}$$

For the case of $a_{30} \neq 0$, after some transformation of (3), we obtain

$$a_3^2(z) = \left(\frac{\sigma_3 a_{10} a_{20}}{\Lambda}\right)^2 \{[4 + R^2\Delta k^2 + 4R\Delta k \times \cos(\Delta\varphi_0)]\sin^2\left(\frac{\Lambda z}{2}\right) + R^2\Lambda^2\cos^2\left(\frac{\Lambda z}{2}\right) + 2R\Lambda\sin(\Delta\varphi_0)\sin(\Lambda z)\}, \tag{6}$$

$$\varphi_3(z) = \frac{\Delta kz}{2} + \varphi_{30} - \arctan\frac{2\cos(\Delta\varphi_0) + R\Delta k}{2\sin(\Delta\varphi_0) + R\Lambda\cot(\Lambda z/2)} \tag{7}$$

The system (2) which is written for the complex amplitudes can be reduced to the differential equation system for the phases of the three interacting waves:

$$\begin{aligned} \frac{d\varphi_1}{dz} &= -\sigma_1 \frac{a_3 a_2}{a_1} \cos[(\varphi_3 - \varphi_1 - \varphi_2 - \Delta kz)], \\ \frac{d\varphi_2}{dz} &= -\sigma_2 \frac{a_3 a_1}{a_2} \cos[(\varphi_3 - \varphi_1 - \varphi_2 - \Delta kz)], \\ \frac{d\varphi_3}{dz} &= -\sigma_3 \frac{a_1 a_2}{a_3} \cos[(\varphi_3 - \varphi_1 - \varphi_2 - \Delta kz)]. \end{aligned} \tag{8}$$

With known z-dependence for the amplitude and phase of the generated wave, the system (8) can be integrated analytically with the same assumption for non-depleting intensity for the fundamental waves. As a result, the phase shift $\Delta\varphi_1 = \varphi_1(L) - \varphi_{10}$ of the fundamental wave "1" at the output ($z = L$) of the nonlinear media is

$$\Delta\varphi_1 = \frac{\sigma_1\sigma_3 a_{20}^2 \Delta kL}{\Lambda^2}\left\{[1 - SR^2 + R\Delta k\cos(\Delta\varphi_0)] \times [1 - \text{sinc}(\Lambda L)] + R\Lambda\sin(\Delta\varphi_0)\left[\frac{1 - \cos(\Lambda L)}{\Lambda L} - \frac{\Lambda}{\Delta k}\cot(\Delta\varphi_0)\right]\right\}. \tag{9}$$

For many practical cases it is necessary to work at relatively high values of the mismatch ($\Delta kL > 3$). In this case we can suggest that $4S/\Delta k^2 \ll 1$ and (9) is simplified to

$$\Delta\varphi_1 = \frac{\sigma_1\sigma_3 a_{20}^2 L}{\Delta k}[1 - \mathrm{sinc}(\Delta kL)] + \frac{\sigma_1 a_{20}}{\Delta k}\frac{a_{30}}{a_{10}}[\sin(\Delta\varphi_0) - \sin(\Delta kL + \Delta\varphi_0)]. \tag{10}$$

Two terms are responsible for the phase shift of the fundamental wave in the case of seeding at generated wavelength: (*i*) a cubic one, based on coupled second-order processes (cascade cubic nonlinearity), that is the same as for zero seeding [3, 8, 15] and (*ii*) single quadratic interaction with participlation of the seeding wave.

2. Comparison with the numerical results and discussions

For evaluating the limits of application of the above derived formulae we solved system (2) numerically by the Runge–Kutta method. Type II second-harmonic generation was chosen for consideration, i.e., we chose $\sigma_3 \cong 2\sigma_1$, $\sigma_1 \cong \sigma_2$. First we compare the analytical solution of the system (2), obtained by the approximation used in [2, 4] and our analytical solution, with the numerical solution for the case of zero input for the generated wave ($a_{30} = 0$), ratio of the intensities of the two (signal and pump) fundamental waves $I_1/I_2 = 0.2$ and ratio of the length of the crystal to the nonlinear interacting length $L/L_{NL} = \sigma|a_{20}|L = 0.6$. In Fig. 1, the curves for the phase shifts $\Delta\varphi_1$ as a function of normalized phase mismatch ΔkL obtained by these three approaches are shown. It is seen that the approximation of fixed intensity used here is much closer to the exact numerical solution (marked solid line) than the previously used approximation (dashed line), even for these relatively high values of input intensities that correspond to 60% conversion coefficient for the generated wave with respect to the signal wave 1 at optimal mismatch ΔkL.

Another advantage of the obtained formula is that it describes SHG and sum-frequency mixing processes in the presence of seeding. In Fig. 2, the phase shift$\Delta\varphi_1$ vs ΔkL for two different values of the input phase difference $\Delta\varphi_0$:$\Delta\varphi_0 = 0,\pi$ is shown. The seeding intensity is 10 times less than the weak signal intensity. Input intensity of the strong (pump) wave and the signal wave are the same as used for Fig. 1. Dashed line represents the numerical calculations for $\Delta\varphi_0 = \pi$ input phase difference. "ΔkL" dependencies for the signal wave phase shift have similar shapes as in the case of no seeding, with this difference that the center of the curve are positioned at different places at ΔkL axis depending on the input phase difference and the seeding intensity, when $\Delta\varphi_0$ is different from $\pi/2$ and $3\pi/2$. The curves with $\Delta\varphi_0 = \pi$ and $\Delta\varphi_0 = 0$ are symmetrically positioned with respect to the point "$\Delta kL = 0$".

The obtained formulae and Fig. 2 clearly show that the presence of seeding helps to obtain phase shift of the fundamental waves at exact phase matching condition ($\Delta k = 0$). In this case the change of the magnitude of the shift can be controlled not only by the amplitude of the pump fundamental wave but also by the input phase of

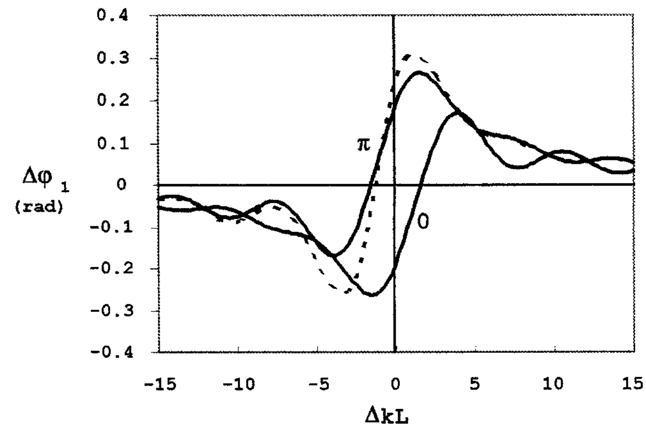

Fig. 2. Phase shift $\Delta\varphi_1$ as a function of ΔkL for two different values of the input phase difference $\Delta\varphi_0$: $\Delta\varphi_0 = 0,\pi$. Seeding intensity $I_{30} = 0.1I_{20}$. Input intensities of the fundamental waves are the same as used for Fig. .1

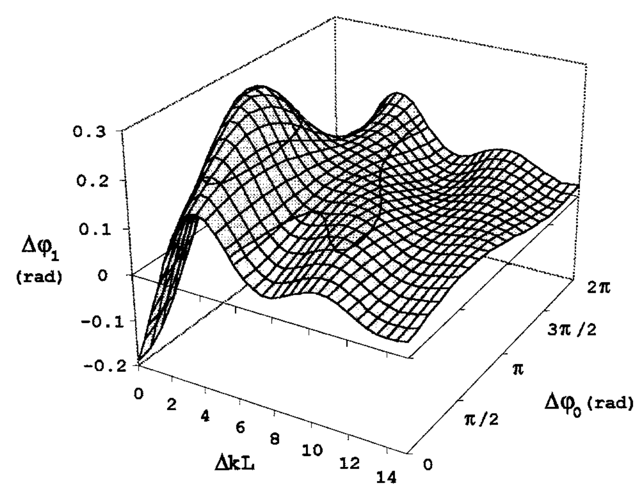

Fig. 3. 3D graph of the output phase shift of the signal wave as a function of the input phase difference $\Delta\varphi_0$ and the mismatch ΔkL. Input intensities of the three interacting waves are the same as for Fig. 2

the seeding wave.

$$\Delta\varphi_1(L) = -\sigma_1\frac{a_{30}a_{20}}{a_{10}}L\cos(\Delta\varphi_0). \tag{11}$$

The induced phase shift in this last case is due to the single second-order process. The sign of the shift depends on the sign of the second-order susceptibility tensor.

The formulae derived here allow one to visualize quickly and optimize the process of phase modulation of the fundamental waves. As an example, Fig. 3 shows the phase shift gained by the signal fundamental wave in type II SHG with non-zero second-harmonic wave input. The parameters of the input waves are the same as in the case of Fig. 2. Maximum phase shift is achieved for normalized wave vector mismatch $\Delta kL = 2$ and input phase difference

$$\Delta\varphi_0 = \varphi_{10} + \varphi_{20} - \varphi_{30} = \frac{4}{5}\pi.$$

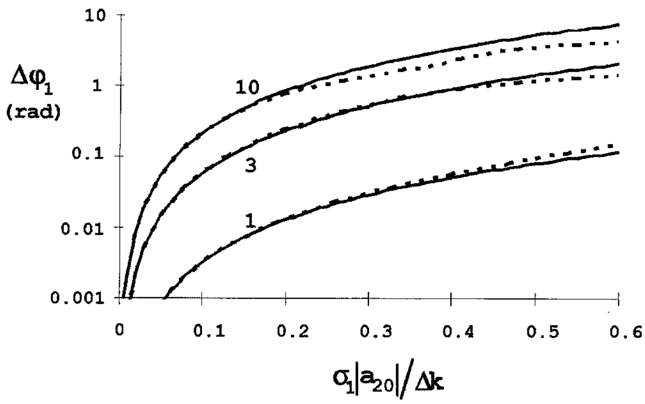

Fig. 4. Comparison of the numerical (*dashed line*) and analytical (*solid line*) approach: output phase shift of signal wave "1" as a function of the normalized amplitude of the pump wave $\sigma a_{20}/\Delta k$ for three different values of the mismatch ΔkL

In Fig. 4, the analytical dependencies of the phase shift of the signal wave as a function of the amplitude of the pump wave normalized to the Δk input are compared for different mismatches $\Delta kL = 1$; 3; 10 with the exact numerical calculations. Analytical curves were obtained with formula (10). The analytical curves are close to the numerical ones up to pump intensities corresponding to $(\sigma_1 a_{20}/\Delta k) = 0.5$. It is seen that at power levels where analytical formulae work well, phase shifts exceeding $\pi/2$ can be obtained. This level of shifts is sufficient to switch all-optical switching devices based on symmetrical Mach Zehnder interferometer [13].

3. Conclusion

We have shown that approximation of fixed intensity can be used to describe analytically the process of low power self- and cross-phase modulation of the waves involved in quadratic nonlinear optical processes as SHG and sum-frequency mixing. The criteria for the validity of the approximation of fixed intensity is that the ratio $4S/\Delta k^2$ be not more than 2. We intend to use these formulae for a description of the process of mode-locking with the so-called nonlinear doubling mirror [18] used recently for mode-looking CW pumped Nd : YAG laser [19]. Intracavity power level in CW pumped solid state lasers is low enough, so the expressions described here are valid.

Acknowledgements. We would like to thank the Bulgarian National Science Foundation for support under contract F405 and Prof.I.Tomov for stimulating discussions.

References

1. H.J. Bakker, P.C.M. Planken, L.Kuipers, A. Lagendijk: Phys. Rev. A **42**, 4085 (1990)
2. R. DeSalvo, D.J. Hagan, M. Sheik-Bahae, G. Stegeman, E.W. Van Stryland, H.Vanherzeele: Opt. Lett. **17**, 28 (1992)
3. G.I. Stegeman, M. Sheik-Bahae, E.W. Van Stryland, G. Assanto: Opt. Lett. **18**, 13 (1993)
4. D.C. Hutchings, J.S. Aitchison, C.N. Ironside: Opt. Lett. **18**, 793 (1993)
5. A.L. Belostotsky, A.S. Leonov, A.V. Meleshko: Opt. Lett. **19**, 856 (1994)
6. A.V. Smith, M.S. Bowers: J. Opt. Soc. Am. B **12**, 49 (1995)
7. H.Tan, G.P. Banfi, A.Tomaselli: Appl. Phys. Lett. **63**, 2472 (1993)
8. S. Nitti, H.M. Tan, G.P. Banfi, V. Degiorgio: Opt. Commun. **106**, 263 (1993)
9. A. Re, C. Sibilia, E. Fazio, M. Bertolotti: J. Mod. Opt. **42**, 823 (1995)
10. G. Assanto, G. Stegeman, M. Sheik-Bahae, E. Van Stryland: Appl. Phys. Lett. **62**, 1323 (1993)
11. G. Assanto, G. Stegeman, M. Sheik-Bahae, E. Van Stryland: IEEE J. QE- **31**, 673 (1995)
12. N.R. Belashenkov, S.V. Gagarskii, M.V. Inochkin: Opt Spectrosc. (USSR) **66**, 1383 (1989)
13. C.N. Ironside, J.S. Aitchison, J.M. Arnold: IEEE J. QE- **29**, 2650 (1993)
14. P.S Russell: Electron. Lett. **29**, 1228 (1993)
15. S.M. Saltiel, K.R. Koynov, I.H. Buchvarov: Bulg. J. Phys. **22**, 39 (1995)
16. D.J. Hagan, Z. Wang, G. Stegeman, E.W. Van stryland, M. Sheik-Bahae, G. Assanto: Opt. Lett. **19**, 1305 (1994)
17. Z.A. Tagiev, A.S. Chirkin: Zh. Eksp. Teor. Fiz. (USSR) **73**, 1271 (1977)
18. K. Stankov: Appl. Phys. B **45**, 191 (1988)
19. M. Danailov, G. Cerullo, V. Magni, D. Segala, S. De silvestri: Opt. Lett. **19**, 792 (1994)